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Cogeneration of Hydro and Geothermal Power

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ABSTRACT

This paper describes the formulation of an optimization problem for the cogeneration of hydro and geothermal resources. The principal elements of the problem are described and formulations for shadow prices (steam values) are derived. A discussion follows with insights into the interplay between the two resource types and the details of what affects the steam values. A vision for an altered role of geothermal energy in future energy systems is presented and linked to the need for better understanding of the variable cost of geothermal energy.

This analysis shows that some flexibility in geothermal power production leads to lower operational cost (less need for make-up wells or pumping) and better use of hydro resources. This is particularly relevant in power systems where hydro spillage is common (e.g. the Icelandic power system). In power systems where wind and solar power generate a large share of the supply, geothermal energy can become of major importance as a load-following resource to balance supply and demand.

1. INTRODUCTION

Traditionally geothermal power plants are designed to run on fixed load (baseload) rather than following demand in the market. This means that electricity from geothermal plants is generally bid into energy markets at low prices to ensure that it is all sold. With the rise of other renewable energy options, such as wind and solar, geothermal producers have run into trouble selling their product as those other renewable sources sometimes have priority into the market or are simply selling energy for a lower price (after all wind and solar energy is not resource limited and thus the variable production cost is close to 0). As the geothermal power producers are in the process of adapting to these changes, by strengthening their load variation controls and strengthening their understanding of the cost of variable production.

Electrical energy in Iceland is produced mainly from hydro (~70%) and geothermal (~30%) power plants. In most cases the hydro plants take care of load balancing and the geothermal plants run at baseload. Recently, however, Landsvirkjun has started using geothermal power plants for longer term (weeks or months) load balancing when the status of nearby hydro plants is favorable (in periods of foreseeable spillage). This results in more efficient use of hydro resources and the geothermal resource is conserved longer for future generations. It also results in lower production cost for the geothermal resources as well decline will be slower, thus reducing need for investment in make-up wells. Although recently put into practice, this idea is not entirely new, as records of similar thoughts can be found in reports by Ingimarsson, (1987) and Stefánsson & Elíasson (1997).

One can speculate about the reason for these ideas not gaining further ground. In Iceland it may be because until recently Landsvirkjun has focused mostly on hydro power, while geothermal power has mostly been generated by other power companies. With no real market platform to trade power between the companies, the incentive for utilizing this benefit has been lost. However, there are energy systems where one would expect it to be important to utilize this opportunity further, for example in New Zealand. But, for some reason, geothermal electricity continues to be supplied to the NZ market at very low prices to ensure stable baseload production. A possible reason for this could be that the variable cost of geothermal production never becomes high enough to justify reducing the load. But understanding the variable cost of production from geothermal resources is not an entirely trivial task, an more elements need to be considered than first meet the eye. Deepening this understanding is the principal element of this paper.

2. FORMULATION

The objective in the cogeneration of hydro and geothermal resource problem, as defined here, is to find the production strategy that maximizes the profit of utilizing these resources over a given time period. A mathematical formulation that facilitates an efficient solution of this problem follows.

2.1 Objective Function

The objective function is the profit of the operation, i.e. the sum of the revenues less the costs. The income for each time period is modeled by the function

$$I\left(P_i, E_i^H, E_i^G\right) = P_i \left(E_i^H + E_i^G\right)\Delta t \tag{1}$$

where P_i, E_i^H and E_i^G denote the energy price and the production rate for the hydro and geothermal resource over period i=(1,2,3...), which starts at time $(i-1)\Delta t$ and ends at time $i\Delta t$.

The operation and maintenance (O&M) cost for the hydro power is modeled as a linear function of production rate, i.e.

$$C_{OM}^H\left(E_i^H\right) = \left(c_f^H + c_E^H E_i^H\right)\Delta t \tag{2}$$

The operation and maintenance (O&M) cost for the geothermal power has a fixed element, but also depends on the number of make-up wells required, the number of wells in operation and the production rate.

$$C_{OM}^{G}\left(N_i^G, N_{i-1}^G, E_i^G\right) = \left(c_f^G + c_N^G N_i^G + c_E^G E_i^G\right)\Delta t + c_W^G\left(N_i^G - N_{i-1}^G\right) \tag{3}$$

In the equations above the c denotes the various unit costs and N_i^G denotes the number of wells operating over time period i.

With these definitions of cost and income the profit function becomes

$$\Pi = \sum_{i=1}^{n}\left[I\left(P_i, E_i^H, E_i^G\right) - C_{OM}^{H}\left(E_i^H\right) - C_{OM}^{G}\left(N_i^G, N_{i-1}^G, E_i^G\right)\right]\mathrm{e}^{-\mathrm{r(i-1)\Delta t}}$$

$$= \sum_{i=1}^{n}\left[\left(P_i\left(E_i^H + E_i^G\right) - c_f^H - c_E^H E_i^H - c_f^G - c_N^G N_i^G - c_E^G E_i^G\right)\Delta t - c_w\left(N_i^G - N_{i-1}^G\right)\right]\mathrm{e}^{-\mathrm{r(i-1)\Delta t}} \tag{4}$$

The integer n denotes the number of time periods (Δt) considered and the discount factor for the investment is given by r.

2.2. Constraints

The profit function is to be maximized, but it is bounded by some physical constraints. First, an energy balance must be maintained for the reservoirs. This is formulated in terms of an energy "stock" (Juliusson & Axelsson 2018).

For hydro resources the following formulation is used

$$S_i^H = S_{i-1}^H - E_i^H \Delta t - Y_i^H \Delta t + R_i^H \Delta t \tag{5}$$

where Y_i^H and R_i^H denote the spillage and the recharge for the hydro resource, respectively. The recharge would typically be a given time series based on historical inflow measurements to the reservoir.

The formulation used for the geothermal resource is

$$S_i^G = S_{i-1}^G - E_i^G \Delta t + R^G\left(S_{i-1}^G\right)\Delta t$$

$$= S_{i-1}^G - E_i^G \Delta t + R_{max}^G \frac{S_{max}^G - S_{i-1}^G}{S_{max}^G}\Delta t \tag{6}$$

where $R^G\left(S_{i-1}^G\right)$ denotes the recharge function for the geothermal resource, which is approximated here by the linear function $\frac{R_{max}^G\left(S_{max}^G - S_{i-1}^G\right)}{S_{max}^G}$.

The power plants will constrain on the production rate as given by the following inequalities

$$E_i^H \leq E_{max}^H \tag{7}$$

$$E_i^H \geq E_{min}^H \tag{8}$$

$$E_i^G \leq E_{max}^G \tag{9}$$

$$E_i^G \geq E_{min}^G \tag{10}$$

The hydro reservoir would also have a restriction on the maximum energy stock that it can hold before spillage occurs

$$S_i^H \leq S_{max}^H \tag{11}$$

The geothermal resource has a restriction which relates to the power capacity of the available wells. This is represented by assuming that all wells provide the same amount of power at each time, and the power capacity depends on the amount of stock available in the resource. Here a linear approximation of the individual well capacity, $E_w\left(S_i^G\right) = \frac{E_{w,max} S_i^G}{S_{max}^G}$, is used. Thus, the well constraint becomes

$$E_i^G \leq N_i^G E_w\left(S_{i-1}^G\right) = N_i^G E_{w,max} \frac{S_{i-1}^G}{S_{max}^G} \tag{12}$$

There also needs to be a constraint that prevents the possibility of selling wells that have been drilled. This is given by the following

$$N_{i-1}^G \leq N_i^G \tag{13}$$

To complete the formulation some assumptions must be made about the demand. One option is to specify the demand by some function $D(t)$ and require the system to produce enough power to satisfy this demand[1]

$$E_i^H + E_i^G = D_i \tag{14}$$

The optimization problem then has the following representation

$$\max_{S^H, E^H, Y^H, S^G, E^G, N^G} \Pi\left(E^H, E^G, N^G\right) \tag{15}$$

subject to

$$S_i^H - S_{\{i-1\}}^H + E_i^H \Delta t + Y_i^H \Delta t - R_i^H \Delta t = 0, \quad i = 1, \dots, n \tag{15a}$$

$$S_i^G - S_{i-1}^G + E_i^G \Delta t - R_{max}^G \frac{S_{max}^G - S_{i-1}^G}{S_{max}^G} \Delta t = 0, \quad i = 1, \dots, n \tag{15b}$$

$$S_i^H - S_{max}^H \leq 0, \quad i = 1, \dots, n \tag{15c}$$

$$E_i^H - E_{max}^H \leq 0, \quad i = 1, \dots, n \tag{15d}$$

$$E_{min}^H - E_i^H \leq 0, \quad i = 1, \dots, n \tag{15e}$$

$$E_i^G - E_{max}^G \leq 0, \quad i = 1, \dots, n \tag{15f}$$

$$E_{min}^G - E_i^G \leq 0, \quad i = 1, \dots, n \tag{15g}$$

$$E_i^G - N_i^G E_{w,max} \frac{S_{i-1}^G}{S_{max}^G} \leq 0, \quad i = 1, \dots, n \tag{15h}$$

$$N_{i-1}^G - N_i^G \leq 0, \quad i = 1, \dots, n \tag{15i}$$

$$E_i^H + E_i^G - D_i = 0, \quad i = 1, \dots, n \tag{15j}$$

$$-Y_i^H \leq 0, \quad i = 1, \dots, n \tag{15k}$$

$$-N_i^G, -S_i^H, -S_i^G \leq 0, \quad i = 0, \dots, n \tag{15l}$$

2. SOLUTION

The problem represented by equation 15 is a linear program with both linear and quadratic constraints. The KKT (Karush-Kuhn-Tucker) conditions provide first order necessary conditions for the optimal solution of the problem. These conditions are (stationarity, primal feasibility, dual feasibility and complimentary slackness) stated for the problem in the following sections.

2.1 Stationarity

The stationarity condition can be written as:[2]

$$\begin{aligned}\nabla \Pi = {} & \sum_{i=1}^{n} \alpha_i \nabla \left(S_i^H - S_{i-1}^H + E_i^H \Delta t + Y_i^H \Delta t - R_i^H \Delta t\right) + \sum_{\{i=1\}}^{n} \beta_i \nabla \left(S_i^G - S_{i-1}^G + E_i^G \Delta t - R_{max}^G \frac{S_{max}^G - S_{i-1}^G}{S_{max}^G} \Delta t\right) \\ & + \sum_{\{i=1\}}^{n} \chi_i \nabla \left(E_i^H - E_{max}^H\right)\Delta t + \sum_{i=1}^{n} \delta_i \nabla \left(E_{min}^H - E_i^H\right)\Delta t + \sum_{i=1}^{n} \epsilon_i \nabla \left(E_i^G - E_{max}^G\right)\Delta t + \sum_{i=1}^{n} \phi_i \nabla \left(E_{min}^G - E_i^G\right)\Delta t \\ & + \sum_{i=1}^{n} \gamma_i \nabla \left(S_i^H - S_{max}^H\right) + \sum_{i=1}^{n} \eta_i \nabla \left(E_i^G - N_i^G E_{w,max} \frac{S_{i-1}^G}{S_{max}^G}\right)\Delta t + \sum_{i=1}^{n} \iota_i \nabla \left(E_i^H + E_i^G - D_i\right)\Delta t\end{aligned}$$

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[1] Another option is to relate the demand function to the energy price, P_i, and let the price determine how much is produced for the market. A function for the revenue under market price conditions could be:

$$P_i\left(E_i^H + E_i^G\right) = \begin{cases} P_0 D_0 + P_{ws}\left(E_i^H + E_i^G - D_0\right), & \text{for } E_i^H + E_i^G \leq D_0 \\ P_0 D_0 + P_0 \left(\dfrac{D_0}{E_i^H + E_i^G}\right)^{\frac{1}{k}} \left(E_i^H + E_i^G - D_0\right), & \text{for } E_i^H + E_i^G \geq D_0 \end{cases}$$

where it is assumed that if the produced quantity is less than the base demand, D_0, then additional power must be supplied from the whole sale market at price P_{ws}. If production exceeds the base demand, then additional power can be sold at a reduced unit price as dictated by the declining market price $P_0 \left(\frac{D_0}{E_i^H + E_i^G}\right)^{1/k}$.

[2] Note that the time step Δt has been factored into all rate related conditions for notational convenience and consistency in the units of the KKT multipliers.

$$+\sum_{i=1}^{n}\kappa_i \nabla\left(N_{i-1}^{G} - N_i^{G}\right) - \sum_{i=1}^{n}\lambda_i \nabla Y_i^{H}\Delta t - \sum_{i=1}^{n}\mu_i \nabla N_i^{G} \; - \sum_{i=1}^{n}\nu_i \nabla S_i^{H} \; - \sum_{i=1}^{n}\theta_i \nabla S_i^{G} \tag{16}$$

where the gradient ∇ is taken w.r.t. each of the decision variables (S^H, E^H, Y^H, S^G, E^G and N^G) and each of the KKT multipliers ($\chi_i, \delta_i, \epsilon_i, \phi_i, \gamma_i, \eta_i, \kappa_i, \lambda_i, \mu_i, \nu_i$ and θ_i). The derivatives w.r.t. the KKT multipliers enforce the constraints, i.e. the Primal Feasibility conditions as outlined in the next subsection. The derivatives w.r.t. the decision variables provide additional conditions which relate the various KKT multipliers, which can also be interpreted as the opportunity cost incurred as a result of the various constraints. To have a feasible solution of the problem, the constraints must either be active and satisfied, or if these constraints are not active, the KKT multipliers must be zero. This is described by the complementary slackness conditions given below.

2.1.1 Stationarity with respect to decision variables

The stationarity conditions with respect to individual decision variables were derived and are written out explicitly for each variable below.

$$\mathrm{S_i^H}: \quad \alpha_i = \alpha_{i+1} + \nu_i - \gamma_i \tag{17a}$$

$$\mathrm{S_n^H}: \quad \alpha_n = \nu_n - \gamma_n \tag{17b}$$

$$\mathrm{E_i^H}: \quad \alpha_i = \left(P_i - c_E^H\right)e^{-r(i-1)\Delta t} - \chi_i + \delta_i - \iota_i \tag{17c}$$

$$\mathrm{Y_i^H}: \quad \alpha_i = \lambda_i \tag{17d}$$

$$\mathrm{S_i^G}: \quad \beta_i = \beta_{i+1}\left(1 - \frac{R_{max}^G}{S_{max}^G}\Delta t\right) + \eta_{i+1} N_{i+1} \frac{E_{w,max}}{S_{max}^G}\Delta t + \theta_i \tag{17e}$$

$$\mathrm{S_n^G}: \quad \beta_n = \theta_n \tag{17f}$$

$$\mathrm{E_i^G}: \quad \beta_i = \left(P_i - c_E^G\right)e^{-r(i-1)\Delta t} - \epsilon_i + \phi_i - \eta_i - \iota_i \tag{17g}$$

$$\mathrm{N_i^G}: \quad \eta_i = \frac{\left(c_N^G\Delta t + c_w^G\left(1 - e^{-r\Delta t}\right)\right)e^{-r(i-1)\Delta t} - \kappa_i + \kappa_{i+1} - \mu_i}{E_w\left(S_{i-1}^G\right)\Delta t} \tag{17h}$$

$$\mathrm{N_n^G}: \quad \eta_n = \frac{\left(c_N^G\Delta t + c_w^G\right)e^{-r(n-1)\Delta t} - \kappa_n - \mu_n}{E_w\left(S_{n-1}^G\right)\Delta t} \tag{17i}$$

2.2 Primal Feasibility and Dual Feasibility

The primal feasibility conditions simply state that the constraints of the objective function must be satisfied. Thus, the primal feasibility conditions are described by equations 15a through 15l.

The dual feasibility conditions simply state that the KKT multipliers related to the inequality constraints must be non-negative, i.e.:

$$\chi_i, \delta_i, \epsilon_i, \phi_i, \gamma_i, \eta_i, \kappa_i, \lambda_i, \mu_i, \nu_i, \theta_i \geq 0, \quad i = 1, \dots, n \tag{18}$$

2.3 Complimentary Slackness

The complimentary slackness conditions state that each inequality constraint must be either inactive or active and satisfied at its boundary. For inactive constraints the KKT multipliers are zero, thereby eliminating the influence of the constraint on the solution. For active constraints, the KKT multipliers are positive and represent the strength with which the respective constraint must influence the objective to maintain a feasible solution. The complimentary slackness conditions for each constraint is written out in equations 19a through 19n.

$$\alpha_i\left(S_i^H - S_{i-1}^H + E_i^H\Delta t + Y_i^H\Delta t - R_i^H\Delta t\right) = 0, \quad i = 1, \dots, n \tag{19a}$$

$$\beta_i\left(S_i^G - S_{i-1}^G + E_i^G\Delta t - R_{max}^G \frac{S_{max}^G - S_{i-1}^G}{S_{max}^G}\Delta t\right) = 0, \quad i = 1, \dots, n \tag{19b}$$

$$\chi_i\left(E_i^H - E_{max}^H\right) = 0, \quad i = 1, \dots, n \tag{19c}$$

$$\delta_i\left(E_{min}^H - E_i^H\right) = 0, \quad i = 1, \dots, n \tag{19d}$$

$$\epsilon_i\left(E_i^G - E_{max}^G\right) = 0, \quad i = 1, \dots, n \tag{19e}$$

$$\phi_i\left(E_{min}^G - E_i^G\right) = 0, \quad i = 1, \dots, n \tag{19f}$$

$$\gamma_i\left(S_i^H - S_{max}^H\right) = 0, \quad i = 0, \dots, n \tag{19g}$$

$$\eta_i\left(E_i^G - N_i^G\, E_{w,max}\frac{S_{i-1}^G}{S_{max}^G}\right) = 0, \quad i = 1, \dots, n \tag{19h}$$

$$\iota_i\left(E_i^H + E_i^G - D_i\right) = 0, \quad i = 1, \dots, n \tag{19i}$$

$$\kappa_i\left(N_{i-1}^G - N_i^G\right) = 0, \quad i = 1, \dots, n \tag{19j}$$

$$\lambda_i Y_i^H = 0, \quad i = 1, \dots, n \tag{19k}$$

$$\mu_i N_i^G = 0, \quad i = 0, \dots, n \tag{19l}$$

$$\nu_i S_i^H = 0, \quad i = 0, \dots, n \tag{19m}$$

$$\theta_i S_i^G = 0, \quad i = 0, \dots, n \tag{19n}$$

2.4 Opportunity Cost

The KKT multipliers can be interpreted as opportunity costs associated with the corresponding constraints. These can also be viewed as the marginal utility of being able to violate these constraints, also known as the shadow price.

The shadow price for the demand constraint, ι_i, is of particular interest as it underlines which terms to consider when deciding whether to produce from the hydro or geothermal resource. Stationarity condition 17c for marginal production from a hydro resource gives

$$\iota_i = (P_i - c_E^H)e^{-r(i-1)\Delta t} - \alpha_i - \chi_i + \delta_i \tag{20}$$

This clearly states that the marginal utility, ι_i, of producing from a hydro resource depends on

- $(P_i - c_E^H)e^{-r(i-1)\Delta t}$, i.e. the NPV of profit made from hydro generation over time interval i,
- α_i, opportunity cost of using hydro stock from the reservoir now vs. storing it for the future (often called the water value),
- χ_i, opportunity cost of production limitations due to maximum power plant capacity,
- δ_i, opportunity cost of production limitations due to minimum power plant capacity.

Similarly, stationarity condition 17g gives the geothermal perspective

$$\iota_i = \left(P_i - c_E^G\right)e^{-r(i-1)\Delta t} - \beta_i - \eta_i - \epsilon_i + \phi_i \tag{21}$$

which shows that the marginal utility of producing from a geothermal resource consists of

- $\left(P_i - c_E^G\right)e^{-r(i-1)\Delta t}$, i.e. the NPV of profit made from geothermal generation over time interval i,
- β_i, opportunity cost of using geothermal stock from reservoir now vs. storing it for the future (can be regarded as the steam value),
- η_i, opportunity cost of production limitations due to insufficient well capacity,
- ϵ_i, opportunity cost of production limitations due to maximum power plant capacity,
- ϕ_i, opportunity cost of production limitations due to minimum power plant capacity.

At the optimal solution, the marginal utility (or shadow price or opportunity cost) ι_i of producing from each resource should be the same for all resources.

The opportunity costs may be somewhat hard to comprehend, and it should be noted that they will be zero unless corresponding constraint is active (this is described by the complementary slackness conditions). For example, the opportunity cost for power plant capacity limits will be zero unless production is at maximum or minimum capacity.

Using equations 17a and 17b, the following expression for the water value can be derived:

$$\alpha_i = \sum_{j=i}^{n} \nu_j - \gamma_j \tag{22}$$

Here ν_i is the opportunity cost of emptying the hydro reservoir and γ_i is the opportunity cost of filling the hydro stock to its maximum capacity. The water value, α_i, can be related to the water spillage, Y_i^H, by looking at the stationarity condition for spillage, 17d, which states that $\alpha_i = \lambda_i$. Combining this with the complementary slackness condition for spillage, $\lambda_i Y_i^H = 0$, leads to the intuitive finding that (at the optimum solution) if there is spillage ($Y_i > 0$) then the water value ($\alpha_i = \lambda_i$) must be 0. In other words, when there is spillage the opportunity cost of using a marginal amount of water from the reservoir to produce power is 0 because that marginal amount of water could not have been stored for future use anyway. [3]

Using equations 17e and 17f, the following expression for the steam value β_i can be derived:

$$\beta_i = \sum_{j=i}^{n} \theta_j \left(1 - \frac{R_{max}^G \Delta t}{S_{max}^G}\right)^{j-i} + \sum_{j=i+1}^{n} \left(\eta_j N_j \frac{E_{w,max}\Delta t}{S_{max}^G}\right)\left(1 - \frac{R_{max}^G \Delta t}{S_{max}^G}\right)^{j-(i+1)} \tag{23}$$

[3] An interesting observation that can be made based on equation 22, is that when $\alpha_i = 0$ then $\sum_{j=i}^{n} \nu_j = \sum_{j=i}^{n} \gamma_j$. In other words, if the hydro reservoir is full, then the future opportunity cost of not having a larger reservoir is equal to the future loss incurred if the reservoir is emptied. Note that these are marginal costs, \$/MWh.)

In this model the geothermal resource recharges faster and produces less as the stock is depleted. Therefore, the reservoir will never empty and $\theta_i = 0$ for all i, as per equation 19n. Thus, the expression for β_i can be further simplified to

$$\beta_i = \sum_{j=i+1}^{n} \left(\eta_j N_j \frac{E_{w,max}\Delta t}{S_{max}^G} \right) \left(1 - \frac{R_{max}^G \Delta t}{S_{max}^G} \right)^{j-(i+1)} \tag{24}$$

What equation 24 tells us is that the steam value, β, is a product of the future opportunity cost of being constrained by well capacity η (i.e. shadow price of well capacity constraints), the future number of wells N, the marginal change in well capacity as a function of the reservoir stock and a power function that depends on the marginal recharge rate to the reservoir. This last factor, i.e. the power function, is between 0 and 1 and will have a larger power exponent as it reaches further into the future. Therefore, this factor works like a discount factor, reducing the impact of current decisions on future performance. This makes sense as the recharge becomes larger as the stock in the reservoir is reduced and thus acts as a balancing factor against the current exploitation that is being evaluated by the steam value (i.e. the steam value is the opportunity cost of utilizing steam now versus saving it for the future).

Now take a closer look at the opportunity cost of production limitations due to insufficient well capacity (see eq. 17h and 17i). We know that $\mu_i = 0$ for all i, since $N_0^G > 0$ and $N_{i-1}^G \leq N_i^G$, i.e. we have

$$\eta_i = \begin{cases} \frac{(c_N^G \Delta t + c_w^G) e^{-r(i-1)\Delta t} - \kappa_i}{E_w(S_{i-1}^G)\Delta t}, & for\ i = n \\ \frac{\left(c_N^G \Delta t + c_w^G (1 - e^{-r\Delta t})\right) e^{-r(i-1)\Delta t} - \kappa_i + \kappa_{i+1}}{E_w(S_{i-1}^G)\Delta t}, & otherwise \end{cases} \tag{25}$$

There is a close relationship between the opportunity cost of production limitations due to insufficient well capacity (η_i) and the opportunity cost of owning an unused well (κ_i). The following expression for κ_i can be derived from equation 25:

$$\kappa_i = \sum_{j=i}^{n} C_j^G e^{-r(j-1)\Delta t} - \eta_j E_w(S_{j-1}^G) \tag{26}$$

where

$$C_i^G = \begin{cases} c_N^G \Delta t + c_w^G, & for\ i = n \\ c_N^G \Delta t + c_w^G (1 - e^{-r\Delta t}), & otherwise \end{cases} \tag{27}$$

Equation 26 shows that the cost of owning an unused well (in terms of \$/well) consists of the operating cost and the investment in the well, but it is reduced by the opportunity cost of being constrained by well capacity for all future time intervals. This is intuitive, i.e. if one has over-invested in wells then the well capacity constraint will seldom be active (η_i will be small) and thus the value of κ_i will be large.

3. DISCUSSION

A recent publication by the International Renewable Energy Agency (IRENA) discusses the electricity storage needs of future energy systems (International Renewable Energy Agency, 2017). It is stated that with the foreseeable growth of variable renewable electricity (from solar and wind), energy will need to be stored over days, weeks or months. An estimate from the same publication is that the total storage capacity for electrical energy appears set to triple by 2030. There appear to be opportunities for geothermal applications in these future energy systems, as geothermal resources are enormous natural storage units. In fact, an artificial geothermal system has recently been created in Germany, above ground, for energy storage (Renewable Energy World, 2019)

To utilize geothermal energy systems to their full potential in these future energy systems a slight change in design and thinking is required. Instead of regarding geothermal energy as a purely baseload resource, it needs to be relabeled as a load-following resource (similar to hydro). In fact, this step has recently been taken with the publication of DOE-GTO's and NREL's GeoVision Study (US Department of Energy, 2019). Some advances in well metering and control would be needed for many of todays geothermal fields, but these are solvable matters. Some upgrades of turbine control systems and electrical transmission systems may also be needed. Finally, geothermal energy producers need a way to evaluate the current cost of production from geothermal resources to determine when to store and when to produce at full capacity.

The formulations presented here are mostly intended to give insight into how to evaluate the cost of production from a geothermal field. As the formulations show, there are more things to be considered that one might expect, e.g. the impact of current production on future performance (steam values) and the opportunity cost of not being able to supply power when prices are high (well capacity constraints). The formulation given here could be expanded to consider wells with variable performance and operation cost with relatively small changes. The main challenge is, however, to put this cost in context with expected performance from other resources in the market, as we have seen in the cogeneration of hydro and geothermal problem. This seems to be a challenging optimization problem that is yet to be solved (the main complication is due to the reverse convex constraint seen in equation 15h).

At Landsvirkjun we have created some pseudo-methods to estimate the variable cost of producing from wells in the Krafla geothermal field. We use these numbers to determine how much should be produced from the field (ranging from 50 to 60 MW) depending on the state of nearby hydro resources, and which wells to turn off when full capacity is not required. Figure 1 shows an example of such cost estimates, broken down into fixed cost per well, cost per energy unit produced, make-up well cost, gas emission cost and brine reinjection cost.

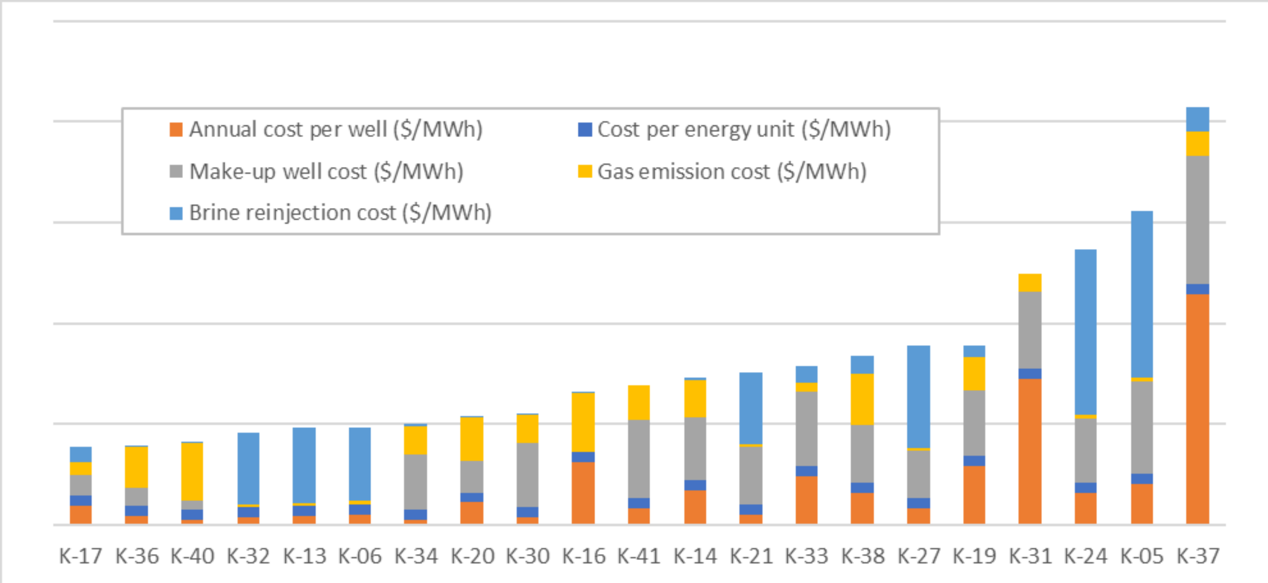

Figure 1: An example of variable cost estimates for production wells in the Krafla geothermal field, broken down into fixed cost per well, cost per energy unit produced, make-up well cost, gas emission cost and brine reinjection cost.

4. SUMMARY

The main result of this work was the formulation of what we have chosen to call the steam values. These are comparable to water values that are generally used to determine when and in which amount to produce from hydro reservoirs. Water values also form the basis of how to set a price for hydroelectric energy in many markets (e.g in Iceland, New Zealand, Brazil, Canada and more). The steam values could serve a similar purpose.

The formulation presented here showed the following:

- When determining whether to produce from hydro or geothermal resources one must consider the water value (α), the steam value (β) and the current opportunity cost of well capacity constraints (η), as illustrated in equations 20 and 21.
- Water values are formed based on the risk of emptying the reservoir and not being able to deliver sold energy versus filling the reservoir and wasting energy that could have been stored for later use.
    - If the hydro reservoir is full the water value is $\alpha_i = 0$.
    - If the hydro reservoir is never emptied the water value is $\alpha_i = 0$.
- Steam values are formed from a number of parameters, including well decline (make-up well requirement), recharge rate and O&M cost. Other parameters may also play a role, such as gas emissions and brine reinjection cost.
    - If no make-up wells are needed for future production the steam value is $\beta_i = 0$.
    - When an energy unit is produced from steam, the future production of wells is reduced (marginally), but this negative effect is discounted by a marginal increase in recharge resulting from lowering the reservoir stock.

Equations 24 through 27 can be used to approximate the steam value (β) and the current opportunity cost of well capacity constraints (η) with iterative simple spreadsheet computations, which should reflect the energy price from a geothermal power plant. To get a more exact figure, however, the optimization problem presented by equation 15 needs to be solved, preferably with a stochastic formulation of unknown parameters such as hydro recharge and future well performance for geothermal resources. To the best knowledge of the authors, this remains an unsolved optimization challenge.

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