Proceedings World Geothermal Congress 2023
Beijing, China, October 9 – 11, 2023

# The importance of experimental thermoporoelastic coefficients for the complete characterization of geothermal reservoirs from low enthalpy to supercritical systems

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**Keywords:** Thermoporoelastic coefficients, experimental thermoporoelasticity, geothermal systems characterization.

## ABSTRACT

Under isothermal conditions, classical elasticity of solids without pores requires only two moduli to describe the mechanical relationship between stresses and strains, which can be obtained in simple 1-dimensional experiments. Thermoporoelasticity of hydrothermal rocks requires around 20 experimental coefficients, which depend on pressure and temperature. Due to the porosity of the rock and the different possible experimental conditions, the measurements that in a solid without pores are made in 1-dimension, in porous rocks can only be made in two-dimensional experiments, under different pressures and temperatures. The main geomechanical parameters to study the behavior of rocks in any geothermal system are: porosity $\varphi$, absolute permeability $k$, variation of the fluid contained in the pores $\zeta$, density $\rho$, Young's modulus $E$, global compressibility $C_B$, Poisson's modulus $\nu$, Lamé coefficient $\lambda$, shear modulus $G$, volumetric modulus $K_B$, reverse poroelastic expansion $H$, restricted and unrestricted reverse specific storage $M$ and $R$, respectively, Skempton's coefficient $B$ and Biot's modulus $b$. Petrophysical properties are measured for each specific geothermal system, their values are unique, heterogeneous and temperature dependent. In addition to their geomechanical behavior, porous rocks also have a natural thermal response characterized by other types of parameters such as their volumetric thermal expansivity $\gamma_B$, and the thermal expansion of the pores $\gamma_\varphi$.

Due to the natural porosity of some geothermal rocks, thermoporoelastic coefficients can be measured experimentally under different conditions: Drained conditions, when the system is open, the fluid can leave the pores, and the solid skeleton fully supports the total compression. Undrained conditions, the system is closed and the fluid in the pores remains constant. There are also different types of compressibility: solid skeleton, pore, volumetric, jacketed, non-jacketed, etc. There is great complexity in the amount and way of measuring thermoporoelastic coefficients whose values explicitly depend on both the confining pressure and the value of the temperature under which the measurement is made. The central purpose of this article is to define the main thermoporoelastic coefficients, how they are measured experimentally, and how they are derived algebraically from a base of only five arbitrary independent coefficients. The existing experimental gaps are clearly pointed out, showing the importance of the coefficients in the correct characterization of all types of geothermal reservoirs, from low enthalpy to supercritical systems.

## 1. INTRODUCTION

Geothermal reservoir rocks are porous, compressible and elastic, they also present poroplastic deformations. The presence of a fluid in motion within pores and fractures modifies the rock geomechanical response. The rock poroelasticity is evidenced by its compression and by the variation of fluid pressure, which reduces or increases the pores volume. The reduction of the pore volume is the main source of fluid release stored in the rock. Poroelasticity explains how the fluid within the pores carries a portion of the load acting on porous rocks. The remaining part of the total load is supported by the rock skeleton, formed by its solid volume. The skeleton is modeled as an elastic solid coupled to the laminar flow of a fluid that obeys Darcy's Law and certain equilibrium and continuity conditions. The classic theory of Maurice Biot (1941) is a mathematical-experimental model of poroelastic rock mechanics. This model consists of a group of equations in partial derivatives, which includes several thermoporoelastic coefficients that can predict the rock poroelastic deformation when is subjected to different internal and external mechanical and thermal forces, as pressure and temperature vary.

This document includes the complete deduction of the experimental thermoporoelastic parameters that support Biot’s theory. Linear thermoporoelasticity requires five independent poroelastic moduli to describe the interactions between strains and stresses. Herein is shown how other coefficients can be derived algebraically with only five known basic parameters. Introducing three thermal volumetric expansion coefficients, one for the fluid and two for the solid skeleton, a complete set of parameters for thermoporoelastic geothermal rocks is obtained. To illustrate the practical use of these parameters, two tables with poroelastic coefficients are presented, one table with data from the literature and a second table with data from the Los Humeros, Mexico geothermal field. Further applications and explanations can be extended to various geothermal phenomena: a) full deduction of classical isothermal Biot theory coupled to thermal stresses, b) how stress changes produce changes in fluid pressure; c) how changes in fluid pressure or temperature or fluid mass content produce changes in the volume of the porous rock; d) how any increase in pore pressure or temperature induces a dilation of the rock. The extreme sensitivity of petrophysical parameters to any change in temperature is shown, making evident the enormous importance of their experimental measurement in subcritical, critical and supercritical conditions, to fully characterize geothermal reservoirs under these respective thermodynamic conditions.

## 2. DESCRIPTION AND MEASUREMENT OF THERMOPOROELASTIC COEFFICIENTS

In experimental poroelasticity, there are four different kinds of bulk moduli and two types of deformations, drained and undrained. Both modes represent limiting responses of the rock (Biot, 1941; Fjaer *et al*., 1992; Wang, 2000; Guéguen and Bouteca, 2004):

1) **Drained conditions**: During the test, the rock is confined and subjected to support an external hydrostatic pressure $\sigma_H$. In this test, the fluid in the pores is allowed to escape and the total stress is entirely supported by the rock skeleton. The deformations are achieved at constant pore fluid pressure $p_f$. Biot (1962) called these conditions, "an open system".
2) **Undrained conditions**: during this test, the deformations are measured at constant fluid mass content $m_f$. The rock is entirely submerged in a fluid in such a way that the external hydrostatic pressure is balanced by the pore pressure $\sigma_H = -p_f$. In this test, the fluid in the pores remains constant; no fluid is allowed to move into or out of the control volume. The fluid remains trapped in the skeleton. For Biot (1962) this was characteristic of a "closed system".

Different types of compressibility can be defined that are crucial to coupling the mass and energy flows to the rock deformation. The experimental relations between stress and volumetric deformations are based on the definition of the following pressures:

- confining lithostatic pressure: $p_k$
- fluid or pore pressure: $p_f = p = -\sigma_H$
- differential pressure: $p_d = p_k - p_f$
- effective pressure: $p_e = p_k - b\, p_f$

Where $b$ is the Biot-Willis coefficient defined later; $p_f$ is the pressure of the fluid filling up the pore space and considered positive. Note that the hydrostatic pressure is always compressive and opposed to the fluid pressure, thus $\sigma_H = -p_f < 0$. When the differential pressure is constant, the corresponding experiment is performed under unjacketed conditions.

## 2.1 Experimental Porosity

Porous rock is formed by the superposition of two interacting continuous media, the solid skeleton and the fluid (Figure 1). The effective pore volume $V_\varphi$ [m$^3$] is the global amount of interconnected pores and fractures that allow flow to occur, $V_S$ is the total volume of solid rock grains, without pores. The volume of pores $V_\varphi$ plus the volume of the fractures $V_F$ make up the total pore volume $V_P$. The global or bulk volume $V_B$ (Fig. 1) is formed by the union of $V_P$ and $V_S$.

$$V_P = V_\varphi + V_F\,, \quad V_B = V_P + V_S \tag{0}$$

The porosity of the rock is the pore space fraction that contains the fluid and its reciprocal, is the fraction of the solid.

$$\varphi_f = \varphi = \frac{V_\varphi}{V_B}\,; \quad \varphi_S = \frac{V_S}{V_B} = 1 - \varphi \;\Rightarrow\; \varphi + \varphi_S = 1\,; \quad \frac{1}{\mathrm{V}_B} = \frac{\varphi}{V_\varphi} = \frac{\varphi_S}{V_S} \tag{1}$$

Where $\varphi_f$ is the effective porosity and $\varphi_S$ is the solid fraction. Both the solid and the fluid volumes expand/ contract when temperature increases/decreases; therefore, porosity is a function of pressure and temperature under which the measurement is made.

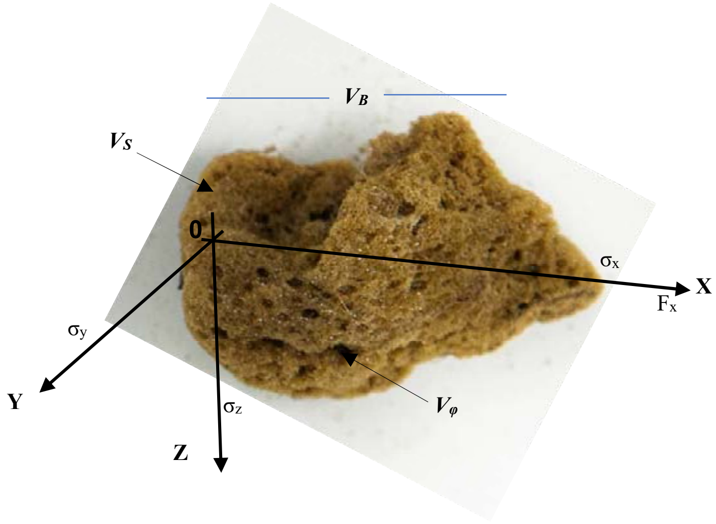

**Figure 1: Reticulite collected at the Hawaiian Volcanic Observatory, clearly illustrating the pores and solid skeleton of the volcanic rock ([www.nature.com/articles/s41598-020-62741-1/](www.nature.com/articles/s41598-020-62741-1/)) *Scientific Reports.***

2.1.1 Differential relations between porosity and volumes

From previous definitions given by equations (0) and (1), it follows that:

$$dV_B = dV_\varphi + dV_S \;\Rightarrow\; \frac{dV_B}{V_B} = \varphi\frac{dV_\varphi}{V_\varphi} + \varphi_S\frac{dV_S}{V_S} \;\Rightarrow\; \frac{d\varphi}{\varphi} = \frac{dV_\varphi}{V_\varphi} - \frac{dV_B}{V_B} \tag{2}$$

### 2.2 Absolute Permeability

The permeability $k$ of a porous rock is its natural ability to allow fluid to be transported through interconnected pores and fractures. If the rock is fractured, then fluid transmission through interconnected fissures of different dimensions must be included. Fluids prefer the path of least resistance to flow, always traveling in paths towards the highest permeability. Absolute permeability is the measure of this capacity when a single fluid, liquid, vapor or gas, is present in the rock. Relative permeability occurs when two or more fluids are present. There are at least four different types of absolute permeability.

a) permeability related to the intergranular spaces of the porous rock (mD)
b) micropermeability associated with interconnected microfractures (mD)
c) permeability related to networks of fractures and macrofractures (D)
d) macropermeability of open faults. (D)

Absolute permeability is estimated with the experimental 1-dimensional (OX) Darcy's law. (D = 1 Darcy = $9.86923 \times 10^{-13}$ $m^2$).

$$v_x = \frac{q}{A} = -\frac{k_x}{\mu_f}\left(\frac{\Delta p}{L}\right) \Rightarrow k_x = \mu_f \, v_x \left(-\frac{\Delta x}{\Delta p}\right) > 0 \tag{3}$$

where $q, k_x, \mu_f, A, L, \Delta p$ are volumetric fluid flow rate, absolute permeability, fluid viscosity, cross-sectional area, length and pressure drop over the length $L = \Delta x$, respectively. There are several types of pressure tests that allow to estimate rock permeability *in situ*. The fluid viscosity depends on both, pressure, and temperature, which results in the same dependency for the permeability.

### 2.3 Stress and Relative Strain

The forces applied to porous rocks modify their dimensions (Fig. 1) by expansion (tension $\sigma_x > 0$), compression ($\sigma_x < 0$) or shear stress ($\sigma_{xy} \neq 0$). The stress acting in the OX direction is defined as the applied force $F_x$ per unit area $A$ where it is applied.

$$\sigma_x = \frac{F_x}{A} \quad \left[\frac{N}{m^2} = Pa\right] \tag{4}$$

The same experimental definition holds for the other cartesian directions OY and OZ. Relative strain is a measure of the deformation of the rock when a stress is applied on it. A cylindrical core of diameter $d_0$ and initial length $x_0$, when subjected to an axial stress $\sigma_x$, its initial length changes to a value $x$, so its relative axial strain in the OX direction is:

$$\varepsilon_x = \frac{x - x_0}{x_0} = \frac{\Delta x}{x_0} = \frac{\partial u_x}{\partial x} \quad [ad] \tag{5}$$

Where $u_x$ represents the displacement of the solid particles. These experimental measurements are extended to the other axes OY and OZ, to define $\sigma_y$, $\sigma_z$, $\varepsilon_y$, $\varepsilon_z$, respectively. If the strain is zero when the stress is removed and the rock returns to its original shape, the phenomenon is elastic. If the back strain is not zero, then it is plastic.

### 2.4 Relative Volumetric Strain

Adding the three longitudinal strains in the OX, OY and OZ axis respectively we obtain a fundamental variable of porous mechanics, the bulk strain $\varepsilon_B$ (in classic elasticity $\varepsilon_B = \varepsilon_{kk} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$):

$$\varepsilon_B = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\Delta x}{x_0} + \frac{\Delta y}{y_0} + \frac{\Delta z}{z_0} = \frac{y_0 z_0 \, \Delta x + x_0 z_0 \, \Delta y + x_0 y_0 \, \Delta z}{x_0 \, y_0 \, z_0} = \frac{\Delta V_B}{V_B^0} = \vec{\nabla} \cdot \vec{u}_B \tag{6}$$

Where $\Delta V_B$ is the change of the initial bulk volume $V^0{}_B$, and $\boldsymbol{u}_B$ is the bulk vector displacement. The ratio of volume change to the original porous rock volume is equal to the sum of the principal strains and is called the volumetric strain. It can be shown that the divergence of the velocity of the rock deformation is equal to the rate of change of the rock volume as a consequence of the rock mass conservation.

### 2.5 Young's Modulus

This poroelastic parameter defines the relationship between longitudinal stress and strain, describing the response of the rock to linear deformation. In a state of uniaxial tension (OX axis) Young modulus is measaured as follows:

$$E_x = \frac{F_x / A}{\Delta x / x_0} = \frac{\sigma_x}{\varepsilon_x} \quad \left[\frac{N}{m^2} = Pa\right] \tag{7}$$

The same definition applies to define $E_y$ and $E_z$ respectively, Young's modulus is a 3D tensor.

### 2.6 Poisson's Modulus

This is another important elastic parameter that measures the rock compressibility perpendicular to applied stress. It is experimentally defined as the ratio of latitudinal (diameter in the OY or OZ axis) to longitudinal strain (OX axis):

$$\nu = \frac{\Delta d / d_0}{\Delta x / x_0} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \quad [ad] \tag{8}$$

Where $d_0$ is the initial length of reference of the porous sample and $\Delta d$ is the change of this length. The minus sign appears because when the sample is dilated in the OX direction, its diameter is reduced and vice versa, if the cylinder is compressed, $\varepsilon_x$ decreases and both perpendicular strains $\varepsilon_y$ and $\varepsilon_z$ are dilated. This modulus is dimensionless.

### 2.7 Bulk Modulus and Compressibility

The Bulk Modulus $K_B$ for drained conditions measures the change in hydrostatic pressure when the rock volume changes with respect to an initial volume $V_0$ which here is the same as $V_B$. Rock compressibility $C_B$ is a key poroelastic coefficient that measures the change in volume when the hydrostatic pressure changes with respect to an initial volume $V_B$; compressibility is the reciprocal of the bulk modulus. When the test is done at constant temperature, $C_B$ is equal to the rock isothermal compressibility:

$$K_B = \frac{-\Delta p}{\Delta V_B / V_B} \quad [Pa] \tag{9}$$

$$C_B = -\frac{1}{V_B}\frac{\Delta V_B}{\Delta p} = \frac{1}{K_B} \quad \left[Pa^{-1}\right] \tag{10}$$

### 2.8 Rigidity Modulus

This parameter measures the resistance of the rock to change in shape. The rigidity modulus describes the response to shear; that is why it is also known as the shear coefficient:

$$G_{xy} = \frac{\text{shear stress}}{\text{shear strain}} = \frac{1}{2}\frac{\sigma_{xy}}{\varepsilon_{xy}} \quad [Pa] \tag{11}$$

### 2.9 Lamé Moduli

The first Lamé modulus expresses the rock response to tension or compression under drained experimental conditions:

$$\lambda_B = \frac{\Delta p}{\Delta V_B / V_B} - \frac{1}{3}\frac{\sigma_{xy}}{\varepsilon_{xy}} = K_B - \frac{2}{3}G_{xy} \quad [Pa] \tag{12}$$

The second Lamé modulus expresses the rock response to tension or compression for undrained experimental conditions:

$$\lambda_U = K_U - \frac{2}{3}G_{xy} = K_U - K_B + \lambda_B \quad [Pa] \tag{13}$$

Where $K_U$ is the rock undrained bulk modulus (Eq. 21). Since there are no shear stresses in the fluid, the shear coefficient must be the same in both conditions drained and undrained, thus $G_{xy} = G_U = G_B$ in any combination of axes ($X$, $Y$, $Z$).

The mechanics of non-porous elastic solids requires only two experimental coefficients to describe the relationships between stresses and strains; either the two Lamé modules ($\lambda$, $G$) or ($E$, $\nu$), since only two elastic coefficients are independent. The porosity in geothermal rock introduces additional experimental complications, due to its dependence on pressure and temperature $\varphi$ ($p$, $T$). Thus, other parameters are necessary to fully describe the porous rock in a geothermal system.

### 2.10 Compressibility of the Solid Skeleton

The elastic modulus $K_S$ measures the resistance of the solid rock against deformations produced by the hydrostatic compression $\sigma_H$ when the differential pressure $p_d$ is constant:

$$\sigma_H = -p_f \quad \Rightarrow \quad K_S = \frac{-\Delta p_f}{\Delta V_S / V_S} = -\left(\frac{\Delta p_f}{\Delta \varepsilon_S}\right)_{p_d} \tag{14}$$

The non-jacketed volumetric compressibility $C_S$ is defined as the undrained deformation obtained experimentally under constant differential pressure conditions, that means: $\Delta p_d = 0$. In this case any change of the confining pressure is similar to the change of fluid pressure, thus $\Delta p_k = \Delta p_f$. The measurements evaluate the changes of the structural volume only when the fluid pressure changes. Therefore, $C_S$ measures the grain compressibility of the solid matrix, which is the inverse of the solid bulk modulus $K_S$:

$$C_S = -\frac{1}{V_S}\left(\frac{\partial V_S}{\partial p_f}\right)_{p_d} = \frac{1}{K_S} \tag{15}$$

### 2.11 Biot-Willis Coefficient

This is a very important parameter in poroelasticity, which is used throughout the theory developed by Biot (Biot & Willis, 1957; Wang, 2000; Guéguen & Bouteca, 2004). This coefficient $b$ is deduce adding both bulk and solid strains:

$$\varepsilon = \varepsilon_B + \varepsilon_S = -\frac{p_k - p_f}{K_B} - \frac{p_f}{K_S} = -\frac{1}{K_B}\left( p_k - \left( 1 - \frac{K_B}{K_S} \right) p_f \right) \tag{16}$$

Where $\varepsilon$ is the overall volumetric strain. Then, the coefficient $b$ is defined as:

$$b = 1 - \frac{K_B}{K_S} = 1 - \frac{C_S}{C_B} \tag{17}$$

Consequently, from the corresponding partial derivative in (Eq. 16) when $\varepsilon$ is constant:

$$-K_B\,\varepsilon = p_k - b\,p_f \;\Rightarrow\; \boxed{b = \left( \frac{\partial p_k}{\partial p_f} \right)_{\varepsilon}} \tag{18}$$

Equations (17) and (18) are derived from the same Eq. (16) for the Biot-Willis parameter. Eq. (18) expresses that $b$ is equal to the change of confining pressure $p_k$ with respect to the fluid pressure change when the total volumetric strain remains constant. From experiments it is found that $C_S \leq C_B$ then $K_B \leq K_S$ , thus: $0 \leq b \leq 1$; and $b = 0$ if and only if there are no pores and $C_S = C_B$. Experimental relationships for porous rocks (Biot & Willis, 1957; Fjaer *et al.*, 1992; Wang, 2000; Bundschuh & Suárez, 2010) showed that $\varphi \leq b \leq 1$. In the case of soils and unconsolidated rocks, it is obvious that $C_B >> C_S$, therefore $K_B << K_S$ and thus $b \approx 1$, which results in the classical Terzaghi equation, which explains that the effective total axial load in a porous medium is determined by the sum of the solid stress and the pore pressure. .

### 2.12 The Variation of the Fluid Mass Content in the Pores

The Let $M_f$ [kg] be the mass of the fluid in the pores and $\rho_f$ [kg/m$^3$] its density, the fluid mass content $m_f$ per unit reference volume $V_B$ of the porous rock is defined as:

$$m_f = \frac{M_f}{V_B} = \frac{M_f V_f}{V_B V_f} = \frac{M_f\,\varphi}{V_f} = \rho_f\,\varphi \qquad \left[ \frac{kg}{m^3} \right] \tag{19}$$

Consequently Biot (1941) introduced a poroelastic dimensionless variable $\zeta$ (*zeta* in Greek) which he defined as "the volume of fluid which enters or exits the pores in a unit volume of bulk material", or "the variation in water content", or for a generic fluid, "the variation of fluid content" (Eq. 19). If $m_0$ and $\rho_0$ are the mass content and the fluid density in a reference state respectively, then the variation of fluid content $\zeta$ in the pores is defined as:

$$\zeta(p,T) = \frac{m_f - m_0}{\rho_0} = \frac{\Delta m_f}{\rho_0} = \frac{\rho_f \Delta\varphi + \varphi \Delta\rho_f}{\rho_0} \qquad [\mathrm{ad}] \tag{20}$$

Therefore, "$\zeta$ is the volume of fluid transported into or out of storage". The variation of $m_f$ occurs by changes in the fluid density and /or in the porosity, and $\zeta = 0$ if and only if both functions are constants.

### 2.13 Undrained Bulk Modulus and Undrained Compressibility

The Bulk Modulus $K_U$ under undrained conditions measures the change in confining pressure when the rock volume changes with respect to an initial volume $V_B$. The undrained compressibility $C_U$ is the inverse of this experimental modulus $K_U$:

$$C_U = -\left( \frac{\varepsilon_B}{\Delta p_k} \right)_{\zeta} \;\Leftrightarrow\; K_U = \frac{1}{C_U} \tag{21}$$

The undrained volumetric compressibility $C_U$ is the strain obtained when the rock is subjected to compressive stresses in all directions and the fluid contained in the pores remains constant. The undrained volumetric modulus $K_U$ measures the global resistance of the rock to deformations produced by the confining pressure when the fluid mass $\zeta$ contained in the pores remains constant.

### 2.14 Compressibility of the Pore Volume

The unjacketed compressibility of the pore volume $C_\varphi$ is defined as the change of pore volume with respect to the pore pressure change per unit volume when $p_d$ remains constant:

$$C_\varphi = -\frac{1}{V_\varphi}\left( \frac{\partial V_\varphi}{\partial p_f} \right)_{p_\mathrm{d}} = -\frac{1}{\varphi}\left( \frac{\partial \varphi}{\partial p_k} \right)_{p_\mathrm{d}} \qquad \left[ Pa^{-1} \right] \tag{22}$$

Equations (15) and (22) define two different forms of unjacketed compressibilities $C_S$ and $C_\varphi$. The compressibility of the pore volume varies with the effective compression, the temperature and the porosity. If the pressure in the pore decreases during the reservoir's fall off, then important reductions in the pore volume can occur, even with partial or total collapse of pores and fractures. Reported effects of pressure in the pore volume (Bundschuh & Suárez, 2010) show that its compressibility decreases when pressure increases within a range of 1 to 550 bar, for temperatures between 24°C and 205°C. For higher pressure values the variation is lower. The effect of temperature in the compressibility of the pore volume is appreciated in experiments with different rocks. Several results show that at 205°C the compressibility can be between 12% and 55% higher than at 24°C. The average of all the samples analyzed by Passmore and Archer (1985) shows a compressibility increment of 21% when the temperature increases inside this range. Other experimental results from the same authors, using sandstone cores, show that compressibility increases with porosity in the range between 19% to 28%. Below 19% porosity there was no trend.

### 2.15 Relations among the Biot-Willis Coefficient, the Bulk Moduli and the Pore Volume

Using the compressibility of the pore volume previously defined (Eq. 22), the total compressibility of the porous rock, for a drained experiment, is equal to the compressibility of the solid grains plus the compressibility of the pore space:

$$K_\varphi = \frac{1}{C_\varphi} \;\Rightarrow\; \frac{1}{K_B} = \frac{1}{K_S} + \frac{\varphi}{K_\varphi} \;\Rightarrow\; 1 = \frac{K_B}{K_S} + \varphi\frac{K_B}{K_\varphi} \tag{23}$$

Therefore, from equation (17) :

$$b = 1 - \frac{K_B}{K_S} = \varphi\frac{K_B}{K_\varphi} = -\left(\frac{\cancel{\varphi}}{\cancel{K}_\varphi}\frac{\Delta V_\varphi}{\Delta \cancel{p}_k}\right) \Bigg/ \left(-\frac{1}{\cancel{K}_B}\frac{\Delta V_B}{\Delta \cancel{p}_k}\right) = \left(\frac{\Delta V_\varphi}{\Delta V_B}\right)_{p_f} \tag{24}$$

Equation (24) gives another definition of $b$ as the ratio of pore volume change to total bulk volume change under drained conditions. Equations (23) and (24) provide two ways of computing the drained bulk modulus of the pore space $K_\varphi$ . These formulae must be obtained from experiments in which pore fluids can flow in or out of the pores to maintain the pore pressure constant

### 2.16 Isothermal Compressibility of the Fluid

The compressibility of the fluid $C_f$, when temperature $T$ and fluid mass $M_f$ are constants, is defined as the change of fluid volume with respect to the effective pressure change per unit volume:

$$C_f = -\frac{1}{V_f}\left(\frac{\partial V_f}{\partial p_f}\right)_T = \frac{1}{\rho_f}\left(\frac{\partial \rho_f}{\partial p_f}\right)_T \tag{25}$$

### 2.17 Expansion and Storage Coefficients

There are two other experimental poroelastic coefficients, $H$ and $R$ introduced by Biot (1941) in his fundamental theory:

$$\frac{1}{H} = \left(\frac{\Delta \varepsilon_B}{\Delta p_f}\right)_\sigma = \left(\frac{\Delta \zeta}{\Delta \sigma}\right)_{p_f} , \quad \frac{1}{R} = \left(\frac{\Delta \zeta}{\Delta p_f}\right)_\sigma \tag{26}$$

Where $1/H$ is a poroelastic expansion coefficient, which describes how much $V_B$ changes when $p_f$ changes while keeping the applied stress $\sigma$ constant; $1/H$ also measures the changes of $\zeta$ when $\sigma$ changes and $p_f$ remains constant. $1/R$ is the unconstrained specific storage coefficient, which represents the changes of $\zeta$ when $p_f$ changes. This parameter is measured when the applied stress $\sigma$ remains constant. Together with the bulk compressibility $C_B$, these three poroelastic coefficients given entirely describe the poroelastic response of the rock for isothermal, isotropic stresses.

2.17.1 Matrix form of the Fundamental Poroelastic Equations in 2D for a 1D Experiment

Writing explicitly the linear relationships for $\sigma$ and $p_f$ using equation (26):

$$\begin{pmatrix} \varepsilon_B \\ \zeta \end{pmatrix} = \begin{pmatrix} C_B & H^{-1} \\ H^{-1} & R^{-1} \end{pmatrix} \cdot \begin{pmatrix} \sigma \\ p_f \end{pmatrix} \;\Leftrightarrow\; \begin{pmatrix} \sigma \\ p_f \end{pmatrix} = \frac{1}{D_b}\begin{pmatrix} R^{-1} & -H^{-1} \\ -H^{-1} & C_B \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_B \\ \zeta \end{pmatrix} \tag{27}$$

Where $D_b$ is called the determinant of Biot in the previous inverse matrix equation (27): $\dfrac{1}{D_b} = \dfrac{K_B R H^2}{H^2 - K_B R} \neq 0 ;\; \sigma = -p_k$ .

The sign conventions are: the stress $\sigma > 0$ in tension and $\sigma < 0$ in compression; the volumetric strain $\varepsilon_B > 0$ in expansion and $\varepsilon_B < 0$ in contraction; $\zeta > 0$ if fluid is added to $\Delta V_B$ and $\zeta < 0$ if fluid is extracted from $\Delta V_B$; the pore pressure $p_f > 0$ if it is larger than the atmospheric pressure. Here $\Delta V_B$ is also representing the bulk control volume. Due to the experimental fact that compressibility is a function of fluid pressure and temperature, then all parameters in equation (27) also depend on specific ($p_f$, $T$) conditions.

### 2.18 Solid/Fluid Coupling Coefficient and Constrained Specific Storage

Biot (1941) introduced a fourth poroelastic coefficient $1/M$, called the constrained specific storage, which is equal to the change of $\zeta$ when $p_f$ changes measured at constant strain. $1/M$ is expressed in terms of the three fundamental ones used in equation (27):

$$\frac{1}{M} = \left( \frac{\Delta \zeta}{\Delta p_f} \right)_{\varepsilon_B} = \frac{1}{R} - \frac{K_B}{H^2} \;\Rightarrow\; M = \frac{R H^2}{H^2 - K_B R} \tag{28}$$

The coefficient $M$ is the inverse of the constrained specific storage, measured at constant strain (Wang, 2000); this parameter characterizes the elastic properties of the fluid because it measures how the fluid pressure changes when $\zeta$ changes. Another coefficient $C$ introduced by Biot (1962), represents the coupling of deformations between the solid grains and the fluid; it depends on both, the fluid properties and the bulk rock properties:

$$C = \frac{1}{D_b H} = \frac{K_B R H}{H^2 - K_B R} = \frac{K_B}{H} M \tag{29}$$

These three parameters $b$, $M$ and $C$ are the core of the fundamental poroelastic equations, and there is a key relationship among them. If we take the limit when $\Delta p_f \to 0$ with $\sigma = - p_k$ , from the definitions of $K_b$ and $H$ (Eqs. 9 and 26), we find three extra fundamental relationships for the Biot's coefficient:

$$\frac{C}{M} = \frac{K_B}{H} = \lim_{\Delta p_f \to 0} \left( \frac{\Delta p_k}{\Delta \varepsilon_B} \frac{\Delta \varepsilon_B}{\Delta p_f} \right) = \left( \frac{\partial p_k}{\partial \varepsilon_B} \frac{\partial \varepsilon_B}{\partial p_f} \right) = \left( \frac{\partial p_k}{\partial p_f} \right)_{\varepsilon} = b \tag{30}$$

Therefore, the Biot-Willis coefficient $b$, can also be interpreted as the change in confining pressure $p_k$ with respect to the change in fluid pressure when the total volumetric strain remains constant. The solid/fluid coupling coefficient $C$ is also computed as follows:

$$b = \frac{C}{M} \;\Leftrightarrow\; \boxed{C = b\,M} \tag{31}$$

### 2.19 Skempton Coefficient

There is an additional important parameter $B$, called the Skempton coefficient (Skempton, 1954; Wang, 2000; Guéguen & Boutéca, 2004), which represents the change in pore pressure when the applied stress changes for undrained conditions. Skempton's coefficient is also defined as the ratio of the induced pore pressure to the change of stress loading under undrained condition. This parameter is calculated directly, using the first part of matrix equation (27):

$$\zeta = \frac{\sigma}{H} + \frac{p_f}{R} \quad\Rightarrow\quad B = \left( \frac{\Delta p_f}{\Delta p_k} \right)_{m_f} = - \left( \frac{\Delta p_f}{\Delta \sigma} \right)_{\zeta} = \frac{R}{H} \tag{32}$$

The presence of fluid in the rock pores adds another extra stress due to hydrostatic pressure, which is identified as the fluid pressure or pore pressure $p_f$, because all the pores are assumed to be interconnected. Biot's linear theory is appropriate for isothermal, homogeneous, and isotropic porous rocks, regardless of temperature variations in geothermal rocks. However, it is obvious that all the poroelastic coefficients introduced so far are dependent on the pressure and temperature at which they are obtained; that is, they depend on the thermodynamic conditions ($p_f$ ,$T$) at which the rock is found. Therefore, it is necessary to introduce additional coefficients for non-isothermal processes to complete or complement the Biot's linear theory.

### 2.20 Volumetric Thermal Expansivity for Expansion/Contraction of the Bulk Rock

The heat absorption is manifested as a dilatation of rock volume size. However, if the rock is cooling its volume decreases. This thermal rock expansion/contraction can be linear, areal or volumetric and it causes internal stresses. The volumetric expansivity concept is applicable to solids, liquids and gases. The bulk expansivity of rocks is measured by the volumetric thermal expansion coefficient at constant confining pressure $p_k$:

$$\gamma_B = \frac{1}{V_B} \left( \frac{\partial V_B}{\partial T} \right)_{p_k} = - \frac{1}{\rho_s} \left( \frac{\partial \rho_s}{\partial T} \right)_{p_k} \qquad \left[ \frac{1}{{}^\circ\mathrm{C}} \right] \tag{33}$$

The general trend is that bulk thermal expansion increases when temperature rises. $\gamma_B$ is about $10^{-5}$ $K^{-1}$ for solids, $10^{-4}$ $K^{-1}$ for liquids and $10^{-3}$ $K^{-1}$ for gases (Bundschuh & Suárez, 2010).

### 2.21 Thermal Expansivity of the Pores

The thermal expansion coefficient of the pores at constant $p_k$ and constant pore pressure $p_f$ is:

$$\gamma_\varphi = \frac{1}{V_\varphi} \left( \frac{\partial V_\varphi}{\partial T} \right)_{p_f} = \frac{1}{\varphi V_B} \left( \frac{\partial (\varphi V_B)}{\partial T} \right)_{p_f} = \frac{1}{\varphi} \left( \frac{\partial \varphi}{\partial T} \right)_{p_f} \tag{34}$$

The last term of equation (34) defines the expansivity of the rock pores, coupling the fluid and heat flow to the rock deformation.

### 2.22 Fluid Thermal Expansivity

The thermal expansion coefficient of the fluid is given by the following equation:

$$\gamma_f = \frac{1}{V_f}\left(\frac{\partial V_f}{\partial T}\right)_{p_f} = -\frac{1}{\rho_f}\left(\frac{\partial \rho_f}{\partial T}\right)_{p_f} \tag{35}$$

### 2.23 Thermal Expansivity of the Fluid Mass

The special thermal expansion coefficient $\gamma_m$ measures the changes in the fluid mass content when the temperature changes and the average stress $\sigma_M$ and fluid pressure $p_f$ are held constant:

$$\gamma_m = \frac{1}{m_f}\left(\frac{\partial m_f}{\partial T}\right)_{p_f} = \frac{1}{\rho_f}\left(\frac{\partial \rho_f}{\partial T}\right)_{p_f} + \frac{1}{\varphi}\left(\frac{\partial \varphi}{\partial T}\right)_{p_f} = -\gamma_f + \gamma_\varphi \tag{36}$$

The thermal expansivity of the fluid mass is equal to the thermal expansion of the pores minus the thermal expansion of the fluid.

## 3. ALGEBRAIC RELATIONSHIPS AMONG DIFFERENT POROELASTIC COEFFICIENTS

There are different coefficients and formulations to represent the porous rock deformation coupled to fluid flow. There are two fundamental poroelastic phenomena and two mechanisms playing a key role in the interaction between the pore fluid and the rock:

1) Solid → Fluid coupling occurs when a change in applied stress produces a change in the fluid pressure or in the fluid mass.
2) Fluid → Solid coupling occurs when a change in the fluid pressure or in the fluid mass produces a change in the volume of the porous rock.
3) An increase of pore pressure induces a dilation of the rock.
4) Compression of the rock causes a rise of pore pressure if the fluid is prevented from escaping the pores.

Many coefficients are needed to support poroelastic theory, but only three of these parameters are actually independent. The three basic poroelastic coefficients can be chosen arbitrarily, but in practice, their selection depends on the ease with which they can be obtained in the laboratory. Equation (27) shows that three basic parameters selected to constitute an experimental reference set were the drained bulk compressibility $C_B$, the expansion coefficient $1/H$ and the unconstrained specific storage $1/R$. Other three possible basic coefficients are the drained bulk modulus $K_B$, the undrained bulk modulus $K_U$, and the Biot coefficient $b$. In principle, the rest of the coefficients can be constructed algebraically from preexisting relations between them, which are deduced from their definition and experimental measurement. However, in practice it is impossible to compute all the coefficients of the poroelastic theory using only classical elastic constants (Eqs. 7-15). Because of porosity, at least five mixed coefficients are necessary for the whole poroelastic coupling, including fluid compressibility, which is not a rock property. A minimum, necessary and sufficient set of mixed poroelastic and independent measurable parameters can be for example: $\{E, G, \varphi, K_S, K_f\}$. With these five experimental modules, we can calculate a complete set of 16 poroelastic coefficients. To complete this task, it is necessary to establish adequate algebraic relationships.

### 3.1 Algebraic Relations between Poroelastic Parameters

From an experimental point of view, it appears that some poroelastic coefficients are more difficult to measure than others are. This is the case of the variation of the fluid mass content. To describe the saturating fluid, we need its pressure and its temperature. It is possible to make other choices of the basic experimental parameters using the relationships developed in this section.

#### 3.1.1 Relationships among the five classic elastic parameters $E$, $\nu$, $K_B$, $G$, $\lambda$

Among the five elastic coefficients of simple solids, only two are independent; the other three are deduced from them. The mutual relationships among these elastic parameters in isotropic rocks are well known:

$$\begin{aligned} K_B &= \lambda + \frac{2}{3}G\,,\ G = \frac{E}{2(1+\nu)}\,,\ \lambda = \frac{E\,\nu}{(1+\nu)(1-2\,\nu)} = \frac{2G\,\nu}{1-2\,\nu} \\ E &= \frac{3\lambda + 2G}{\lambda + G}G\,,\ \nu = \frac{\lambda}{2(\lambda+G)} = \frac{E}{2G} - 1,\ K_B = \frac{E}{3\,(1-2\,\nu)} \end{aligned} \tag{37}$$

These relations are valid for the corresponding poroelastic parameters under drained or undrained experimental conditions (Mavko *et al*., 2003; Dill, 2007). The algebraic relationships that follow between the different poroelastic coefficients are deducible from their respective experimental definitions with simple algebraic operations.

#### 3.1.2 Relationships among $K_B$, $K_U$, $M$, $R$, $H$.

$$K_U = \frac{M}{R}K_B = \frac{K_B\,H^2}{H^2 - K_B\,R} = K_B\left(1 - \frac{K_B}{H}\frac{R}{H}\right)^{-1} \tag{38}$$

### 3.1.3 Relationships among $b$, $B$, $K_B$, $K_U$, $M$.

$$K_U = \frac{K_B}{1-bB} \;\Rightarrow\; B = \frac{K_U - K_B}{b K_U} = \frac{bM}{K_U} \tag{39}$$

### 3.1.4 Definition of the undrained fluid pressure $p_U$ using $b$, $B$, $K_B$, $K_U$.

$$\varepsilon_B = -\frac{p_k - b\,p_U}{K_B} = -\frac{p_k}{K_U} \;\Rightarrow\; p_U = -\frac{K_U - K_B}{b}\,\varepsilon_B = B\,p_k \;\Rightarrow\; B = \left(\frac{\partial p_U}{\partial p_k}\right)_{\zeta} \tag{40}$$

### 3.1.5 Relationships among $b$, $B$, $K_B$, $K_U$, $M$, $R$, $C$.

$$K_B = K_U\left(1 - bB\right) = K_U - \frac{K_B}{R} M\,b\,B = K_U - \frac{K_B}{R} C\,B \;\Rightarrow\; C = \left(K_U - K_B\right)\frac{R}{B\,K_B} \tag{41}$$

### 3.1.6 Relationships among $C$, $b$, $K_B$, $K_U$, $M$, $H$.

$$K_B = K_U - \frac{K_B}{H} C = K_U - b\,C = K_U - b^2 M = K_U - \frac{C^2}{M} \tag{42}$$

### 3.1.7 Relationships among $K_S$, $K_B$, $b$, $K_U$, $C$, $M$.

$$1 = b + \frac{K_B}{K_S} \;\Rightarrow\; K_S = \frac{K_B}{1-b} = \frac{K_U - b\,C}{1-b} = \frac{K_U - b^2 M}{1-b} = \frac{M\,K_U - C^2}{M - C} \tag{43}$$

### 3.1.8 Relationships among the Lamé Moduli and $K_U$, $K_B$.

$$\frac{2}{3} G = K_U - \lambda_U = K_B - \lambda_B \;\Rightarrow\; \lambda_U = K_U - K_B + \lambda_B \;\Rightarrow\; \boxed{\lambda_U = \lambda_B + b\,C} \tag{44}$$

### 3.1.9 Construction of the undrained Poisson's coefficient using $\nu_B$, $b$, $B$.

$$\lambda_B = \frac{E\,\nu_B}{\left(1+\nu_B\right)\left(1-2\,\nu_B\right)} \;\Rightarrow\; \nu_U = \frac{3\,\nu_B + b\,B\left(1-2\nu_B\right)}{3 - b\,B\left(1-2\,\nu_B\right)} \tag{45}$$

The relationship given by the first part of equation (45) is valid for both, drained and undrained modulus. Since there are no shear stresses in the fluid, the shear coefficient and the Young modulus *E* must be the same in both situations.

### 3.1.10 Relationships among $\varphi$, $K_S$, $K_f$, $b$, $M$.

$$\varphi\left(\frac{1}{K_S} - \frac{1}{K_f}\right) = \frac{b}{K_S} - \frac{1}{M} \;\Rightarrow\; M = \frac{K_f\,K_S}{\varphi\,K_S + \left(b - \varphi\right)K_f} \tag{46}$$

### 3.1.11 Relationships among the bulk moduli, $b$, $C$, $M$, $\varphi$.

$$b = \frac{C}{M} \;\Rightarrow\; C = M\left(1 - \frac{K_B}{K_S}\right) = \frac{K_f\,K_S}{\varphi K_S + \left(b - \varphi\right)K_f}\left(1 - \frac{K_B}{K_S}\right) \tag{47}$$

### 3.1.12 Expression relating Biot parameters $C$, $M$, the bulk moduli and $\varphi$.

$$C = \frac{K_f}{\varphi}\left(1 - \frac{K_B}{K_S}\right) \Big/ \left(1 - \frac{K_f}{K_S} + \frac{K_f}{\varphi K_S}\left(1 - \frac{K_B}{K_S}\right)\right) \;\Rightarrow\; M = \frac{C}{1 - K_B / K_S} \tag{48}$$

### 3.1.13 Construction of the Biot tangent modulus in terms of $C$, $M$, $b$, $K_f$, $K_S$, and $\varphi$.

From these coefficients, Biot (1962) derived another parameter called the tangent modulus *N*, defined as the pressure variation with respect to the porosity variation when both strain and temperature are held constant (Coussy, 2004).

$$\frac{1}{N} = \frac{1}{M} - \frac{\varphi}{K_f} = \frac{b}{C} - \frac{\varphi}{K_f} = \frac{b - \varphi}{K_S} \tag{49}$$

### 3.2 The Low-Frequency Gassmann-Biot Equation

Biot extended his theory to poroelastodynamic phenomena in two parts: low and high frequency ranges, respectively (Biot, 1956a,b). The low frequency theory developed by Gassmann (1951) and Biot (1956a) allows to predict the resulting increase in the undrained bulk modulus $K_U$ of a saturated rock when an increment of pore pressure is induced from a passing seismic wave (Mavko *et al.*, 2003). The main result of this theory is condensed in the so called Gassmann-Biot equation, which combines porosity and all the poroelastic bulk moduli, in a single formula, the Biot coefficient $b$ is given by equation (47). From equations (42) and (48):

$$K_U - K_B = b^2 M \quad \Rightarrow \quad \boxed{K_U = K_B + \frac{b^2}{\dfrac{\varphi}{K_f} + \dfrac{b-\varphi}{K_S}}} \tag{50}$$

3.2.1 Relationships among $K_U$, $K_B$, $K_f$, $K_S$. Another formulation of equation (50) is:

$$K_U = K_B + \frac{K_f}{\varphi}\left(1 - \frac{K_B}{K_S}\right)^2 \frac{1}{1 + \dfrac{K_f}{K_S}\left(\dfrac{1}{\varphi}\left(1 - \dfrac{K_B}{K_S}\right) - 1\right)} \tag{51}$$

3.2.2 The reciprocal form of equation (51). This equation is obtained by computing $(K_U - K_B) / (K_U K_B)$:

$$\frac{1}{K_U} = \frac{1}{K_B} - \frac{\left(\dfrac{1}{K_B} - \dfrac{1}{K_S}\right)^2}{\dfrac{1}{K_B} - \dfrac{1}{K_S} + \varphi\left(\dfrac{1}{K_f} - \dfrac{1}{K_S}\right)} \tag{52}$$

3.2.3 Another form of Eq. (50). After doing some algebra another equivalent formulation of equation (50) is obtained:

$$\frac{K_U}{K_S - K_U} = \frac{K_B}{K_S - K_B} + \frac{K_f}{\varphi\left(K_S - K_f\right)} \tag{53}$$

3.2.4 Relationships among porosity and bulk moduli. From equation (53), we obtain a formula for the inverse of porosity in terms of the bulk moduli:

$$\frac{1}{\varphi} = \frac{K_S - K_f}{K_f}\left(\frac{K_U}{K_S - K_U} - \frac{K_B}{K_S - K_B}\right) \tag{54}$$

From equation (50), we obtain another useful formula for the porosity, which includes the Biot coefficients and the Skempton parameter:

$$\varphi = \frac{K_f}{K_S - K_f}\left(\frac{K_S}{M\,(1 - b\,B)} - \frac{b^2}{(1-b)} - b\right) \tag{55}$$

### 3.3 Useful Algebraic Relationships among the Main Poroelastic Functions

Writing explicitly the inverted relations for σ and $p_f$ in the matrix equation (27) and using equation (29):

$$\sigma = \frac{K_B H^2}{H^2 - K_B R}\,\varepsilon_B - \frac{K_B R H}{H^2 - K_B R}\,\zeta = \frac{K_B}{R} M\,\varepsilon_B - C\,\zeta \tag{56}$$

$$p_f = \frac{R H^2}{H^2 - K_B R}\,\zeta - \frac{K_B R H}{H^2 - K_B R}\,\varepsilon_B = M\,\zeta - C\,\varepsilon_B \tag{57}$$

$$\zeta = \frac{p_f}{M} + b\,\varepsilon_B \tag{58}$$

Using the definition of the undrained bulk modulus (Eq. 21) in equation (56), we deduce that:

$$K_U = -\left(\frac{\Delta p_k}{\Delta \varepsilon_B}\right)_\zeta = \left(\frac{\Delta \sigma}{\Delta \varepsilon_B}\right)_\zeta = \frac{K_B}{R} M = \frac{K_B H^2}{H^2 - K_B R} = K_B\left(1 - \frac{K_B}{H}\frac{R}{H}\right)^{-1} \Rightarrow \boxed{K_U = \frac{K_B}{1 - b\,B}} \tag{59}$$

And therefore:

$$\boxed{B = \frac{K_U - K_B}{b\,K_U} = \frac{b\,M}{K_U}} \tag{60}$$

From these relationships, we can define the fluid pressure in the undrained regime, when ζ remains constant. From (Eq. 30) and the definition of $K_U$:

$$\varepsilon_B = -\frac{p_k - b\,p_U}{K_B} = -\frac{p_k}{K_U} \Rightarrow p_U = -\frac{K_U - K_B}{b}\varepsilon_B = B\,p_k \tag{61}$$

Thus, the fluid pressure variation for undrained conditions is:

$$\left(\frac{\partial p_U}{\partial p_k}\right)_{\zeta} = B \tag{62}$$

Using equation (58) we obtain the variation of the fluid content in the drained regime:

$$\zeta = b\,\varepsilon_B + \frac{p_f}{M} = -b\frac{p_k}{K_U} + \frac{b\,p_f}{B\,K_U} = \frac{b}{B\,K_U}\left(p_f - B\,p_k\right) \tag{63}$$

The bulk moduli $K_B$ and $K_S$ of the porous rock were already defined. From (Eq. 59), we can construct several relationships between these three different bulk moduli and the other poroelastic parameters.

$$K_B = K_U\left(1 - b\,B\right) = K_U - \frac{K_B}{R}M\,b\,B = K_U - \frac{K_B}{R}C\,B \tag{64}$$

$$K_B = K_U - \frac{K_B}{H}C = K_U - b\,C = K_U - b^2 M = K_U - \frac{C^2}{M} \tag{65}$$

We can include $K_S$ from equation (16):

$$1 = b + \frac{K_B}{K_S} \Rightarrow K_S = \frac{K_B}{1-b} = \frac{K_U - b\,C}{1-b} = \frac{K_U - b^2 M}{1-b} = \frac{M\,K_U - C^2}{M - C} \tag{66}$$

## 4. NUMERICAL VALUES OF SOME POROELASTIC COEFFICIENTS

Tables (1) and (2) present both experimental and computed numerical values of the poroelastic parameters herein defined. Note that *C*, *H* and some other coefficients are not included in these Tables, but they can be easily calculated using the formulae introduced in previous sections.

**Table 1.- Poroelastic parameters measured and calculated for different types of rocks. _**

| **Rock Type** | $\varphi$ **(%)** | $E$ **GPa** | $\nu$ | $\nu_U$ | $G$ **GPa** | $\lambda$ **GPa** | $\lambda_U$ **GPa** | $K_B$ **GPa** | $K_U$ **GPa** | $K_S$ **GPa** | $B$ | $b$ | $M$ **GPa** | $R$ **GPa** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **[0]** | *65.0* | *0.18* | *0.01* | *0.49* | *0.09* | *.001* | *6.1* | **0.062** | **6.2** | $10^5$ | **0.99** | **1.0** | *6.1* | *0.06* |
| **[1]** | **26.0** | *9.7* | **0.15** | **0.31** | **4.2** | *1.8* | *7.3* | **4.6** | *10.1* | **42.0** | *0.61* | *0.89* | *6.9* | *3.2* |
| **[2]** | **19.0** | *13.1* | **0.17** | **0.30** | **5.6** | *2.9* | *10.9* | **6.6** | *14.6* | **28.9** | *0.71* | **0.77** | *13.4* | *6.1* |
| **[3]** | **13.0** | *30.5* | **0.26** | **0.32** | **12.1** | *13.1* | *23.2* | **21.2** | **31.2** | **72.6** | **0.46** | **0.71** | *20.0* | *13.6* |
| **[4]** | **2.0** | *60.0* | **0.25** | **0.27** | **24.0** | *24.2* | *28.3* | **40.0** | **44.3** | **50.0** | **0.49** | **0.19** | *107.3* | *97.0* |
| **[5]** | **0.5** | *42.2* | **0.42** | **0.42** | **14.9** | *73.9* | *74.0* | **83.8** | **83.9** | **85.0** | **0.06** | **0.01** | *384.3* | *384.0* |

**Notice**: The rock types are: [0]- Clay, ($K_f$ = 3.9 GPa); [1].- Boise sandstone, ($K_f$ = 2.0 GPa); [2].- Berea sandstone ($K_f$ = 3.5 GPa); [3].- Indiana limestone, ($K_f$ = 3.1 GPa), (Wang, 2000). [4].- Tennessee marble, ($K_f$ = 3.5 GPa), (Detournay and Cheng, 1993). [5].- Rock with celestite, ($K_f$ = 2.0 GPa), (Mavko *et al.*, 2003). Rocks of Type [0] and [5], are the two limit cases of linear poroelastic theory, for $b \approx 1$ and $b \approx 0$, respectively. The figures in italics were calculated using the poroelastic formulas.

**Table 2.- Poroelastic parameters of the Los Humeros, Mexico geothermal field. _**

| $\rho_r$ kg/m$^3$ | $\varphi$ (%) | $E$ GPa | $\nu$ | $\nu_U$ | $G$ GPa | $\lambda$ GPa | $\lambda_U$ GPa | $K_B$ GPa | $K_U$ GPa | $K_S$ GPa | $B$ | $b$ | $M$ GPa | $R$ GPa |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2770** | **3.8** | **34.8** | **0.29** | **0.33** | **13.5** | **18.5** | **25.2** | **27.5** | **35.2** | **50.1** | **0.49** | **0.45** | **38.0** | **29.6** |
| **2500** | **8.5** | **35.8** | **0.31** | **0.32** | **13.7** | **21.7** | **24.8** | **30.8** | **33.9** | **49.7** | **0.24** | **0.38** | **21.6** | **19.6** |
| **2340** | **16.3** | **30.0** | **0.29** | **0.31** | **11.6** | **16.4** | **19.0** | **24.2** | **26.8** | **45.2** | **0.21** | **0.47** | **11.9** | **10.7** |
| **2210** | **20.9** | **25.3** | **0.30** | **0.32** | **9.70** | **15.1** | **17.1** | **21.5** | **23.5** | **40.0** | **0.19** | **0.46** | **9.4** | **8.64** |
| **2210** | **20.9** | **25.3** | **0.30** | 0.31 | **9.70** | **15.1** | 15.5 | **21.5** | 22.0 | **40.0** | 0.04 | **0.46** | 2.0 | 1.95 |

**Notice**: Basic experimental data set used: $\{E, G, \varphi, K_S$ and $K_f\}$. The rocks are andesites. The measured coefficients were obtained under drained conditions, except for $K_S$. The confining pressure was 250 bar, the ambient temperature was constant and equal to 25°C. The bulk modulus of water was constant, $K_w$ = 2.1 GPa. The last row was computed with a lower bulk modulus of water corresponding to $T$ = 290°C and $p_f$ = 110 bar is $K_w$ = 0.42 GPa. This last example illustrates the influence of the temperature on the poroelastic coefficients. For cold water, the estimated value of $\zeta \approx 2.1\times10^{-3}$, while for hot water $\zeta \approx 11.1\times10^{-3}$. Therefore, the variation of the fluid content is much higher in geothermal reservoirs than in isothermal aquifers.

## 5. CONCLUSIONS

Geothermal reservoirs rocks are porous, compressible, elastic, and plastic. The presence of a moving fluid inside pores, fractures and faults modifies the geomechanical response of the rock. To fully understand this response requires experimental measurement of several thermoporoelastic coefficients, which depend on pressure and temperature. However, it is not necessary to measure all these coefficients experimentally. Once a base of at least four independent coefficients for the porous rock is measured in the laboratory, plus the fluid compressibility, all the other coefficients can be calculated algebraically using the formulas presented in this article, which can be assumed to be valid in a wide range of different pressures and temperatures.

The poroelasticity of the rock is manifested by compression resulting from the decrease in fluid pressure, which reduces the volume of the pores. This reduction in pore volume is the main cause of the release of fluid stored in the rock. Poroelasticity explains how the fluid inside the pores supports a part of the load acting on the porous rocks. The remaining part of the total load is supported by the solid skeleton. The linear non-isothermal thermoporoelastic model, requires thermoporoelastic coefficients to support the theory.

In this document a brief but complete description of the main thermoporoelastic coefficients, the way in which they are measured and defined and the way in which they can be deduced algebraically from a base of only five independent experimental coefficients, has been made. But for real non-isothermal conditions, at various pressures and temperatures, there are many voids and few experimental data available. Emphasis should be placed on this lack of information to determine all the coefficients for different non-isothermal conditions. This experimental activity will allow a correct characterization of geothermal reservoirs, from classical and low enthalpy systems to huge supercritical systems.

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