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Numerical Simulators for Two-Phase Wellbore Flow and Well Test Analysis of Kyushu University

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ABSTRACT

Numerical simulators for wellbore flow and pressure transient were developed. Numerical simulator for wellbore flow can be used for pressure and temperature profiles by giving wellhead condition such as pressure, steam and water flow rates, and well specification. Production well deliverability can also be simulated by coupling wellbore flow and reservoir flow under steady state. On the basis of the line source solution, numerical simulator for determining optimal values of transmissivity and storativity was developed on Excel worksheet by using ad-in Solver. Input data of pressure transient, flowrate of active wells are input on the work sheet.

1. INTRODUCTION

In geothermal development production wells connect reservoir and surface facilities such as turbine for power generation by producing steam and water mixture in water dominated systems. Wellbore flow generally consists of steam-water two-phase flow part and liquid water single-phase flow part. Several wellbore simulator have been developed and reported their characteristics and their capacity (Bjornsson and Bodvarsson, 1987; Hadgu and Freeston, 1990). Wellbore flow model also can be used for evaluating well deliverability or productivity by coupling a reservoir flow model in the vicinity of wells (Itoi et al., 1987: Hadgu et al., 1995). The well deliverability is one of the basic and important information for evaluating reservoir potential and for designing power plant capacity. At the same time, permeability of reservoir penetrated by wells also control the well productivity. Reservoir permeability and porosity can be evaluated by analyzing pressure transient test such as build-up test and interference test.

We have developed numerical simulators for geothermal wellbore flow named as WellKyu (Wellbore Flow Simulator of Kyushu University) and pressure well test analysis on Excel as KyuPEx (Kyushu University Pressure Well Test Analysis Simulator on Excel). Wellbore flow model which have been used in the exercise for undergraduate student as well as their research projects. In this paper, we introduce these simulators and example of simulated results.

2. WELLBORE FLOW SIMULATOR

Geothermal well drilled in water dominated systems generally produce steam-water two-phase mixture. Fluid flow in wellbore consists of two-phase flow and liquid water single-phase flow to fully two-phase flow. Fluid conditions at the feed point where the well is connected to the reservoir can be either water single-phase or steam-water two-phase. Flow amount into the well is controlled by permeability of reservoir. Thus, a model need to be coupled with wellbore flow and reservoir flow.

2.1 Model description

A well bore flow model assumes:1)Well is vertical with constant diameter, 2) Flow in the well is steady state and homogeneous for steam-water two-phase, 3) Heat exchange between fluid in the well and surrounding formation is neglected, 4) Potential energy and acceleration energy are neglected in energy conservation equation, thus isenthalpic flow, 5) Fluid is single component of water without any gas nor dissolved solid, 6) Single feed zone at the well bottom. Then, the momentum equation for the flow in wellbore can be expressed as:

$$\mathrm{d}p + \frac{\mathrm{d}w^2}{2v} + \frac{\lambda w^2}{2Dv}\mathrm{d}H + \mathrm{g}\frac{\mathrm{d}H}{v} = 0 \quad (1)$$

where D is the diameter of well (m), p is the pressure (kg/ (ms^2)), v is the specific weight of fluid (m^3/kg), w is the fluid velocity (m/s), H is the vertical length of well (m), g is the gravitational acceleration (m/s^2), λ is the friction factor of wellbore surface (-). Equation (1) is used for both steam-water two-phase flow and liquid-water single-phase flow.

Equation (1) is numerically evaluated by the finite difference method or the numerical integration from well head to well bottom or vice versa. Here, numerical integration is adopted from the well bottom to the well head (Itoi et al., 1987).

$$\frac{1}{j}\int_{p_{wb}}^{p_{wh}} \frac{dp}{jv + \frac{g}{jv}} + \frac{2jD}{\lambda}\int_{p_{wb}}^{p_{wh}} \frac{dv}{jv + \frac{g}{jv}} + H_{12} = 0 \quad (2)$$

$$j = \frac{G}{F}\sqrt{\frac{\lambda}{2D}}$$

where p_{wh} is the wellhead pressure (kg/ (ms^2)), p_{wb} is the well bottom pressure (kg/ (ms^2)), H_{12} is the depth of wellbore from wellhead to well bottom (m), F is the cross sectional area of the well (m^2), G is the mass flow rate (kg/s).

For steam-water two-phase flow in the well, specific volume of the two-phase fluid is calculated by Eq.(3) using void fraction:

$$v = \frac{1}{\alpha\rho_s + (1-\alpha)\rho_w} \tag{3}$$

where ρ is the density (kg/m^3), α is the void fraction (-). Subscripts s and w denote steam and water, respectively. Smith's formula is used for calculating α (Smith, 1969-1970). On the other hand, specific volume of liquid water single-phase is used that of saturated water with respect to the saturation pressure at the flash starting point. Steam quality is calculated using specific enthalpies of liquid water and steam of saturated water and steam.

λ is calculated using the Karman-Nikuradse equation valid for turbulent flow in a pipe. λ for two-phase flow is given as 1.1 times as large that for water single-phase flow.

For water single-phase flow which present below the depth of flash starting depth in the well, thermodynamic properties of saturated water with respect to reservoir fluid temperature are given.

Fluid velocity can be calculated as:

$$w=Gv/F$$

In the case of fluid flowing into wellbore at the feed zone as liquid water single-phase, liquid water flows upward from the feed zone and then starts flashing where it reaches its saturation pressure with respect to reservoir fluid temperature. Steam-water two-phase flow presents from this depth up to the wellhead by increasing steam fraction.

Pressure profile of two-phase flow region in the wellbore can be obtained by numerically evaluating Eq.(2) from the wellhead pressure, p_{wh}, to the p_{wh}+dp , dp being increment of pressure, then depth from the wellhead to its pressure is calculated. This process is repeated until the pressure reach the saturation pressure. Below this depth, fluid flow in the well is of water single-phase, then Eq.(2) can be simplified by assuming the specific weight of fluid being constant as that of saturated water.

In order to evaluate well deliverability, a combined model of wellbore flow and reservoir flow is required. Flow in the reservoir is steady state of radial coordinate system. Phase conditions in the reservoir is either 1) liquid-water single-phase, 2) liquid water single-phase followed by steam-water two-phase. The second condition appears as fluid pressure decreases while flowing in the reservoir to the wellbore and drops down to the saturation pressure, then fluid starts flashing and two-phase condition remains until the fluid reaches to the wellbore. In this case, steam-water two-phase fluid flows from the reservoir into the wellbore and flows upward in the wellbore to the wellhead.

As a boundary condition at the well bottom, we need to specify a well bottom pressure and mass flow rate. Here, we introduce calculation method for the mass flow rate.

Darcy velocity in reservoir can be expressed as:

$$u = -\frac{k}{v}\frac{\partial p}{\partial r} \tag{4}$$

where u is the Darcy velocity (m/s), k is the permeability (m^2), ν is the kinematic viscosity (m^2/s) and r is the radial distance (m). Kinematic viscosity for two-phase fluid, ν_t, can be expressed as:

$$\frac{1}{v_t} = \frac{k_{rw}}{v_w} + \frac{k_{rs}}{v_s} \tag{5}$$

where k_{rw} and k_{rs} are the relative permeability of water and steam, respectively (-). ν_w and ν_s are the kinematic viscosity of water and steam, respectively (m^2/s). X curve is used for the relative permeability curve.

Mass flow rate from reservoir into wellbore can be expressed for steam-water two-phase as:

$$G = 2\pi kh\frac{\int_{p_{wb}}^{p_{sat}}\frac{\partial p}{v_t}}{\ln(r_{sat}/r_{wb})} \tag{6}$$

where r_{sat} is the radius where flashing start in reservoir. Integral part in Eq.(6) need to be numerically evaluated as ν_t is a function of relative permeabilities and kinematic viscosities of water and steam which varies with pressure. For liquid water single-phase as:

$$G = \frac{2\pi kh(p_e - p_{wb})}{v_w \ln(\frac{r_e}{r_{wb}})} \tag{7}$$

where p_e is the outer boundary pressure (kg/(ms^2)), r_e is the outer boundary radius (m).

Mass flow rate can be determined by specifying well bottom pressure, p_{wb}, then p_{wb} and G are given as boundary conditions for wellbore flow simulation and an unknown value of well head pressure is numerically calculated using Eq.(2).

We named this simulator as Wellbore Flow Simulator of Kyushu University (WellKyu). WellKyu can be used for simulating pressure and temperature profiles in a production well and for well deliverability. For simulating temperature and pressure profiles, well specification and wellhead conditions are given as input data. For well deliverability simulation, well specification and reservoir conditions are given.

2.1 Example of simulation

2.1.1 Pressure and temperature profiles

Simulation is carried out from wellhead to well bottom by giving input data: well depth, diameter, mass flow rate of steam and water at the wellhead, wellhead pressure, surface roughness of pipe. The simulator can handle two different diameter of the well, D_1 and D_2, and length for respective diameter: H_1 and H_2 as shown in Fig.1. If the well has a single diameter, H_2 is given as 0. Table 1 shows an example of input data for Well A14 with two different size of wellbore diameter, which is given as a text file.

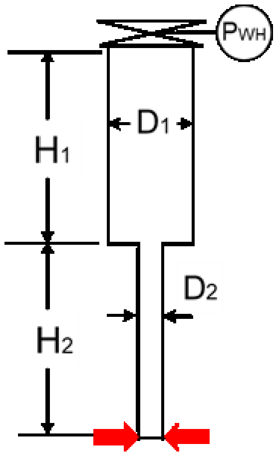


Figure 1: Well diameter and length. Feed zone is located at the well bottom.

Table 1: Input data for pressure and temperature profiles of Well A14.

'A14': Well name

7.6 :Wellhead pressure, p_{wh} (bar)

43: Steam flowrate at wellhead, G_s (t/h)

211 : Water flowrate at wellhead, G_w (t/h)

674 : Length of upper part of well, H_1 (m)

396 : Length of lower part of well, H_2 (m)

0.201 : Diameter of upper part of well, D_1 (m)

0.138 : Diameter of lower part of well, D_2 (m)

0.0457 : Roughness of pipe surface, ε (mm)

Simulated results are output in a text file and should be graphed by using other software such as Excel as shown in Fig.2. Specific enthalpy of produced fluid is calculated using a steam quality at the well head and specific enthalpies of steam and water under the wellhead pressure. Saturation pressure can be found in the steam table for saturated water having this specific enthalpy. In this case, fluid of water single-phase flows into the well from the reservoir at the feed zone at 66.5 bar and flows upward in the wellbore to the flash point depth of 762m at 37.0 bar. Above this depth in the wellbore, steam-water two-phase continues up to the wellhead.

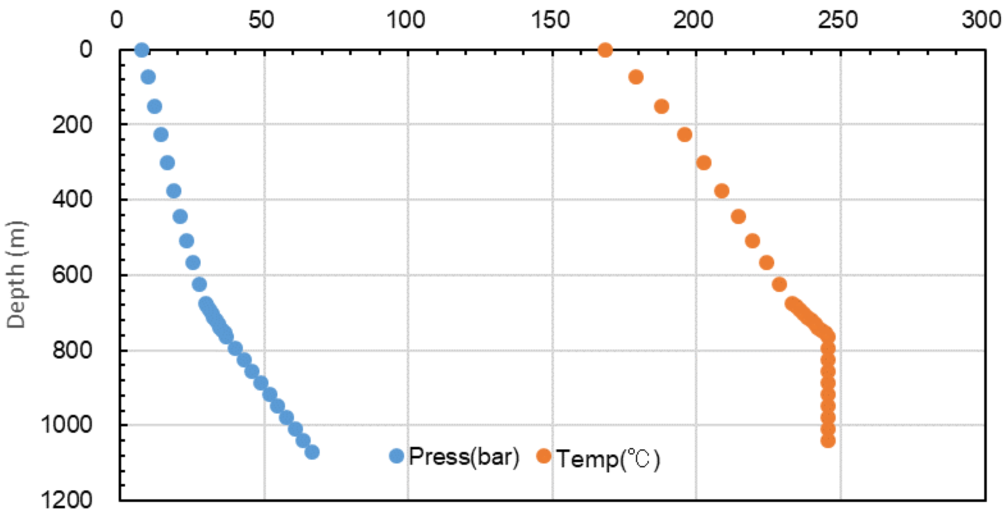


Figure 2: Simulated results for pressure and temperature profiles of Well A14.

2.1.2 Well deliverability

Input data are given in Table 2. In this simulation we give conditions of well and reservoir as input data. For reservoir conditions, following parameter values are given: reservoir pressure (p_r, bar), reservoir fluid temperature (T_r, °C), permeability-thickness product (kh, m^3), initial water saturation in reservoir(S_w, -). For wellbore conditions, lengths of wellbore for respective dimeters, and roughness of wellbore surface are given.

Table 2 Input data for well deliverability.

1000.0,500.0,0.25,0.150:H_1,H_2,D_1,D_2

0.0457: ε

90.0,240.0: p_r, T_r

1.0: S_w

0.5E-11: kh

Other than these values, r_e=500r_w is given. p_e is given the same as p_r.

In the simulation, well bottom pressure, p_{wb}, is automatically specified by subtracting a unit size of pressure from reservoir pressure,p_r then mass flow rate into the wellbore, G, is calculated using either Eq.(6) or Eq.(7). Then, both G and p_{wb} are given as the boundary condition of wellbore flow, and simulation in wellbore is carried out and unknown wellhead pressure is determined so as to satisfy Eq.(2). Simulated results give wellhead pressure, steam and water flowrates, G_s and G_w, at the wellhead. This result expresses well deliverability: relationship between wellhead pressure and flowrates of steam and water as shown in Fig.3.

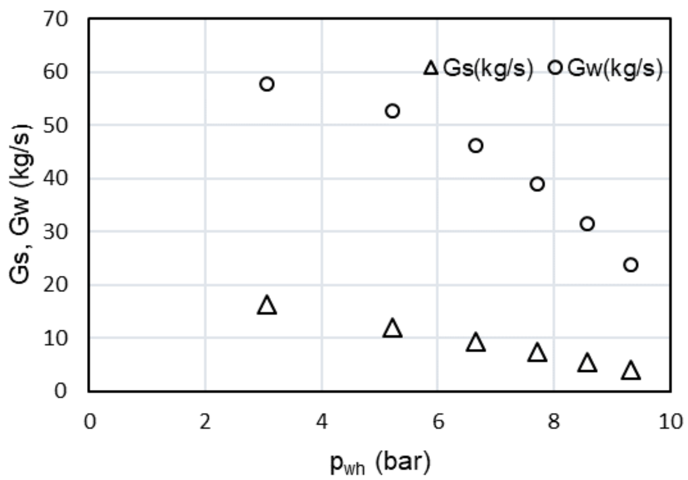


Figure 3: Simulated well deliverability.

3. WELL TEST ANALYSIS SIMULATOR

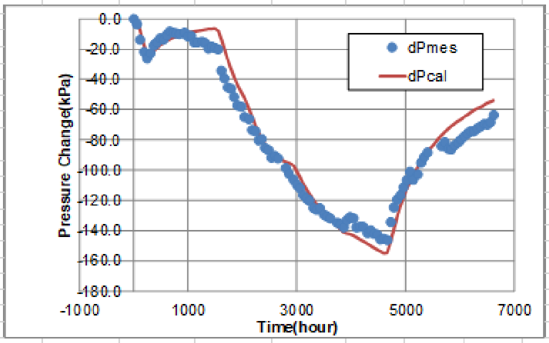
Pressure transients in observation wells due to production and/or reinjection at active well(s) provide reservoir properties such as transmissivity ($T=kh/\mu$, h: thickness of reservoir, μ:viscosity of fluid) and storativity ($S=\phi ch$, ϕ:porosity, c:compressivity) when analyzed. The line source solution on the basis of homogeneous porous media in a radial coordinate system is employed for analysis. The principle of superposition is applied for multiple active wells and their flow rate history. The least squares method is applied for estimating unknown values of T and S by minimizing the residual sums of squares of measured pressure and calculated ones. Microsoft Excel has add-in program, Solver for finding an optimal value for a formula in one cell. This is called the objective cell. We developed a worksheet on which pressure transient data such as observed pressure data, flow rate history of active wells are saved on the cells. The residual sum of squares with respect to transient pressures which is calculated with Excel function of SUMXMYZ, is stored in the objective cell. There are cells for storing optimal values obtained after running Solver (Microsoft).

A worksheet for simulation is presented in Fig.4 and consists of group of cells: Values of T and S for optimal values, objective cell, flow rate history of active wells, pressure change in observation well for measurement and simulated. In this figure, interference well test with one active (production) well was analyzed (Itoi et al., 1993).

	Exponent	Value
Storativity(m/Pa)	-6.897	1.27E-07
Transmissivity(m3/Pa·s)	-7.034	9.25E-08
Residual Sum of Squares	8144.1552	

	Observation Well	Production Well
	WO	P1
X(m)	-1799.8	-1617.54
Y(m)	-1097.5	-2299.8
Distance(m)		1216.04

Flow Rate(m3/hour)			Pressure Change(kPa)			
Time (hour)	P1		Time (hour)	dPmes	dPcal	dPP1
0	0		0	0.0	0.00	0.00
0	-210		64	-3.5	-1.87	-1.87
190.9	-210		124	-13.9	-8.82	-8.82
190.9	-302		184	-21.8	-16.62	-16.62
191.9	-302		244	-28.1	-23.08	-23.08
191.9	-328		304	-22.8	-23.48	-23.48
193	-328		364	-17.5	-21.59	-21.59
193	0		424	-15.4	-19.50	-19.50
1487.5	0		484	-13.0	-17.62	-17.62
1487.5	-195		544	-13.7	-16.01	-16.01
2019	-195		604	-10.8	-14.64	-14.64
2019	-280		664	-8.4	-13.47	-13.47
2292	-280		724	-9.2	-12.47	-12.47
2292	-220		784	-9.4	-11.60	-11.60
2337	-220		844	-9.9	-10.84	-10.84
2337	-280		934	-9.3	-9.87	-9.87
2473	-280		994	-11.8	-9.31	-9.31
2473	-195		1054	-11.4	-8.81	-8.81
2490	-195		1114	-15.4	-8.38	-8.38
2490	-280		1174	-15.5	-7.95	-7.95
2498	-280		1252	-15.2	-7.48	-7.48
2498	-190		1312	-16.1	-7.15	-7.15
2879.5	-190		1372	-19.8	-6.85	-6.85
2879.5	-275		1432	-18.4	-6.58	-6.58
3310	-275		1492	-19.5	-6.32	-6.32
3310	-192		1552	-20.4	-7.88	-7.88
3335	-192		1612	-34.3	-14.12	-14.12
3335	-273		1672	-39.6	-21.16	-21.16
3840	-273		1732	-45.5	-27.73	-27.73
3840	-187		1792	-48.1	-33.65	-33.65
3883	-187		1852	-51.6	-38.98	-38.98
3883	-255		1912	-57.1	-43.79	-43.79
4605	-255		1972	-58.1	-48.16	-48.16
4605	0		2032	-64.8	-52.16	-52.16
4737	0		2118	-66.2	-58.90	-58.90
			2178	-73.5	-64.61	-64.61



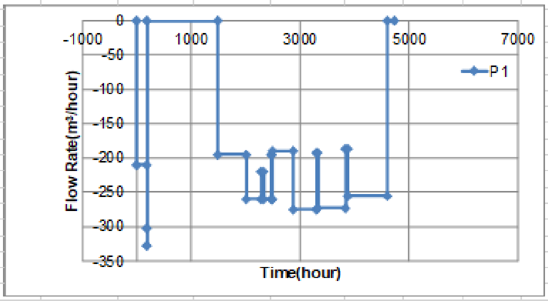


Figure 4: Excel worksheet for pressure interference test analysis

Optimal values of T and S are stored in the cells titled "Exponent" and "Value" as a result of simulation. The objective cell has a title of "Residual sum of squares". Pressure change at an observation well due to production and/or reinjection is calculated using a private function for calculating the exponential integral (Ei) function on Visual Basic as indicated in the column dPP1 for Production well P1. When other production and/or reinjection wells present, pressure change due to these wells are calculated such as dPP2, dPP3 and DPR1 for production wells P2 and P3, and for reinjection well R1, then these pressure changes are summarized for a total pressure change at an observation well as dPcal.

Flow rate should be expressed in a step wise manner in m^3/h and negative for production and positive for reinjection. Well locations are expressed on X-Y coordinate in m and distances between active wells and observation well are calculated. A single well test can be analyzed for T and S by giving well radius as a well distance between active well and observation well, but skin factor and wellbore storage are not to be analyzed.

3.1 Procedure of simulation

After providing all required data on the work sheet, click the Solver command on the Data tab, then the Solver command appears on the screen. In the Set Objective box, enter a cell reference as D5 where the data of the residual sum of squares is stored to Minimum by selecting the Min. button. In the Subject of Constraints box, upper limit of S and T are given by selecting the cell name such as D2 for S being <= 1E-5 and D3 for T being <=1E-5. Then choose the solving method for Generalized Reduced Gradient (GRG) Nonlinear which is used for problems that are smooth nonlinear. The last step is to click the Solve button for solution. Summary of Solver solution can be saved when the minimization was successful. Optimal values of S and T obtained by Solver are indicated in the cell of Value such as 1.53×10^{-7} for S and 8.95×10^{-8} for T as shown in Fig.4. Figure 4 also presents graphs on the comparison between measure and simulated pressures with time. The simulated pressures, dPcal, calculated using the optimal T and S show a fairly good match with measured pressure, dPmes.

4. CONCLUSION

We have developed two numerical simulators for geothermal wellbore flow, WellKyu and pressure well test analysis on Excel as KyuPEx. Wellbore flow model which have been used in the exercise for undergraduate student as well as their research projects. Their validities were presented by analyzing example data.

REFERENCES

Bjornsson,G. and Bodvarsson, G.S.: A Multi-feedzone Wellbore Simulator, Geothermal Resources Council, *Trans.* 11(1987), 503-507

Hadgu,T. and Freeston, D.H.: A Multipurpose Wellbore Simulator, Geothermal Resources Council, *Trans.* 14, PartII (1990), 1279-1286

Hadgu,T., Zimmerman, R.W., Bodvarsson, G.S.: Coupled Reservoir-Wellbore Simulation of Geothermal Reservoir Behavior, *Geothermics*, 24,2,(1995), 145-166.

Itoi, R., Kakihara, Y., Fukuda, M., Koga, A.: Numerical Simulation of Well Characteristics Coupled with Steady Radial Flow in a Geothermal reservoir, Int. Symp. on Geothermal Energy, Kumamoto and Beppu, 201-204(1987)

Itoi, R., Fukuda, M., Jinno, K., Gotoh, H.: Interference Test Analysis at the Takigami Geothermal Field, Japan, *Proceedings*, 18th Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, CA(1993)

Microsoft: https://support.office.com/en-gb/article/define-and-solve-a-problem-by-using-solver-5d1a388f-079d-43ac-a7eb-f63e45925040

Smith,S.L.: Void Fractions in Two-Phase Flow: A Correlation Based upon an Equal Velocity Head Model, *Proceedings*, .Institute Mechanical Engineers, London, 184, Par 1, 36, 647-664