Proceedings World Geothermal Congress 2026, Calgary, Canada, June 8—11, 2026

MATHEMATICAL MODELING OF FRACTURE INTERFACIAL SLIP WITH LUBRIFICATION

Lorenzo Fusi[1,2]*, Antonio Giovinetto[1,2], Giordano Montegrossi[2,3]

[1] Dipartimento di Matematica e Informatica “Ulisse Dini”, Universitá degli Studi di Firenze, Viale Morgagni 67/a, 50134 Firenze, Italy
[2] INSTM - Consorzio Interuniversitario Nazionale per la Scienza e Tecnologia dei Materiali, Via G. Giusti 9, Firenze, Italy
[3] CNR-Institute of Geosciences and Earth Resources, Via G. La Pira, 4, 50121 Florence, Italy
lorenzo.fusi@unifi.it

INTRODUCTION

Hydraulically generated fractures can result from both natural fluid processes and induced circulation during resource exploitation. The driving mechanism typically involves advective flow, which transports heat from deeper strata at elevated temperatures and various acidic components (depending on the origin of the deep fluid, e.g., seawater, CO_2-rich crustal fluids, or volcanic fluids).

When these fluids are undersaturated with respect to the minerals they are in contact with, mineral dissolution occurs. This can enlarge the fracture aperture or increase the porosity of the fracture infill material. Of particular interest is the porosity increase resulting from the dissolution of carbonate minerals when exposed to acidic fluids. This process is common in subsurface environments and highly relevant to both geothermal and hydrothermal (volcanic) systems. The degradation of rock strength plays a central role in failure criteria such as Hoek-Brown and Mohr-Coulomb. In these models the strength properties of intact rock is reduced by incorporating degradation of rock mass quality. The random distribution of pores strongly correlates with key mechanical and engineering properties, including unconfined compressive\tensile strength (UCS, UTS), Young’s modulus, shear strength, and sonic velocity. It has been observed that as porosity increases—especially due to calcite dissolution—the stress threshold for pore collapse significantly decreases. This leads to reduced material cohesion (lubrification effects) and a lower elastic modulus. However, the effects of degradation cannot be solely attributed to increased porosity; microstructural damage, such as the progressive breakdown of intergranular cementation, also plays a crucial role. Fracture -filling materials, similar to host rock, are also subject to degradation. This phenomenon can be modelled using a Strength Degradation Parameter (SDP). A chemical kinetic model can be employed to describe the weathering of infill material, accounting for both consolidation (e.g., precipitation from a supersaturated fluid) and degradation (e.g., dissolution in acidic fluids). Carbonates, such as calcite, are the most common chemistry-dependent materials, while others, like quartz, are primarily temperature-dependent. Some minerals, like gypsum, exhibit hybrid behavior.

In this study, we develop a mathematical model that captures all the above mechanisms. Specifically, we formulate a poroelastic fracture model in which the solid structure of the porous matrix undergoes degradation due to chemical reactions. The poroelastic medium is assumed to behave linearly, and the reactive fluid flow is driven by a prescribed pressure drop. After nondimensionalizing the governing equations using appropriate characteristic quantities, we solve the coupled mechanical and chemical degradation problem numerically via a pseudo-spectral collocation method. We present simulation results that illustrate the onset and evolution of failure regions within the fracture.

C. Atkinson, R. Craster, Theoretical aspects of fracture mechanics, *Progress in Aerospace Sciences* **31** (1) (1995) 1–83.

J. Shang, J. Hu, K. Zhou, X. Luo, M. M. Aliyu, Porosity increment and strength degradation of low-porosity sedimentary rocks under different loading conditions, *International Journal of Rock Mechanics and Mining Sciences* **75** (2015) 216–223.

MATHEMATICAL MODEL

We consider a fracture formed by a poroelastic layer confined between two solid rocks. We study the poroelastic layer that extends infinitely in one direction so that we may confine our analysis on a cross section perpendicular to it (Fig. 1). We only consider macroscopically-averaged-quantities. The fluid within the pores saturates the porous material and a pressure gradient is applied on the upper and lower part of the domain. The governing equations are mass and momentum balance. The system is solved numerically via Spectral Collocation Methods.

$$\frac{1}{M^*}\frac{\partial p^*}{\partial t^*} + \alpha\,\nabla^*\cdot\,\dot{\mathbf{u}}^* + \,\nabla^*\cdot\mathbf{q}^* = 0 \qquad \boldsymbol{\sigma}^{e^*} = 2\mu^*\boldsymbol{\epsilon} + \lambda^*\,\mathrm{tr}\,(\boldsymbol{\epsilon})\boldsymbol{I}$$

$$\mathbf{q}^* = -\frac{k^*}{\eta^*}\nabla^* p^* \qquad \mathrm{div}\,\boldsymbol{\sigma}^* = 0 \qquad \boldsymbol{\sigma}^* = \boldsymbol{\sigma}^{e^*} - \alpha p^*\boldsymbol{I}$$

$$E^* = E_o^*\Big(1 - d(\phi)\Big) \qquad k^* = k_o^*\left(\frac{\phi(t)}{\phi_o}\right)^3$$

$$d = -0.94\exp\left(-\frac{\phi}{0.42}\right) + 0.47$$

Porosity variation as results of e.g. calcite dissolution in a CO_2 rich fluid.

$$\mathrm{CO_{2(aq)}} + \mathrm{CaCO_3} \leftrightarrows \mathrm{Ca^{+2}} + 2\,\mathrm{HCO_3^-}$$

$$\frac{d}{dt}\left[\phi A^*)\right] = -K^*\left[\phi A^*\right]\left[(1-\phi)\,C^*\right]$$

$$\frac{d}{dt}\left[(1-\phi)C^*\right] = -K^*\left[\phi A^*\right]\left[(1-\phi)\,C^*\right]$$

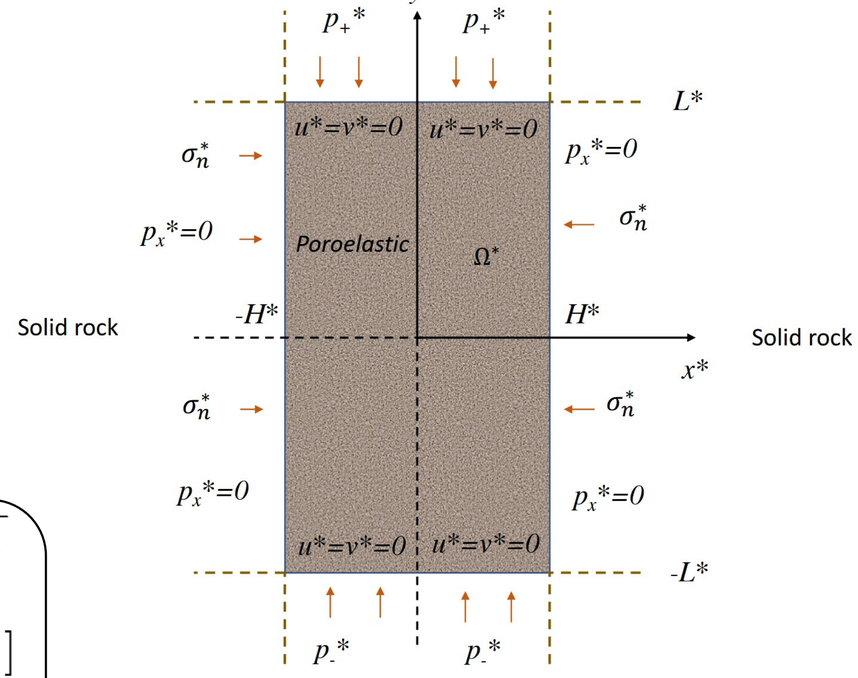


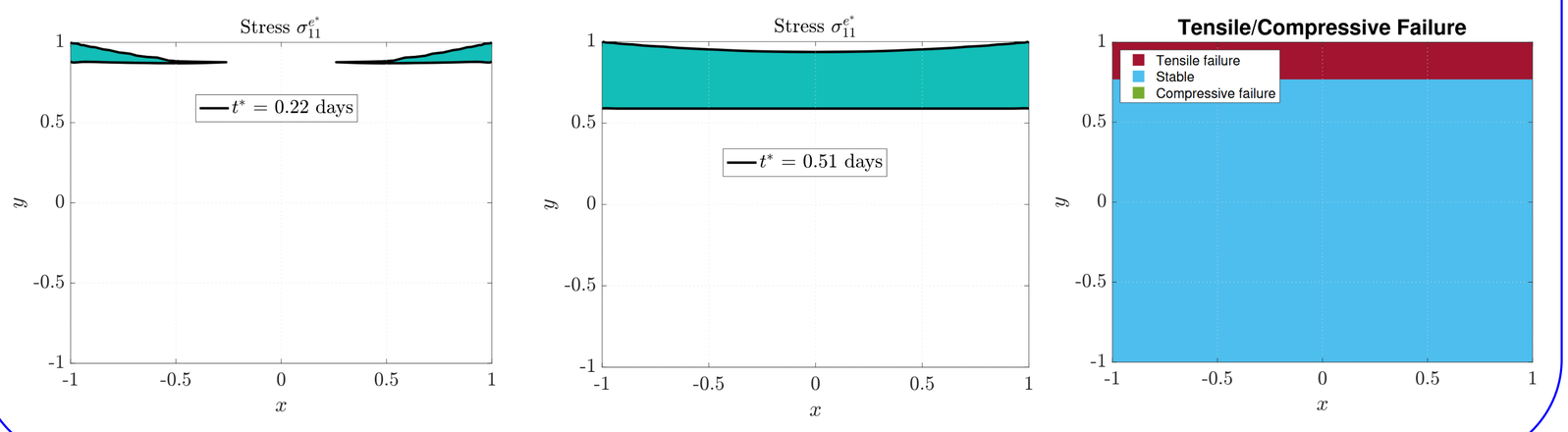
Figure 1. Sketch of the poroelastic layer

RESULTS

The Hoek-Brown criterion was developed as a means to estimate rock mass strength by scaling the relationship derived according to the geological conditions present. The criterion is based on Hoek’s experiments with brittle rock failure and his use of a parabolic Mohr envelope derived from Griffith’s crack theory to define the relationship between shear and normal stress at fracture initiation. Moreover, we investigated the Mohr-Coulomb slip failure criterion.

Compressive-Tensile failure criterion

$$-\,\mathrm{UCS}^* \leqslant \sigma_{11}^{e^*} \leqslant \mathrm{UTS}^* + C^*$$



Shaded area: $\sigma_e^{11} > \mathrm{UTS}$

Mohr-Coulomb failure criterion

$$\mu = \frac{\tau^*}{\Big[-\sigma_{11}^{e^*} + \mathcal{H}(\sigma_{11}^{e^*})C^*\Big]\mathcal{H}(C^* - \sigma_{11}^{e^*}) + \mathcal{H}(\sigma_{11}^{e^*} - C^*)\dfrac{\tau^*}{\mu_c}}$$

