Proceedings World Geothermal Congress 2020+1
Reykjavik, Iceland, April - October 2021

Modelling the Thermal Response of Geothermal Heat Exchangers Using Asymptotic Expansion Techniques

Miguel Hermanns

Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Spain

miguel.hermanns@upm.es

Keywords: geothermal heat exchangers, geothermal boreholes, heat conduction, asymptotic expansion techniques

ABSTRACT

The correct designing and sizing of geothermal heat exchangers plays a crucial role in the successful harnessing of geothermal energy for the energy-efficient heating and cooling of buildings. This designing and sizing is done during the building's design phase using theoretical models to analyze the thermal response of the geothermal heat exchanger during the whole lifespan of the building. Hence, the accuracy, flexibility, and speed of these models have a direct impact onto the analysis and its outcome. Most models in use nowadays exploit certain disparities in time and length scales of the problem to significantly reduce their mathematical complexity, allowing the aforementioned analysis to be performed in feasible amounts of time. Nevertheless, the most accurate and flexible ones (e.g. Superposition Borehole Model) still require several hours or days of computing time, while the faster ones (e.g. *g*-function model) introduce certain artificial simplifying assumptions that hinder their flexibility and accuracy. Since 2011, the author is pursuing a new modeling approach based on the use of asymptotic expansion techniques. These mathematical methods naturally exploit the large disparity in time and length scales, delivering analytical models without the need of artificially introduced simplifying assumptions. The resulting models present the accuracy and flexibility levels of the Superposition Borehole Model with a speed level more in line with the *g*-function model. A brief overview of the on-going work is given in the present paper, and a comparison with the state of the art is carried out to showcase the potential of the models under development.

1. INTRODUCTION

To reduce the environmental impact of mankind's activities, the European Union approved in 2010 the Energy Performance of Buildings Directive that requires all new buildings to be nearly zero-energy by the end of 2020 (European Union, 2010). A building is considered nearly zero-energy when its energy requirements are below certain thresholds, and when the consumed energy is mainly covered by renewable energy sources. For the heating and cooling of such buildings, which accounts for an important fraction of the aforementioned energy requirements, low-enthalpy geothermal energy is often harnessed due to its widespread availability and its high energy-efficiency potential.

A typical geothermal heating, ventilation, and air conditioning (HVAC) system consists in a water-to-water heat pump connected to a geothermal heat exchanger, like the one shown in Figure 1, composed of multiple vertical geothermal boreholes. Each borehole is equipped with several pipes through which a heat carrying liquid flows and exchanges heat with the surrounding ground. The heat carrying liquid flows to/from the heat pump through a network of horizontal pipes, to which boreholes are normally connected in parallel. The geometrical and thermal characteristics of the boreholes and their placement in the available land plot are decided during the building's design phase, for which theoretical models for the thermal response of the geothermal heat exchanger and its surrounding ground are used to forecast its performance during the whole lifespan of the building. Consequently, the accuracy, flexibility, and speed of the employed models have a direct impact onto the design process and its outcome.

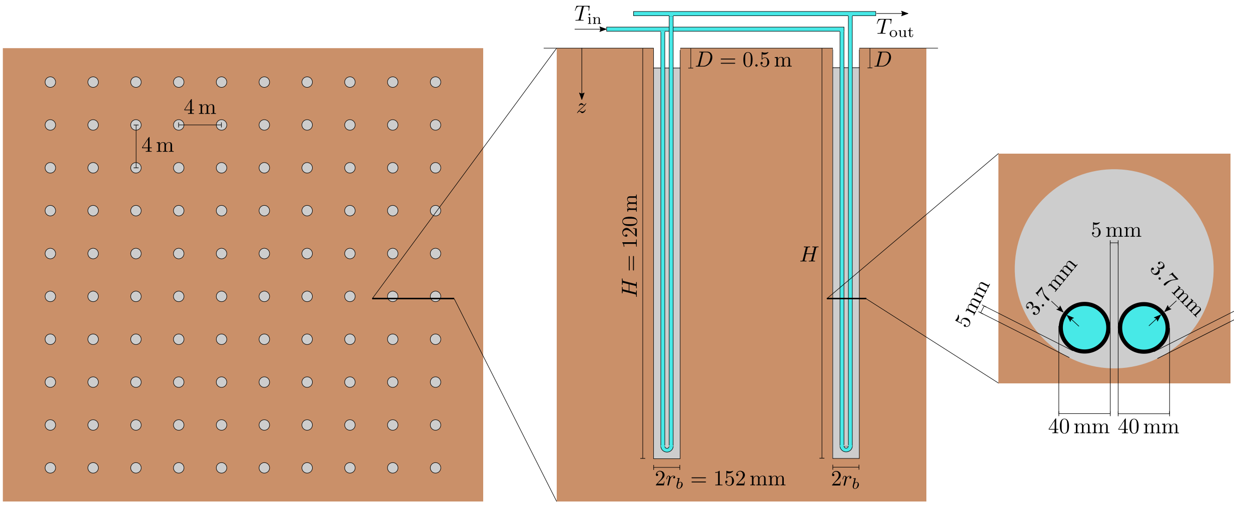

Figure 1: Sketch of a typical geothermal heat exchanger composed of 100 vertical geothermal boreholes arranged in a 10x10 array. Each borehole is equipped with a U-shaped probe and filled up with grout, and a heat carrying liquid is fed to them through a network of horizontal pipes, to which the boreholes are connected in parallel.

Most theoretical models in use nowadays exploit certain disparities in time and length scales of the problem to significantly reduce their mathematical complexity, allowing the aforementioned analysis to be performed in feasible amounts of time. Two of these models deserve special mention as they have dominated the research field for the past 33 years: The Superposition Borehole Model (Eskilson, 1986; Eskilson, 1987; Eskilson and Claesson, 1987) and the g-function model (Eskilson, 1987). While the Superposition Borehole Model (SBM) excels in terms of accuracy and flexibility, the g-function model outstands for its speed, but none of them presents all desired features at the same time. To achieve that, a new modelling approach is required.

Motivated by a large Spanish construction company, *Sacyr Industrial*, the author initiated in 2011 a review from scratch of the complete heat transfer problem that represents the thermal response of a geothermal heat exchanger and its surrounding ground. The triggering idea was the exploit of the large disparity in time and length scales of the problem by means of asymptotic expansion techniques (Kevorkian and Cole, 1981 and 1996; Lagerstrom, 1988). These mathematical methods, which extract the most relevant information from a given problem with large or small parameters, define a set of simplified problems whose sequential solution delivers the sought result.

The theoretical models under development can already be used for the analysis of real-world installations as evidenced by the results presented in Section 3. But significant work remains to be done in order to achieve the desired levels of accuracy in all regimes of operation of the geothermal heat exchanger, as discussed in Section 2. In terms of flexibility, the theoretical models under development are almost on par with the SBM, with their computational cost being, however, significantly lower. As an example, the actual implementation of the developed models requires two minutes of computing time on a single core of an Intel Core i7-7700K processor to simulate 100 years of operation of a real-world geothermal heat exchanger composed of 100 irregularly placed boreholes.

2. REGIMES OF OPERATION OF A GEOTHERMAL HEAT EXCHANGER

Three relevant regimes of operation for a geothermal heat exchanger have been identified and defined as part of the reviewing process initiated in 2011 (Hermanns, 2020). The aim is to develop tailored theoretical models for each of them. The identified regimes of operation have also been mated with the three main criteria driving the designing and sizing of geothermal heat exchangers (Hermanns, 2020). These are the assuring of a prescribed minimum energy efficiency during the whole lifetime of the geothermal HVAC system, the assuring of an acceptable payback time for the additional investment costs incurred by the construction of the geothermal heat exchanger, and the assuring of admissible maximum and minimum temperatures for the heat carrying liquid during the peak heating and cooling demands of the building.

2.1 Long-Term Thermal Response

Most of the work performed so far has been on the first criteria, for which the long-term thermal response of the geothermal heat exchanger is of relevance. Instead of numerically time marching a theoretical model through the whole lifespan of the geothermal HVAC system, as done for instance by the SBM and the g-function model, the author proposed to approximate the long-term thermal response by a time-periodic response (Hermanns, 2020). This is a conservative approach to the design problem, as it represents the thermal response of the geothermal HVAC system after an infinite number of years of operation. The proposed time-periodic approximation also simplifies the modelling work to perform, as the problem at hand can be expanded in Fourier series and solved independently for each harmonic (Hermanns and Ibáñez, 2019b and 2020; Ibáñez and Hermanns, 2018 and 2020).

The Fourier series expansion of a time-periodic variable, for instance the heat injection rate $Q(t)$, is given by

$$Q(t)=\sum_{n=-\infty}^{\infty}\hat{Q}_n\, e^{i\omega_n t}, \tag{1}$$

where $\hat{Q}_n$ is the complex-valued nth harmonic, $i = \sqrt{-1}$, $\omega_n = 2\pi n/\mathcal{T}$ is the angular frequency of the nth harmonic, being $\mathcal{T}$ the one year period, and t is time. The summand of the expansion with $n = 0$ represents a mean annual heat injection rate to which the geothermal heat exchanger and its surrounding ground respond, in the context of the time-periodic approximation, in a steady-state manner. On the other hand, the summands of the expansion with $n \neq 0$ represent harmonic heat injection rates with subannual periods to which the geothermal heat exchanger and its surrounding ground respond in a complex-valued time-harmonic way.

Theoretical models have already been developed for both types of thermal responses. For the case of single vertical boreholes, analytical expressions for their steady-state and time-harmonic thermal responses have been obtained (Ibáñez and Hermanns, 2018; Hermanns and Ibáñez, 2019b). For thermally-interacting boreholes, however, fully analytical expressions have only been obtained for the time-harmonic thermal response (Hermanns and Ibáñez, 2020), whereas semi-analytical results are proposed, for the time being, for the steady-state thermal response of geothermal heat exchangers (Hermanns, 2018). How the developed models are combined to analyze the time-periodic thermal response of a geothermal heat exchanger is explained in Ibáñez and Hermanns (2019).

2.2 Short-Term Thermal Response

The construction of a geothermal heat exchanger entails high investment costs which are afterwards recovered by the property owner through the higher energy savings of the resulting geothermal HVAC system. How long it takes to reach the break-even point is what defines the payback time. In Spain, a payback time of less than 10 years is in general considered acceptable for a geothermal HVAC system. Thus, to assess the economic viability of a given installation it is sufficient to simulate its first 10 to 15 years of operation.

The advantage of having to concentrate only on the first years of operation of a geothermal heat exchanger is that certain simplifications are possible which ease the modelling work to perform. In fact, the asymptotic analysis carried out in Hermanns and Ibáñez (2019b) and Hermanns and Ibáñez (2020) for the time-harmonic thermal response of geothermal heat exchangers can easily be extended to their unsteady thermal response by using the Laplace transform for the time dependence of the problem. In fact, the results shown in Subsection 3.2 for the thermal response of the geothermal heat exchanger are obtained by following this procedure.

Although the achieved accuracy may be acceptable for engineering purposes, especially for the first 10 years of operation of the geothermal HVAC system, it is not satisfactory from a scientific point of view, reason why the results shown in Section 3 are tagged there as “preliminary”. It is possible to improve the theoretical model for the unsteady thermal response of geothermal heat exchangers by further deepening into the heat transfer problem and its mathematical solution. This work has already been completed for the case of a single geothermal borehole (Hermanns, 2021), leading to a model that delivers accurate results for the whole lifespan of the borehole. The extension to multiple thermally-interacting boreholes will hopefully be completed in 2022.

2.3 Transient Thermal Response

The heat carrying liquid in a geothermal HVAC system usually reaches its maximum and minimum operating temperatures during the peak cooling and heating demands of the building, which normally occur during the hottest and coldest moments of the year. Due to safety requirements, structural integrity reasons, and environmental concerns an admissible temperature range is normally defined for the heat carrying liquid. Hence, it is crucial to ensure that the aforementioned maximum and minimum operating temperatures are always inside the prescribed temperature range.

Normally, the described peak heating and cooling demands only last for a few hours or days. This drastically changes the way in which boreholes and their surrounding ground respond, rendering important the thermal inertia of the heat carrying liquid, the grout, and the ground located close to the boreholes (Hermanns, 2020). To analyze and model this thermal response, baptized “transient thermal response”, a completely different approach has to be followed than for the previous two regimes of operation.

An enhanced version of the well-known multipole method (Bennet et al., 1987; Claesson and Hellström, 2011) has been developed at the author’s research group. This enhanced multipole method solves the unsteady heat conduction equation in the grout and the ground taking so into account their thermal inertia in the description of the thermal response of the borehole. Although the corresponding theoretical work has already been completed and its publication is on the way (Rivero and Hermanns, 2021; Hermanns and Rivero, 2021), no results can be shown at the time of writing the present paper.

3. THERMAL RESPONSE OF A GEOTHERMAL HEAT EXCHANGER COMPOSED OF 100 BOREHOLES

To showcase the capabilities of the models developed so far, consider the geothermal heat exchanger represented in Figure 1. Each of the 100 identical boreholes, that are arranged in a 10x10 array with a 4 m spacing between adjacent boreholes, has a total depth of 120 m, a diameter of 152 mm, and a buried depth of 0.5 m. The land plot that hosts the geothermal heat exchanger is located in a geographical location with a mean annual temperature of 14 ºC, a geothermal heat flux of 0.06 W/m^2, and a light-sand type ground with a thermal conductivity of 1.4 W/(m K) and a thermal diffusivity of $8.68\cdot10^{-7}$ m^2/s.

Each borehole contains a U-shaped probe whose pipes present an outer diameter of 40 mm, a wall thickness of 3.7 mm, and a thermal conductivity of 0.42 W/(m K). The placement of the U-shaped probe inside the borehole is as shown in Figure 1, with a gap of 5 mm between the two legs of the U-shaped probe and between each leg and the borehole wall. The remaining space of the borehole is filled with grout up to the aforementioned buried depth. The employed grout has a thermal conductivity of 1.5 W/(m K) and a thermal diffusivity of $4.63\cdot10^{-7}$ m^2/s.

The U-shaped probes are connected in parallel to the network of distribution pipes so that the same inlet temperature results for each borehole. The employed heat carrying liquid is in this case pure water with a density of 999 kg/m^3, a specific heat capacity of 4184 J/(kg K), a thermal conductivity of 0.577 W/(m K), and a dynamic viscosity of $1.138\cdot10^{-3}$ kg/(m s). The mass flow rate of each borehole is 0.150 kg/s, which is enough to keep the flow turbulent inside the U-shaped probes.

In order to meet the heating and cooling needs of the building, the HVAC system imposes onto the geothermal heat exchanger the heat injection rates given in Table 1. For the sake of simplicity, constant values are assumed for each month and the proposed heat injection rates are repeated every year (365 days) for the whole lifespan of the building.

Table 2: Heat injection rates imposed onto the geothermal heat exchanger by the HVAC system of the building. For simplicity, constant values are assumed for each month, which become negative when the heat is extracted from the ground.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Q [kW]	-200	-175	-100	-25	75	175	250	200	125	25	-75	-175

Using Gnielinski’s correlations for the evaluation of convective heat transfer coefficients (Gnielinski, 2015), the formula in Hermanns and Pérez (2014) for the computation of pipe’s inner thermal resistances, and the multipole method for the determination of the borehole’s network of thermal resistances (Bennet et al., 1987; Claesson and Hellström, 2011; Hermanns and Pérez, 2014; Ibáñez and Hermanns, 2018), the following values for the borehole’s outer thermal resistance R_b, borehole’s inner thermal resistance R_a, and borehole’s thermal skewness parameter S result:

$$R_a = 0.353(\mathrm{m K})/\mathrm{W}, \qquad R_b = 0.145(\mathrm{m K})/\mathrm{W}, \qquad S = 0. \tag{2}$$

The just described test case is analyzed next using a large selection of theoretical models. A brief description of the chosen models is given in Subsection 3.1 followed by a discussion of the obtained results in Subsections 3.2 and 3.3. All considered models will employ the very same input parameters, including the just computed values for the thermal resistances R_a and R_b and for the thermal skewness parameter S.

3.1 Considered Models for the Comparison

The theoretical models used for the analysis of the described test case can be classified into three large groups according to the modelling approach/philosophy they follow.

3.1.1 Superposition Borehole Model

The first modelling approach to consider is the one followed by the Superposition Borehole Model (Eskilson, 1987; Eskilson and Claesson, 1987). It consists in solving for each borehole the axisymmetric unsteady heat conduction equation in the ground and particularize its outcome at the borehole wall to obtain the so-called mean azimuthal borehole wall temperature T_b that varies along the borehole. The influence of adjacent boreholes is obtained through superposition of their solutions at the time of computing this temperature. The resulting mean azimuthal borehole wall temperature is then used to solve the quasi-steady energy conservation equations that describe the heat transfer inside each borehole and in the ground located close to each borehole.

Three different realizations of the described modelling approach are used in the present model comparison. The first one is the original implementation due to Eskilson (Eskilson, 1986). It uses first-order finite differences to numerically discretize the spatial derivatives of the energy conservation equations, and an explicit Euler scheme for the time marching of the resulting system of ordinary differential equations. To ensure grid independence in the results, mesh refinement studies have been performed leading to a vertical mesh with NZMESH = 15 and a radial mesh with 103 elements and a minimum size of 0.1 m. A slightly improved version of the original FORTRAN code is used here that allows the thermal resistances R_a and R_b to be directly specified as input parameters to the program (Pahud et al., 1996; Pahud, 2012), allowing so the very same values for R_a and R_b to be used by all models to compare.

The other two realizations of the Superposition Borehole Model (SBM), implemented by the author, are based on the superposition of point sources of heat to solve for each borehole the axisymmetric unsteady heat conduction equation in the ground. This approach, that has also been used by others (Cimmino, 2015, 2018a, and 2018b), is not mathematically equivalent to the original one, but the numerical differences are very small (Cimmino and Bernier, 2014; Lamarche, 2017; Lazzarotto and Björk, 2016). Two realizations of the SBM have been developed and implemented, one for each regime of operation of the geothermal heat exchanger. For the short-term thermal response, the Laplace transform has been used to handle the time dependence of the problem (Hermanns, 2019). For the long-term thermal response, which is assumed to be time-periodic, a Fourier series expansion of the problem is used instead (Hermanns, 2018; Hermanns and Ibáñez, 2020; Ibáñez and Hermanns, 2020). Grid independence in the results has been enforced in both cases.

3.1.2 g-Function Model

The second modelling approach to consider is the one followed by the g-function model (Eskilson, 1987). It consists in uncoupling the unsteady heat conduction problem in the ground from the quasi-steady heat transfer problem inside and around the boreholes through the introduction of a convenient, but artificial, simplification: A certain quantity is assumed to be uniform along the boreholes. The advantage of this modelling approach is that the computationally expensive unsteady thermal response of the ground, known in the literature as g-function, becomes universal. Hence, it only needs to be computed once for each spatial arrangement of boreholes, which is what provides the g-function model with its speed advantage.

Six different realizations of the described modelling approach are used in the present model comparison. The first one is the original realization due to Eskilson (Eskilson, 1987). It computes the aforementioned g-functions using a modified version of the original SBM in which the mean azimuthal borehole wall temperature is forced to be uniform along the boreholes. This is accomplished by setting LOMODE = 2, instead of LOMODE = 5, in the input file for the program (Eskilson, 1987; Blocon, 2019b). The quasi-steady heat transfer problem inside and around the boreholes is then taken into account through Hellström's *effective thermal resistance* R_b^* (Hellström, 1991). The value of R_b^* results from solving the governing equations in the boreholes. This can be done analytically thanks to the introduced assumption of uniform mean azimuthal borehole wall temperature T_b or uniform heat injection rate per unit borehole length q (Hellström, 1991). For the present comparison, the assumption of uniform T_b is used. Last, the influence of the geothermal heat flux is taken into account by letting the reference temperature for the ground to be equal to the unperturbed ground temperature at the mid-depth of the borehole, as proposed by Eskilson in his seminal Ph.D. Thesis (Eskilson, 1987).

The second realization of the g-function model is the well-known commercial simulation tool Earth Energy Designer (Blocon, 2010). In it, the values given in (2) for the thermal resistances R_a and R_b are specified through the "Borehole thermal resistance" dialog in which also the inclusion of the internal heat transfer between the upward and downward pipes is selected. From the results returned by the program, the inlet temperature to the geothermal heat exchanger is computed following Blocon (2019a).

The remaining realizations of the g-function model compute the g-functions using a modified version of the author's SBM and take then into account the heat transfer inside and around the boreholes through the analytical solution to the corresponding governing equations (Hermanns and Ibáñez, 2020). When computing the g-function, two different options are considered for the uniform quantity along the boreholes: The mean azimuthal borehole wall temperature T_b or the heat injection rate per unit borehole length q. Both choices are common in the literature (Cimmino and Bernier, 2014; Priarone and Fossa, 2016). When combined with the two regimes of operation of the geothermal heat exchanger, and thus with the two realizations of the author's SBM, four additional realizations for the g-function model result.

3.1.3 Asymptotic Solutions

The third modelling approach to consider is the one followed by the author. It consists in using asymptotic expansion techniques to exploit the presence of large and/or small parameters in the problem, allowing so the derivation of analytical or semi-analytical expressions for the thermal response of geothermal heat exchangers. As explained in Section 2, tailored theoretical models have been developed, or are in development, for the different regimes of operation of a geothermal heat exchanger. For the present model comparison, the short-term and long-term thermal responses of the described test case will be considered, so that the theoretical models presented in Subsections 2.1 and 2.2 will be used.

3.2 Short-Term Thermal Response

Figure 2 shows the inlet temperature T_{in} to the considered geothermal heat exchanger during its first 10 years of operations, assuming the commissioning of the building takes place on a January 1st. As explained in Subsection 2.2, this is the relevant period of time for the assessment of the economic viability of the geothermal HVAC system. Clearly to see in the plotted results are the monthly variations in the inlet temperature, caused by the monthly changes in the heating and cooling needs of the building, and the year-over-year increase in the inlet temperature, caused by the 73 MWh per year of net injection of thermal energy into the ground.

More interesting than the aforementioned temperature variations, at least for the present work, is the comparison between the results of the different theoretical models. First of all, minimal differences exist between the original SBM due to Eskilson (black line) and the author's SBM (blue line), making the corresponding curves almost indistinguishable in Figure 2. The same occurs with the results obtained using the g-function model due to Eskilson and Hellstöm (cyan line) and the author's g-function model with uniform mean azimuthal borehole temperature T_b along the boreholes (red line). These two conclusions are very important as they validate the author's understanding of the state of the art and his implementations of it, which are used throughout his papers to assess the merits of the new models under development.

The results in Figure 2 also evidence that the assumption of uniform mean azimuthal borehole wall temperature T_b along the boreholes (red line) leads to more accurate results for the g-function model than the assumption of uniform heat injection rate per unit borehole length q along the boreholes (green line). The observed discrepancies between the two assumptions have already been reported in the literature (Cimmino and Bernier, 2014; Malayappan and Spitler, 2013; Monzó et al., 2013; Priarone and Fossa, 2016). Thus, the assumption of uniform mean azimuthal borehole wall temperature T_b along the boreholes is to be preferred, except when the buried depth of the boreholes is zero in which case it leads to a mathematically ill-posed problem (Hermanns and Ibáñez, 2019a).

Unexpected are the results returned by the Earth Energy Designer (EED), represented by black hollow circles in Figures 2 and 3. In theory, they should be very close to the results obtained from using the g-function model due to Eskilson and Hellström (cyan line). This, however, is not happening and further work is required to figure out the reason for the observed differences.

Finally, the actual state of the author's model for the short-term thermal response of geothermal heat exchangers (orange line) performs worse than the g-function model with uniform mean azimuthal borehole wall temperature T_b along the boreholes (red line), but better than the g-function model with uniform heat injection rate per unit borehole length q along the boreholes (green line). When compared to the SBM (blue line), the model under development presents a difference of less than 1 ºC in the considered period of time, which is acceptable for engineering purposes. Nevertheless, this difference is not satisfactory from a scientific point of view, reason why the actual state of the model is tagged as "preliminary". Further theoretical work is on the way to improve its accuracy.

3.3 Long-Term Thermal Response

Figure 3 shows the inlet temperature T_{in} to the considered geothermal heat exchanger during its 100th year of operation, assuming again the commissioning of the building takes place on a January 1st. As explained in Subsection 2.1, this is the relevant period of time for the assessment of the lowest energy efficiency sustained by the geothermal HVAC system. After 100 years of operation, the year-over-year increase in the inlet temperature has almost reached its maximum, which is around 5 ºC above the inlet temperature of the first year of operation (see Figure 2).

A spreading of the results returned by the considered theoretical models is observed in Figure 3, caused by the accumulation of their small differences over the 100 years of operation of the geothermal heat exchanger. Despite that, the results returned by the original SBM (black line) and by the author's SBM (blue line) are still very close to each other, with an observed difference of less than 0.1 ºC. The same applies to the results returned by the g-function model due to Eskilson and Hellström (cyan line) and by the author's implementation of the g-function model with uniform mean azimuthal borehole wall temperature T_b along the boreholes (red line).

The aforementioned spreading raises to almost 0.6 ºC the observed differences between the original SBM (black line) and the author's g-function model with uniform mean azimuthal borehole wall temperature T_b along the boreholes (red line). Using Eskilson's SBM (black line) as benchmark, the observed differences are also higher for the remaining models. So, the observed differences reach 4 ºC for the g-function model with uniform heat injection rate per unit borehole length q along the boreholes (green line) and 3 ºC for the author's preliminary model for the short-term thermal response of geothermal heat exchangers (orange line).

Represented with dashed lines are also the results obtained by assuming the long-term thermal response of the geothermal heat exchanger to be time-periodic. As explained in Subsection 2.1, this assumption is equivalent to looking at the thermal response of the geothermal HVAC system after an infinite number of years of operation. Consequently, the shown results represent the outcome of the considered models after such a long period of time. This explains, for instance, why the time-periodic assumption worsens the accuracy of the g-function models (dashed red and green lines), as the validity of their simplifying assumptions decreases over time.

When comparing the inlet temperatures returned by Eskilson's SBM (black line) and by the author's time-periodic SBM (dashed blue line), differences of only 0.3 ºC are observed. This means two things. First, the geothermal heat exchanger has almost reached a time-periodic behavior after 100 years of operation. Second, the proposed time-periodic approximation is a convenient modelling choice as it delivers accurate results while at the same time avoids the computationally-expensive time marching of a theoretical model through the whole lifespan of the geothermal HVAC system.

Finally, the outcome of the author's time-periodic model for the long-term thermal response of geothermal heat exchangers is discussed. The computed inlet temperature, represented by a dashed orange line, perfectly overlaps with the results obtained from the author's time-periodic SBM (dashed blue line). Thus, when compared to Eskilson's SBM (black line), the observed differences are of 0.3 ºC as well, which demonstrates the accuracy potential of the modelling approach being pursued by the author.

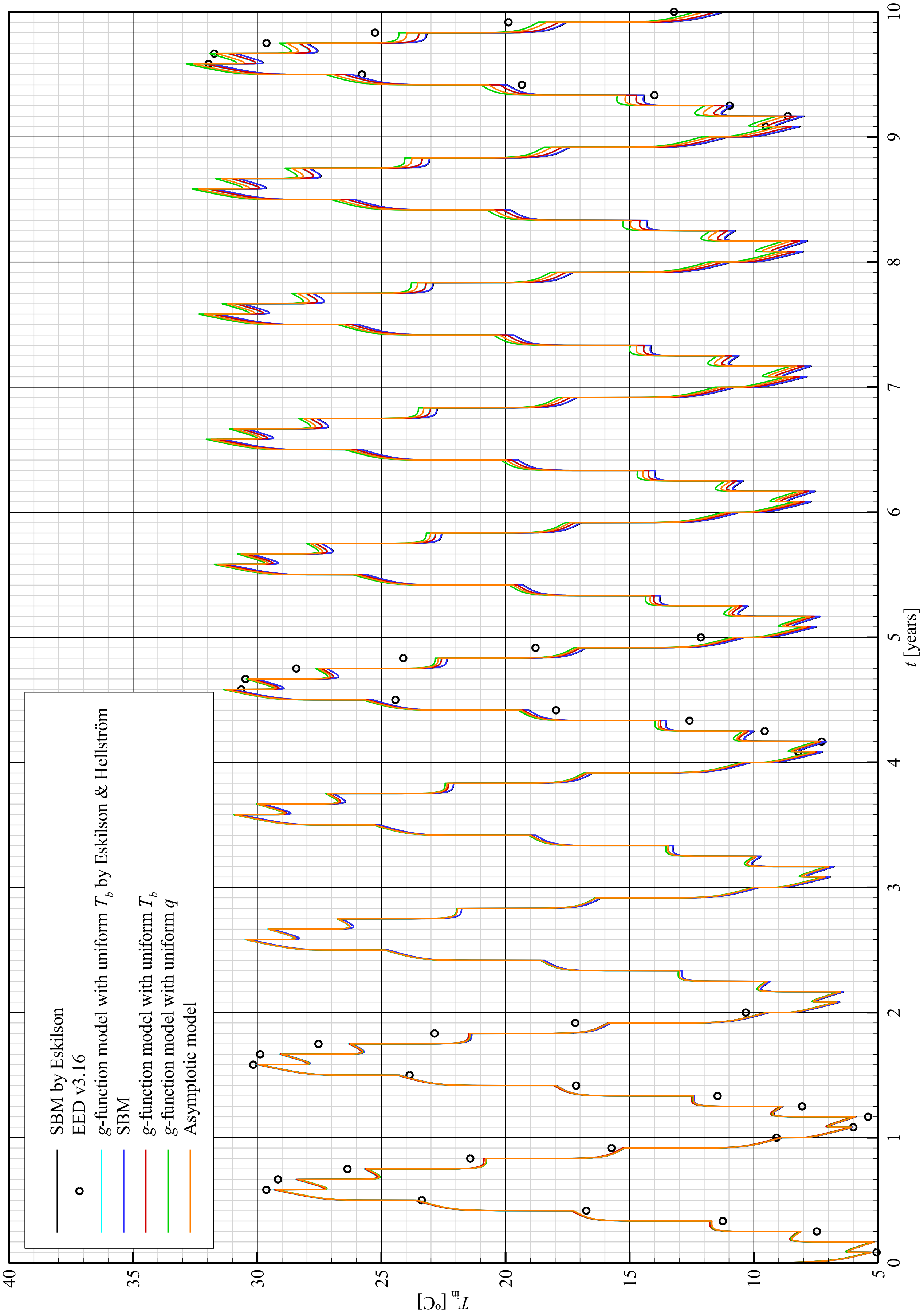

Figure 2: Inlet temperature T_{in} to the geothermal heat exchanger during its first 10 years of operation, assuming the commissioning of the building takes place on a January 1st. As explained in the text, this is the relevant period of time for the assessment of the economic viability of the geothermal HVAC system. Shown are the results returned by the different theoretical models being compared.

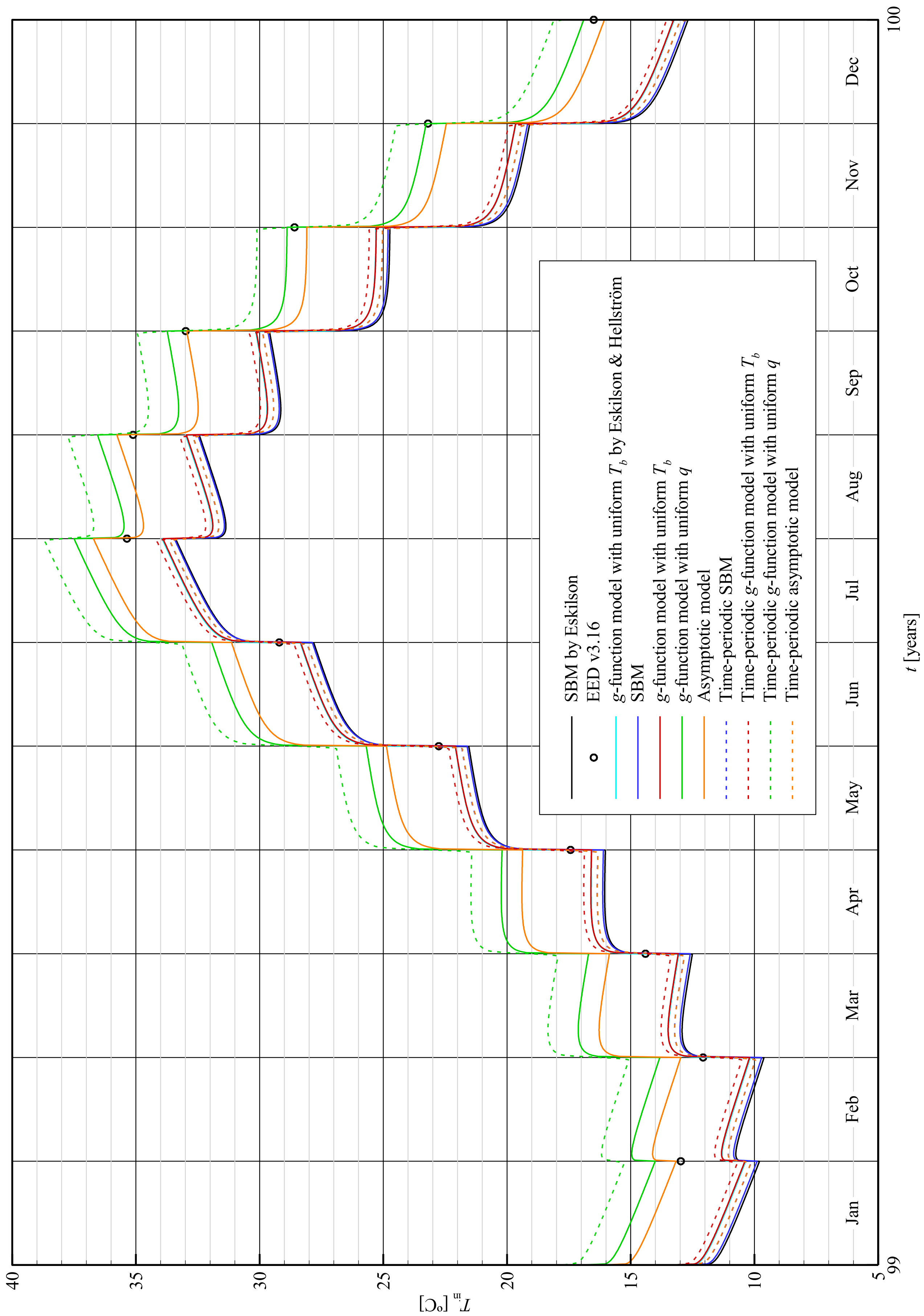

Figure 3: Inlet temperature T_{in} to the geothermal heat exchanger during its 100th year of operation, assuming the commissioning of the building takes place on a January 1st. As explained in the text, this is the relevant period of time for the assessment of the lowest energy efficiency sustained by the geothermal HVAC system. Shown are the results returned by the different theoretical models being compared.

3.4 Computational Cost

Also the computational cost of the theoretical models under development shall be mentioned. The results presented in the previous two subsections, which were obtained without exploiting any similarities between the boreholes or any spatial symmetries in the geothermal heat exchanger, required 125 seconds of computing time on a single core of an Intel Core i7-7700K processor. While the time-periodic results corresponding to the long-term thermal response of the geothermal heat exchanger required 27 seconds, the theoretical model for the short-term thermal response required 98 seconds.

4. CONCLUSIONS

The successful harnessing of low-enthalpy geothermal energy, for the energy-efficient heating and cooling of buildings, strongly relies upon the correct designing and sizing of the geothermal heat exchanger during the building's design phase. Theoretical models are used for that which serve to forecast the thermal response of the envisioned geothermal heat exchanger and its surrounding ground during the whole lifespan of the building. Therefore, the accuracy, flexibility, and speed of the employed models have a direct impact onto the aforementioned design process and its outcome.

To achieve the desired levels of accuracy, flexibility, and speed, the author started in 2011 a review from scratch of the whole heat transfer problem underlying the thermal response of geothermal heat exchangers and their surrounding grounds. With the aid of asymptotic expansion techniques, the presence of small and/or large parameters in the problem is exploited to obtain analytical, or semi-analytical, expressions for the solution to the heat transfer problem of interest.

A brief overview of the author's work has been given in the present paper, including a description of the models developed so far, and of the ones still under development, and their mating with the different regimes of operation of geothermal heat exchangers. To showcase the capabilities of the pursued modelling approach, a comparison against the state of the art has been presented as well. A total of 11 theoretical models have been pitted against each other using as benchmark the thermal response of a geothermal heat exchanger composed of 100 boreholes. Included in the comparison are the original Superposition Borehole Model, developed and implemented by Eskilson, and the commercially available Earth Energy Designer.

The performed comparison reveals that the theoretical models under development are already competitive in terms of accuracy, with temperature discrepancies below 1 ºC when using Eskilson's Superposition Borehole Model as benchmark. Also their speed is already competitive with a total computing time of 125 seconds on a single core of an Intel Core i7-7700K processor. The quoted computing time is for the thermal response of a geothermal heat exchanger composed of 100 irregularly-placed heterogeneous boreholes over a period of time of 100 years of continuous operation.

ACKNOWLEDGEMENTS

This work was funded by the Spanish *Ministerio de Ciencia, Innovación y Universidades* through Project ENE2015-68703-C2-2-R.

REFERENCES

Bennet, J., Claesson, J., and Hellström, G.: *Multipole method to compute the conductive heat transfer flows to and between pipes in a composite cylinder*, technical report, Department of Building Technology and Mathematical Physics, Lund Institute of Technology, Lund, Sweden (1987).

Blocon AB: *Earth Energy Designer Version 3.16*, Lund, Sweden (2010).

Blocon AB: *Earth Energy Designer Version 4.20 – Update Manual*, Lund, Sweden (2019a).

Blocon AB: *Earth Energy Designer Version 4.20 – Frequently Asked Questions*, Lund, Sweden (2019b).

Cimmino, M.: The effects of borehole thermal resistances and fluid flow rate on the *g*-functions of geothermal bore fields, *International Journal of Heat and Mass Transfer*, **91**, (2015), 1119-1127.

Cimmino, M.: A finite line source simulation model for geothermal systems with series- and parallel-connected boreholes and independent fluid loops, *Journal of Building Performance Simulation,* **11**, (2018a), 414-432.

Cimmino, M.: Fast calculation of the *g*-functions of geothermal borehole fields using similarities in the evaluation of the finite line source solution, *Journal of Building Performance Simulation,* **11**, (2018b), 655-668.

Cimmino, M., and Bernier, M.: A semi-analytical method to generate *g*-functions for geothermal bore fields, *International Journal of Heat and Mass Transfer*, **70**, (2014), 641-650.

Claesson, J., and Hellström, G.: Multipole method to calculate borehole thermal resistances in a borehole heat exchanger, *HVAC&R Research*, **17**, (2011), 895-911.

Eskilson, P.: *Superposition Borehole Model, Manual for Computer Code*, Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden (1986).

Eskilon, P.: *Thermal Analysis of Heat Extraction Boreholes*, Ph.D. thesis, Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden (1987).

Eskilson, P., and Claesson, J.: Simulation model for thermally interacting heat extraction boreholes, *Numerical Heat Transfer*, **13**, (1987), 149-165.

European Union: Directive 2010/31/EU of the European Parliament and the Council of 19 May 2010 on the energy performance of buildings, Brussels, Belgium (2010).

Gnielinski, V.: Turbulent heat transfer in annular spaces – a new comprehensive correlation, *Heat Transfer Engineering*, **36**, (2015), 787-789.

Hellström, G.: *Ground Heat Storage: Thermal Analyses of Duct Storage Systems, I. Theory*, Ph.D. thesis, Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden (1991).

Hermanns, M.: An order 10^2 speedup in the computation of the steady-state thermal response of geothermal heat exchangers, *AIP Conference Proceedings*, **2040**, (2018), 150002.

Hermanns, M.: Fast inverse Laplace transform for the unsteady thermal response of geothermal heat exchangers, *AIP Conference Proceedings*, **2186**, (2019), 170002.

Hermanns, M.: New generation of theoretical models for the thermal response of geothermal heat exchangers, *IOP Conference Series: Earth and Environmental Science*, **410**, (2020), 012042.

Hermanns, M.: Unsteady thermal response of slender geothermal boreholes in the limit of slowly varying heat injection rates, *In preparation for SIAM Journal on Applied Mathematics*, (2021).

Hermanns, M., and Ibáñez, S.: On the ill-posedness of the *g*-function model for the thermal response of geothermal heat exchangers, *International Journal of Thermal Sciences*, **138**, (2019a), 285-292.

Hermanns, M., and Ibáñez, S.: Thermal response of slender geothermal boreholes to subannual harmonic excitations, *SIAM Journal on Applied Mathematics*, **79**, (2019b), 230-256.

Hermanns, M., and Ibáñez, S.: Harmonic thermal response of thermally-interacting geothermal boreholes, *SIAM Journal on Applied Mathematics*, **80**, (2020), 262-288.

Hermanns, M., and Pérez, J.M.: Asymptotic analysis of vertical geothermal boreholes in the limit of slowly varying heat injection rates, *SIAM Journal on Applied Mathematics*, **74**, (2014), 60-82.

Hermanns, M., and Rivero, J.M.: On the symmetry properties of the network of thermal resistances representing the thermal response of slender geothermal boreholes, *Geothermics*, **94**, (2021), 102078.

Ibáñez, S., and Hermanns, M.: On the steady-state thermal response of slender geothermal boreholes, *SIAM Journal on Applied Mathematics*, **78**, (2018), 1658-1681.

Ibáñez, S., and Hermanns, M.: Analysis of the long-term thermal response of geothermal heat exchangers by means of asymptotic expansion techniques, *Science and Technology for the Built Environment*, **26**, (2020), 400-413.

Kevorkian, J., and Cole, J.D.: *Perturbation Methods in Applied Mathematics*, Springer, New York (1981).

Kevorkian, J., and Cole, J.D.: *Multiple Scale and Singular Perturbation Methods*, Springer, New York (1996).

Lagerstrom, P.A.: *Matched Asymptotic Expansions: Ideas and Techniques*, Springer, New York (1988).

Lamarche, L.: *g*-function generation using a piecewise-linear profile applied to ground heat exchangers, *International Journal of Heat and Mass Transfer*, **115**, (2017), 354-360.

Lazzarotto, A., and Björk, F.: A methodology for the calculation of response functions for geothermal fields with arbitrarily oriented boreholes – part 2, *Renewable Energy*, **86**, (2016), 1353-1361.

Malayappan, V., and Spitler, J.D.: Limitations of using uniform heat flux assumptions in sizing vertical borehole heat exchanger fields, *Proceedings*, Clima 2013 World Conference, Prague, Czech Republic (2013).

Monzó, P., Acuña, J., Fossa, M., and Palm, B.: Numerical generation of the temperature response factors for a borehole heat exchanger, *Proceedings*, European Geothermal Congress 2013, Pisa, Italy (2013).

Pahud, D., Fromentin, A., and Hadorn, J.C.: *The Superposition Borehole Model for TRNSYS (TRNSBM), User manual for the November 1996 version*, internal report, Laboratoire de Systemes Energetiques, École Polyechnique Fédérale de Lausanne, Lausanne, Switzerland (1996).

Pahud, D.: *The Superposition Borehole Model for TRNSYS 16 or 17 (TRNSBM), User manual for the April 2012 version*, internal report, Dipartimento Ambiente Costruzioni e Design, Scuola Universitaria Professionale della Svizzera Italiana, Lugano, Switzerland (2012).

Priarone, A., and Fossa, M.: Temperature response factors at different boundary conditions for modelling the single borehole heat exchanger, *Applied Thermal Engineering*, **103**, (2016), 934-944.

Rivero, J.M., and Hermanns, M.: Enhanced multipole method for the transient thermal response of slender geothermal boreholes, *International Journal of Thermal Sciences,* **164**, (2021), 106531.