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A Unified Theoretical Model for the Thermal Interaction of Geothermal Boreholes with Aquifers across all Peclet Numbers

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ABSTRACT

The presence of aquifers has significant impact on the thermal response of geothermal boreholes. Therefore, theoretical models used in the design and sizing of geothermal heat exchangers must account for their influence. Existing models in the literature satisfactory address this effect in two different flow regimes, namely, creeping groundwater flows and strong groundwater flows. However, intermediate regimes, which frequently occur in real-world scenarios, are not well addressed by existing models. By using asymptotic expansion techniques, a new theoretical model for the thermal interaction of geothermal boreholes and aquifers across all groundwater flow regimes has been developed. It is not only physically sound and mathematically rigorous but also demonstrates promising performance and accuracy compared to detailed numerical simulations.

1 INTRODUCTION

Heating and cooling of buildings represents a large share of the world's total energy consumption, almost 23% (International Energy Agency 2021). Consequently, improving the efficiency of these systems is crucial for achieving a sustainable future. Reducing energy demand in buildings not only alleviates the pressure on limited resources but also mitigates environmental impacts. Heating, Ventilation and Air Conditioning (HVAC) systems that harness low-temperature geothermal energy are among the most promising solutions to achieve such reductions.

Geothermal HVAC systems consist on a water-to-water heat pump connected to a network of vertical boreholes, like the one shown in Figure 1, which together form the geothermal heat exchanger. Each of these boreholes contains one or more coaxial or U-shaped probes through which a heat carrying liquid flows, exchanging heat with the surrounding ground. To enhance this heat exchange and prevent potential cross-contamination between aquifers, the space around pipes is filled with impermeable grout.

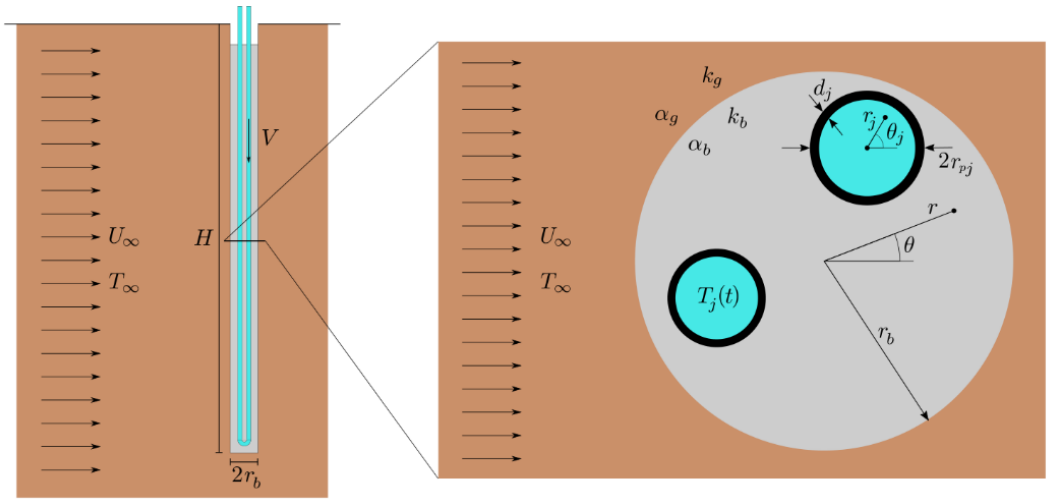


Figure 1. Sketch of a typical geothermal borehole, of depth H and radius r_b and with a single U-shaped probe inside, immersed in a groundwater stream of effective seepage velocity U_∞ and temperature T_∞.

Adequate sizing of geothermal heat exchangers is critical to ensure that geothermal HVAC systems achieve both economic viability and their intended energy efficiency over the lifespan of a building, approximately one hundred years. Achieving this requires trustworthy predictions of the long-term thermal response of the geothermal heat exchangers, which cannot be obtained through detailed numerical simulations due to their excessive computational cost (Raymond et al 2011). Consequently, simplified theoretical models are employed instead, as they are accurate, flexible, and fast.

Numerous theoretical models have been proposed to account for both heat transfer mechanisms occurring in the ground, namely, heat conduction and heat convection due to groundwater flows (Zhao et al 2022). In the literature, the qualitative differences between these models arise from the simplifying assumptions made to describe the thermal response of the borehole and ground under two different groundwater flow regimes, namely, creeping groundwater flows (Rico and Hermanns 2024) and strong groundwater flows (Rico and Hermanns 2025). However, intermediate flow regimes represent a challenge for these models as none of the simplifying assumptions are valid. Therefore, a unified theoretical model capable of accurately describing the ongoing heat transfer across all possible groundwater flow regimes is highly desirable. In this work, asymptotic expansion techniques are employed to develop such a model, which is both mathematically rigorous and physically-sound, providing accurate and reliable forecasts of the thermal response of geothermal boreholes across different groundwater flow regimes.

2 FORMULATION OF THE PROBLEM

Typical geothermal boreholes exhibit extreme slenderness, reaching depths of around 100-200 m while having diameters of only 10-20 cm. Such large disparity in length scales, along with the typical time scales for the heating and cooling demands of buildings, allows neglecting axial heat conduction in comparison to radial heat conduction (Rico and Hermanns 2024). Furthermore, since groundwater in aquifers mostly flows perpendicular to the borehole (Rico and Hermanns 2024), the heat transfer problem in the ground can be formulated in two-dimensional planes perpendicular to the borehole, which are coupled only through the governing equations in pipes.

To avoid the need to formulate and solve the conservation equations within the complex void geometry of the ground, the porous medium approach is employed. Additionally, assuming local thermal equilibrium between the groundwater and the solid matrix allows the use of a single conservation equation, with effective thermal properties, to describe the heat transfer phenomenon in ground. Consequently, ground temperature T must obey the following energy conservation equation, where α_g denotes the effective thermal diffusivity of the ground, (r, θ) are polar coordinates centered at the borehole, and t represents time:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha_g \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right]. \tag{1}$$

In the presence of aquifers, groundwater movement introduces convective terms on the left hand side of the energy conservation equation. The associated velocity field, described by its components (v_r, v_θ) in the polar coordinate system centered at the borehole, can be derived analytically from the classical solution for fluid flow

around a circular cylinder in a porous medium. These components are written in terms of the effective seepage velocity U_∞ of the aquifer and the borehole radius r_b as follows (Rico and Hermanns 2024):

$$v_r = U_\infty \left(1 - \frac{r_b^2}{r^2}\right)\cos(\theta) \qquad \text{and} \qquad v_\theta = -U_\infty \left(1 + \frac{r_b^2}{r^2}\right)\sin(\theta). \tag{2}$$

The present work focuses solely on the temperature field in ground, therefore a general temperature $T_b(\theta, t)$ is prescribed at the borehole wall:

$$r = r_b: \quad T(r, \theta, t) = T_b(\theta, t). \tag{3}$$

Finally, the solution to the formulated problem must tend to the unperturbed ground temperature T_∞ far away from the borehole and at the beginning of the problem:

$$r \to \infty: \quad T \to T_\infty. \tag{4}$$

$$t = 0: \quad T = T_\infty. \tag{5}$$

3 ASYMPTOTIC SOLUTION

Since no exact solution is known to the formulated problem, approximate, albeit accurate, solutions are sought instead. In this work, asymptotic expansion techniques are employed to build such solution by exploiting the presence of small parameters in the governing equations, which allow the problem to be systematically decomposed into simpler ones (Lagerstrom 1988). In particular, the small parameter corresponds to the ratio between the characteristic transversal diffusion time $t_b \sim r_b^2/\alpha_g$ and the characteristic variation of the heating and cooling needs of the building t_q, being $t_b/t_q \ll 1$ (Rico and Hermanns 2024).

The presence of aquifers introduces another characteristic time, namely, the residence time of the groundwater flow in the vicinity of the borehole $t_c \sim r_b/U_\infty$. The ratio between t_b and t_c is the Peclet number of the flow,

$$\frac{t_b}{t_c} \sim Pe = \frac{U_\infty r_b}{\alpha_g}, \tag{6}$$

which measures the relative importance between convective and conductive terms in the energy conservation equation. This dimensionless number allows to distinguish between creeping groundwater flows, where $Pe \ll 1$, and strong groundwater flows, where $Pe \sim 1$ or $Pe \gg 1$. Since the goal of the present work is to accurately describe all flow regimes, no assumption is made regarding the order of magnitude of the Peclet number.

Therefore, the only small parameter that arises in the problem is t_b/t_q. So, the approximate solution is found by substituting the ground temperature T by the following regular perturbation expansion (Lagerstrom 1988):

$$T = T^{(0)} + T^{(1)}\frac{t_b}{t_q} + O\left(\frac{t_b^2}{t_q^2}\right), \tag{7}$$

and then solving, term by term, the sequence of simpler problems that emerge. The complete derivation of the leading-order solution $T^{(0)}$ is provided in Rico and Hermanns 2026.

4 NUMERICAL EXAMPLES

The capabilities of the proposed model are demonstrated by comparing its results against detailed numerical simulations performed using the commercial software package COMSOL (Comsol 2024). A borehole of diameter $2r_b = 152\ \text{mm}$, surrounded by a ground of effective thermal conductivity $k_g = 1.8\ \text{W/(m K)}$ and effective thermal diffusivity $\alpha_g = 7.83 \cdot 10^{-7}\text{m}^2/\text{s}$, is considered for that purpose. The borehole, at a uniform wall temperature
$T_b = 30\ °\text{C}$, starts to exchange heat with the ground at $t = 0$. The groundwater flows with an effective seepage velocity of $U_\infty = 0.356\ m/day$ and an upstream temperature of $T_\infty = 15\ °\text{C}$, leading to a Peclet number of $Pe = 0.4$ for its thermal interaction with the borehole.

Figures 2, 3, and 4 show the time evolution of the ground temperature that results from the thermal interaction of the borehole with the groundwater flow. The solid-colored maps correspond to the reference solution obtained with COMSOL while the black isolines correspond to the proposed model. Left plots focus on the vicinity of the borehole while right plots show the results further away from the borehole.

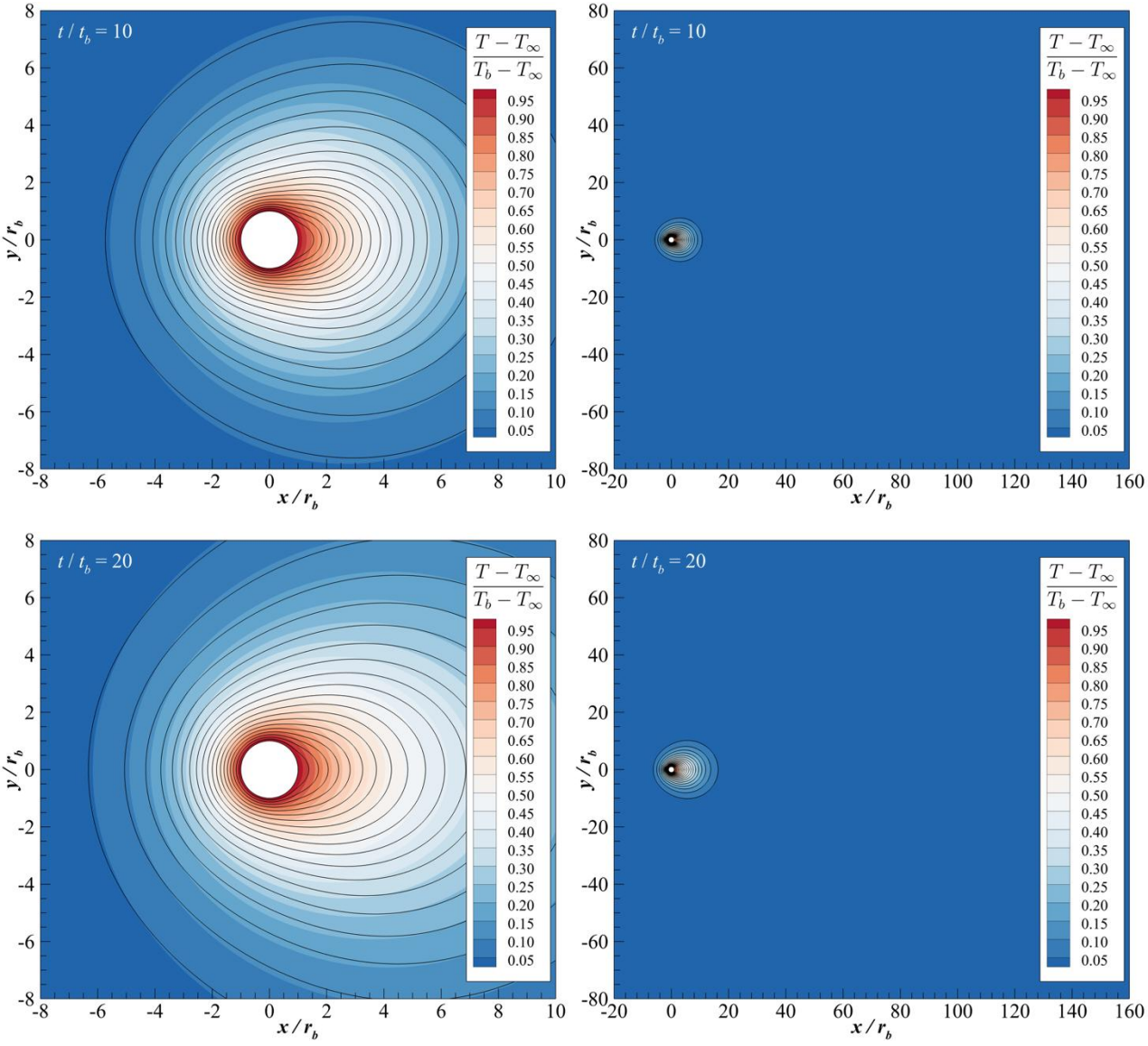
At initial stages of the interaction, for which t and t_b are not sufficiently apart, the proposed model fails to accurately reproduce the perturbed ground temperature in the vicinity of the borehole. Nonetheless, the overall

temperature map, especially how far the thermal perturbation protrudes into the ground, is correctly reproduced by the model. As time passes, the proposed model improves in accuracy. So, for $t\ /\ t_b = 50$ the predicted ground temperature already gets close to the reference solution, especially upstream of the borehole. However, downstream of the borehole the convergence of the model towards the reference solution takes longer due to a fierce competition between the decaying errors introduced by the simplifications of the model and an exponential factor, present in the formulae of the model, that amplifies these errors in certain regions downstream of the borehole (Rico and Hermanns 2026). Thankfully, the decaying errors end up winning the competition, and for longer times the model matches perfectly the reference solution.

5 CONCLUSION

The presence of aquifers significantly affects the thermal response of geothermal heat exchangers. When weak or strong groundwater flows are present, adequate theoretical models for the thermal interaction exist in the literature. But for intermediate regimes no conclusive models can be found. In the present work such a conclusive model has been presented. In fact, the proposed model is valid for all kind of groundwater flows, but its interest is greatest for those elusive intermediate flows for which existing models are less accurate.

The proposed model results from the exploit of the disparity of time scales in the problem, which allow the governing equations to be simplified. By convenient manipulation of these simplified equations, analytical expressions result which constitute the proposed model. Comparisons of the proposed model with detailed numerical simulations of the complete heat transfer problem, obtained with the commercial software package COMSOL, reveal the capabilities and limitations of the performed work.



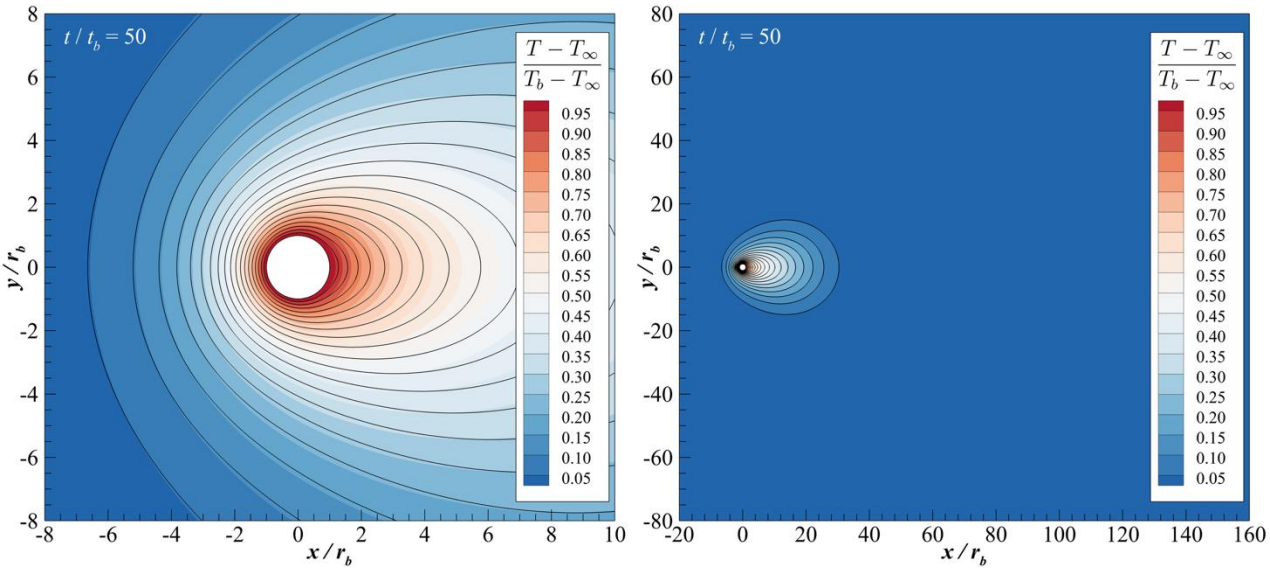


Figure 2. Different time instants of the ground temperature in the thermal interaction of a geothermal borehole with a groundwater flow. The solid-colored maps correspond to the reference solution obtained with COMSOL while the black isolines correspond to the developed asymptotic model. Left plots focus on the vicinity of the borehole while right plots show the results further away from the borehole.

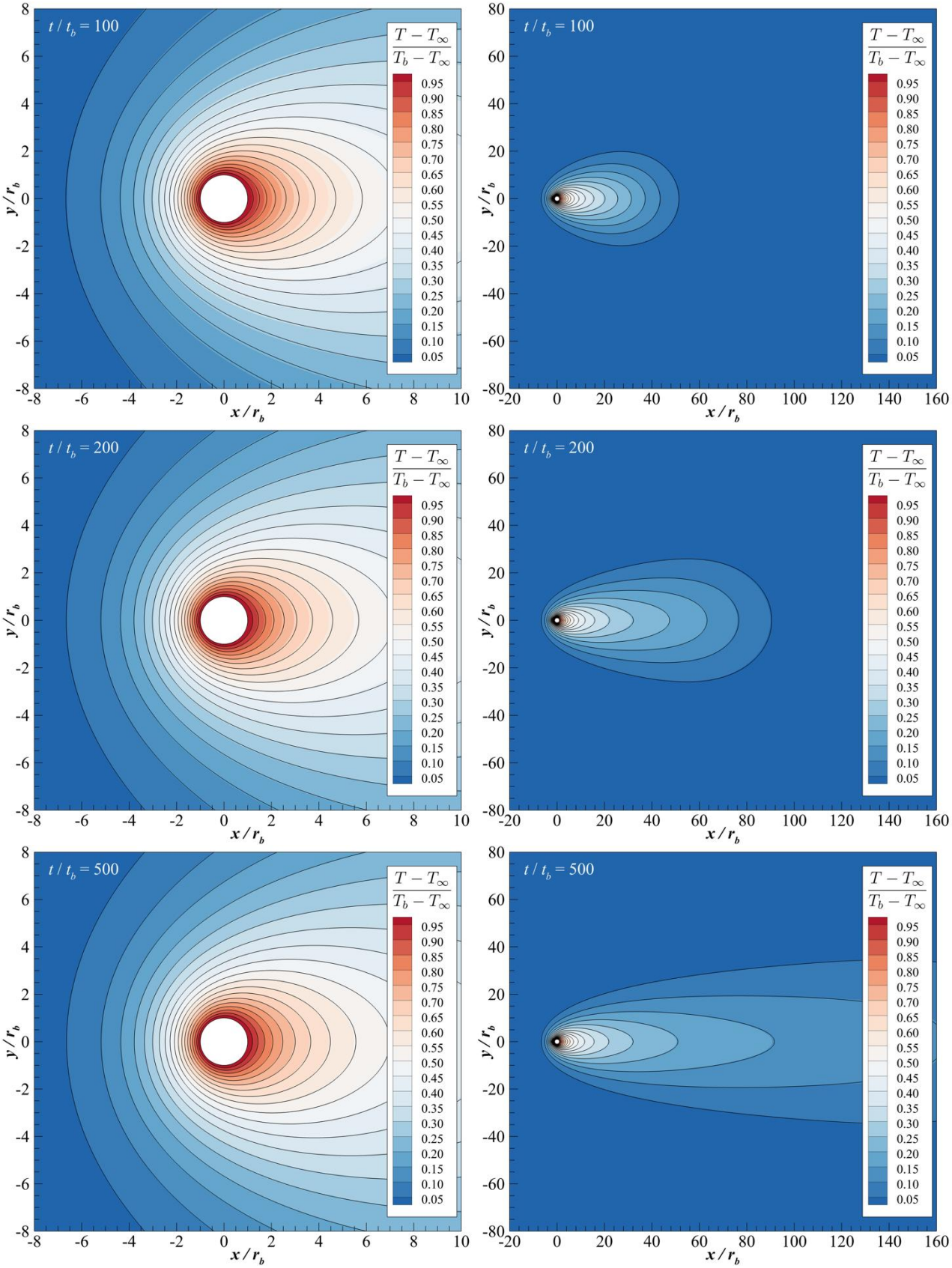


Figure 3. Different time instants of the ground temperature in the thermal interaction of a geothermal borehole with a groundwater flow. The solid-colored maps correspond to the reference solution obtained with COMSOL while the black isolines correspond to the developed asymptotic model. Left plots focus on the vicinity of the borehole while right plots show the results further away from the borehole.

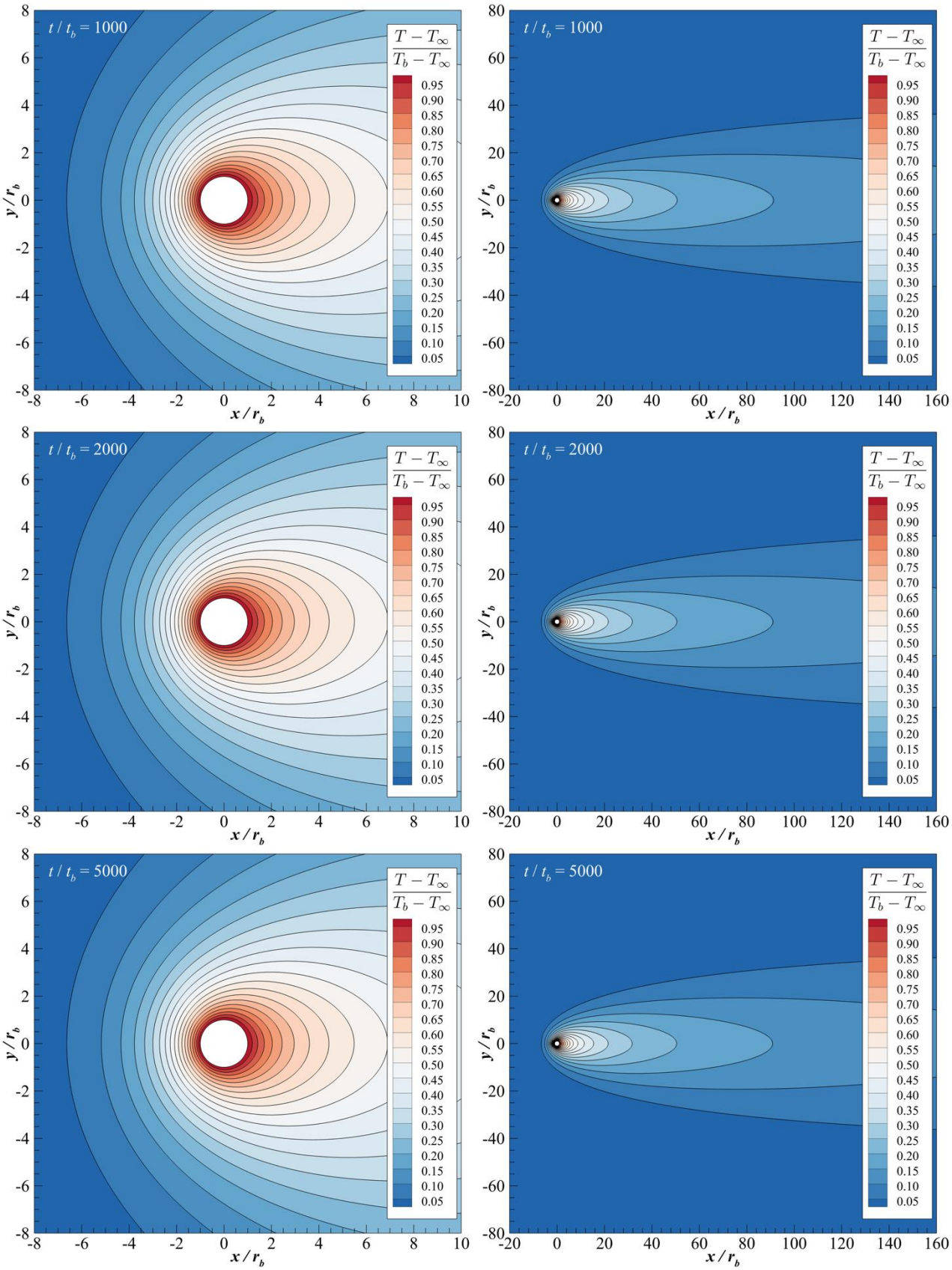


Figure 5. Different time instants of the ground temperature in the thermal interaction of a geothermal borehole with a groundwater flow. The solid-colored maps correspond to the reference solution obtained with COMSOL while the black isolines correspond to the developed asymptotic model. Left plots focus on the vicinity of the borehole while right plots show the results further away from the borehole.

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