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Flexible Lumped Parameter Modelling for Low-Temperature Reservoir Assessment

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ABSTRACT

Reservoir models are crucial for the estimation of a geothermal field's response to different operation scenarios. Where field data is scarce, Lumped Parameter Reservoir Models (LPRM) are often a first choice. These can perform particularly well when operations include only a single observation site or can be approximated as such. We present a flexible LPRM scheme consisting of an arbitrary number of wells and reservoirs. This scheme can furthermore incorporate instantaneous changes of the parameters, as can be observed after earthquakes or altered utilization. Despite the increased model complexity, the set of extrinsic parameters can be optimized analytically, leaving only the intrinsic model properties subject to non-linear optimization. This LPRM method is tested on Icelandic geothermal fields, some strongly affected by the South-Iceland earthquakes of 2000.

1. INTRODUCTION

Lumped Parameter Reservoir Models (LPRM) have been used to describe liquid-dominated low-temperature geothermal fields with quite some success. One such model was described by Axelsson (1985, 1989), implemented in the program LUMPFIT (Axelsson and Arason, 1992), and was used in many studies throughout the last decades (see, e.g., Axelsson *et al.*, 2005).

We extend the classical LPRM by two new concepts, "wells" and "eras", and show the effect of these on the model's capabilities. The final model equations are expressed in a form suitable for semi-analytical optimization, which we will take advantage of when finding the set of optimal model parameters during the inverse modelling phase.

Finally, the capabilities of our LPRM will be illustrated by four examples, where we focus on illustrative qualitative analysis and leave any focus on quantitative analysis for later studies.

Before we start our derivations, we shall clarify the notation used in the equations of the following sections. Matrices are denoted by bold faced uppercase symbols, vectors by bold faced lowercase symbols, scalars as regular faced symbols. The transpose of matrix or vector is denoted as a superscript T. A matrix can be constructed from other matrices or column-vectors by listing these within square brackets, separated by commata. The result is a matrix build up from the columns of the specified matrices in the specified order. In an identical notation row-vectors are constructed from other row-vectors or scalars. The scope of summation and integration is the following term if not indicated otherwise using parentheses. Round brackets in superscript indicate a label, while square brackets in subscript index the "era" (explained in detail later). The dimensions of matrices and vectors are stated only if not clear from the context. Lowercase Greek and Latin letters are used to index vector or matrix elements, while a subscript 0 is not to be understood as such an index. The Kroenecker delta is denoted by $\delta_{i,j}$. A dotted symbol denotes its derivative with respect to time.

2. LUMPED PARAMETER RESERVOIR MODEL

The general LPRM approximates a complex system of aquifers by a finite number of reservoirs. These reservoirs have vertical walls, contain some amount of liquid, and are connected to each other by conductors, through which a pressure driven mass exchange of liquid is realized. From here on we shall only consider isothermal systems and non-compressible liquids. Hence, mass is proportional to volume and pressure differences proportional to differences of liquid levels. We arbitrarily decide to use the latter to parametrize our system.

The volumetric flow rate q_{ij} from reservoir i into reservoir j can be calculated as

$$q_{ij} = \sigma_{ji}\left(l_i - l_j\right),$$

introducing the volumetric conductance σ_{ji} with symmetry condition $\sigma_{ji} = \sigma_{ij}$ and $\sigma_{ii} = 0$ by definition. Each of the N reservoirs is furthermore connected to an infinitely large reservoir of constant liquid level $l_i^{(\mathrm{b})}$through a conductor of conductance ω_i. From this, the volumetric flow rate q_i from reservoir i into its background reservoir is given as

$$q_i = \omega_i\left(l_i - l_i^{(\mathrm{b})}\right).$$

Considering an additional external flow of rate f_i out of reservoir i, the net change of volume in the reservoir is found as

$$\dot{V}_i \;\; = -f_i - q_i - \sum_{k=1}^{N} q_{ik} \;,$$

or equivalently, using the reservoir's base area A_i, as

$$\boldsymbol{A_i \dot{l}_i} \;= -\boldsymbol{f_i} - \boldsymbol{\omega_i}\left(\boldsymbol{l_i} - \boldsymbol{l_i^{(b)}}\right) - \sum_{\boldsymbol{k=1}}^{\boldsymbol{N}} \boldsymbol{\sigma_{ki}}\,(\boldsymbol{l_i} - \boldsymbol{l_k})\,. \tag{1}$$

These equations can be rearranged into

$$A_i \dot{l}_i + \left(\omega_i + \sum_{k=1}^{N} \sigma_{ki}\right) l_i - \sum_{k=1}^{N} \sigma_{ki}\, l_k = -f_i + \omega_i l_i^{(\mathrm{b})}$$

and, using the vector notation $\mathbf{x} = (x_1, x_2, \ldots, x_N)^T$, combined into the matrix equation

$$\mathbf{A\dot{l}} + \mathbf{Cl} = \mathbf{\Omega l}^{(\mathrm{b})} - \mathbf{f}$$

with the symmetric matrices $\mathbf{A}$, $\mathbf{C}$, and $\mathbf{\Omega}$ defined as

$$A_{ij} = \delta_{i,j} A_i, \quad \Omega_{ij} = \delta_{i,j}\omega_i, \quad \text{and} \quad C_{ij} = -\sigma_{ij} + \delta_{i,j}\left(\omega_i + \sum_{k=1}^{N} \sigma_{ki}\right).$$

The solution for initial values $\mathbf{l}(t_0, \mathbf{f}) = \mathbf{l_0}$ is found as

$$\mathbf{l}(t, \mathbf{f}) = \mathbf{l_0} + \int_{t_0}^{t} \mathrm{d}\tau\; \boldsymbol{e}^{-(t-\tau)\mathbf{R}} \mathbf{A}^{-1}\left[\mathbf{\Omega l}^{(\mathrm{b})} - \mathbf{Cl_0} - \mathbf{f}(\tau)\right],$$

with $\mathbf{R} = \mathbf{A^{-1}C}$. Using the generalized eigenvalue problem

$$\mathbf{CT} = \mathbf{AT\Lambda} \tag{2}$$

with $\mathbf{T}^T\mathbf{AT} = \mathbf{TT}^T\mathbf{A} = \mathbf{1}$, we can rewrite this in computationally simpler spectral form as

$$\begin{aligned}\mathbf{l}(t, \mathbf{f}) \;&= \mathbf{l_0} + \mathbf{T}\int_{t_0}^{t} \mathrm{d}\tau\; \boldsymbol{e}^{-(t-\tau)\mathbf{\Lambda}}\mathbf{T}^T\left(\mathbf{\Omega l}^{(\mathrm{b})} - \mathbf{Cl_0} - \mathbf{f}(\tau)\right) \\ &= \mathbf{l}^{(\mathrm{i})}(t) + \mathbf{l}^{(\mathrm{f})}(t, \mathbf{f})\end{aligned}$$

which we furthermore separated into "intrinsic" and "flow" contributions.

The "intrinsic" component, $\mathbf{l}^{(\mathrm{i})}$, corresponds to the initial reservoir levels $\mathbf{l_0}$ "relaxing" towards their equilibrium state dictated by the background reservoir levels.

$$\begin{aligned}\mathbf{l}^{(\mathrm{i})}(t) \;&= \mathbf{l_0^{(i)}} + \mathbf{T}\int_{t_0}^{t} \mathrm{d}\tau\; \boldsymbol{e}^{-(t-\tau)\mathbf{\Lambda}}\mathbf{T}^T\left(\mathbf{\Omega l}^{(\mathrm{b})} - \mathbf{Cl_0^{(i)}}\right) \\ &= [\mathbf{1} - \mathbf{TM}(t - t_0, \mathbf{\Lambda})\mathbf{T}^T\mathbf{C}]\mathbf{l_0} + \mathbf{TM}(t - t_0, \mathbf{\Lambda})\mathbf{T}^T\mathbf{\Omega l}^{(\mathrm{b})}\end{aligned}$$

with the diagonal matrix $\mathbf{M}(t - t_a, \mathbf{\Lambda})$ defined as

$$M_{ii}(t - t_0, \mathbf{\Lambda}) = \mu(t - t_0, \lambda_i) = \frac{1 - e^{-\lambda_i(t-t_0)}}{\lambda_i}. \tag{3}$$

The "flow" contribution, $\mathbf{l}^{(\mathrm{f})}$, describes the response of a model to the external flow, with initial levels and all background levels being zero. Expanding the flow $\mathbf{f}(\tau)$ as the power series $\sum_{k=0}^{\infty} \mathbf{f}^{(k)}\,\tau^k$ we find for this contribution

$$\begin{aligned}\mathbf{l}^{(\mathrm{f})}(t, \mathbf{f}) = \mathbf{l_0^{(f)}} \;&-\mathbf{TM}(t - t_0, \mathbf{\Lambda})\mathbf{T}^T\left(\mathbf{Cl_0^{(f)}} + \mathbf{f}^{(0)}\right) \\ &-\mathbf{T}\sum_{k=1}^{\infty}\left[\int_{t_0}^{t} \mathrm{d}\tau\; \mathbf{e}^{-(t-\tau)\mathbf{\Lambda}}\tau^k\right]\mathbf{T}^T\mathbf{f}^{(k)}\,.\end{aligned}$$

For a piecewise constant function with $\mathbf{f}(t) = \mathbf{f}(t_a)$ on interval $[t_a; t_{a+1})$, all terms of higher order than $\mathbf{f}^{(0)} = \mathbf{f}(t_a)$ vanish. The "flow" component to the reservoir levels at some time t with $t_0 \le t_a < t \le t_{a+1}$ is therefore given by

$$\mathbf{l}^{(\mathrm{f})}(t, \mathbf{f}) \;= \mathbf{l}^{(\mathrm{f})}(t_a, \mathbf{f}) - \mathbf{TM}(t - t_a, \mathbf{\Lambda})\mathbf{T}^T\left(\mathbf{Cl}^{(\mathrm{f})}(t_a, \mathbf{f}) + \mathbf{f}(t_a)\right),$$

where we note that $\mathbf{l}^{(\mathrm{f})}(t_0, \mathbf{f}) = 0$ and $\mathbf{l}^{(\mathrm{f})}(t_a, \mathbf{f})$ for $a > 0$ is calculated as the final levels on the previous interval.

Having established the defining equations we introduce a shorthand notation that allows us to use a more compact form in the remaining following sections. We define the matrix-valued functions

$$\begin{aligned}\mathbf{G}^{(\mathrm{f})}(x,x_0) &= \mathbf{T}\mathbf{M}(x-x_0,\mathbf{\Lambda})\mathbf{T}^T\\ \mathbf{G}^{(\mathrm{c})}(x,x_0) &= \mathbf{G}^{(\mathrm{f})}(x,x_0)\mathbf{\Omega}\\ \mathbf{G}^{(\mathrm{d})}(x,x_0) &= \left[\mathbf{1}-\mathbf{G}^{(\mathrm{f})}(x,x_0)\mathbf{C}\right]\end{aligned}$$

so that the reservoir response due to the flow can be expressed as

$$\mathbf{l}^{(\mathrm{f})}(t,\mathbf{f}) = \mathbf{G}^{(\mathrm{d})}(t,t_a)\mathbf{l}^{(\mathrm{f})}(t_a,\mathbf{f}) - \mathbf{G}^{(\mathrm{f})}(t,t_a)\mathbf{f}(t_a)$$

and our final expression for the reservoir levels reads

$$\mathbf{l}(t,\mathbf{f}) = \mathbf{l}^{(\mathrm{f})}(t,\mathbf{f}) + \mathbf{G}^{(\mathrm{c})}(t,t_0)\mathbf{l}^{(\mathrm{b})} + \mathbf{G}^{(\mathrm{d})}(t,t_0)\mathbf{l_0}\ . \tag{4}$$

2.1 Wells

The volume of a well is orders of magnitude smaller than that of the aquifer it is drilled into and may well be neglectable for many considerations. We will now introduce "wells" into our model, in the form of volume-less reservoirs. To distinguish N_R regular reservoirs from N_W wells, we index the latter using Greek letters and replace their σ, ω, and $l^{(\mathrm{b})}$ by ζ, υ, and $l^{(\mathrm{h})}$, respectively. We furthermore now use the labels "(r)" and "(w)" to emphasize whether a level is that of a well or a reservoir and assume that no two wells are connected to each other directly.

For $A_\alpha = 0$, Eq. 1 is no longer dependent on the time derivative. The well level can be calculated directly as a linear combination of the well flow, the well's background level, $l_\alpha^{(\mathrm{h})}$, and the remaining reservoir levels as

$$l_\alpha^{(\mathrm{w})}(t,\mathbf{f}) = -\frac{1}{\mu_\alpha}f_\alpha^{(\mathrm{w})}(t) + \frac{\upsilon_\alpha}{\mu_\alpha}l_\alpha^{(\mathrm{h})} + \sum_{k=1}^{N_\mathrm{R}}\frac{\zeta_{k\alpha}}{\mu_\alpha}l_k^{(\mathrm{r})}(t) \tag{5}$$

with

$$\mu_\alpha = \upsilon_\alpha + \sum_{k=1}^{N_\mathrm{R}}\zeta_{k\alpha}\ .$$

Separating the wells from the reservoirs in Eq. 1, we get for the remaining reservoir levels

$$A_i\dot{l}_i^{(\mathrm{r})} = -f_i^{(\mathrm{r})} - \omega_i\left(l_i^{(\mathrm{r})} - l_i^{(\mathrm{b})}\right) - \sum_{k=1}^{N_\mathrm{R}}\sigma_{ki}\left(l_i^{(\mathrm{r})} - l_k^{(\mathrm{r})}\right) - \sum_{\alpha=1}^{N_\mathrm{W}}\zeta_{i\alpha}\left(l_i^{(\mathrm{r})} - l_\alpha^{(\mathrm{w})}\right)$$

and after substitution of the well levels

$$\begin{aligned}A_i\dot{l}_i^{(\mathrm{r})} = &-\left[f_i^{(\mathrm{r})} + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}f_\alpha^{(\mathrm{w})}\right] - \sum_{k=1}^{N_\mathrm{R}}\left[\sigma_{ki} + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}\zeta_{k\alpha}\right]\left(l_i^{(\mathrm{r})} - l_k^{(\mathrm{r})}\right) \quad \ldots\\ &-\left[\omega_i + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}\upsilon_\alpha\right]\left(l_i^{(\mathrm{r})} - \left[\frac{\omega_i l_i^{(\mathrm{b})} + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}\upsilon_\alpha l_\alpha^{(\mathrm{h})}}{\omega_i + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}\upsilon_\alpha}\right]\right)\\ = &-\left[f_i^{(\mathrm{r})} + \sum_{\alpha=1}^{N_\mathrm{W}}\frac{\zeta_{i\alpha}}{\mu_\alpha}f_\alpha^{(\mathrm{w})}\right] - \tilde{\omega}_i\left(l_i^{(\mathrm{r})} - \tilde{l}_k^{(\mathrm{b})}\right) - \sum_{k=1}^{N_\mathrm{R}}\tilde{\sigma}_{ki}\left(l_i^{(\mathrm{r})} - l_k^{(\mathrm{r})}\right)\ .\end{aligned} \tag{6}$$

This equation turns out to be equivalent to Eq. 1, in other words, the reservoir system behaves just as derived in the previous section, merely with different parameters and an additional component to the flow.

Of particular interest is a well that is only connected to a single reservoir, i.e., $\upsilon_\alpha = 0$ and with only $\zeta_{i\alpha}$ non-zero. For $f_\alpha^{(\mathrm{w})} = 0$, as would be in "observation wells" in a real system, Eq. 5 shows the well level $l_\alpha^{(\mathrm{w})}$ to equal the reservoir level $l_i^{(\mathrm{r})}$. For $f_\alpha^{(\mathrm{w})} \neq 0$, as in a "production well", it follows from Eq. 6 that all the well flow adds to the reservoir flow $f_i^{(\mathrm{r})}$. Using these properties, we can from here on require all reservoir flow to be constant, i.e., $\mathbf{f}^{(\mathrm{r})}(t) = \overline{\mathbf{f}}$ and all remaining, variable flow, $\mathbf{f}(t)$, to be happening through a well, i.e., $\mathbf{f}^{(\mathrm{w})}(t) = \mathbf{f}(t)$. We will furthermore only be concerned with well levels as the primary output of the model, i.e., the property to be compared to observed levels in applications of the LPRM.

This does not limit the model in any way, as its original form can always be restored by adding one observation and one production well to each reservoir. However, as a nice side effect, this formalism brings the model in line with the real-life situation, in which the wells act as the interface between the operational part (production/injection and observation) and the hydrological part (reservoir response to operation).

Keeping in mind that our model is merely a very simple approximation to the real system, we may just as well leave its strict foundations for a moment and create a related model with both additional features and a less stiff interdependence of its parameters. Again here, if desired, it is possible to restore the model as derived above, by choosing the right parameters.

As the reservoir parameters will in most cases be found by inverse modelling, we can include the effect of wells on the model parameters right from the start, i.e., we simply drop the tildes in Eq. 6. The defining equations for the reservoir model hence read

$$\mathbf{A}\dot{\mathbf{l}}^{(\mathrm{r})} + \mathbf{C}\mathbf{l}^{(\mathrm{r})} = \mathbf{\Omega}\mathbf{l}^{(\mathrm{b})} - \left(\bar{\mathbf{f}} + \mathbf{Z}\mathbf{f}\right)$$

with $Z_{i\alpha} = \zeta_{i\alpha}/\mu_\alpha$. For the well levels, we furthermore add a global shift for all wells, $l^{(\mathrm{g})}$, a shift for each of the wells separately, $\mathbf{l}^{(\mathrm{s})}$, and replace the term of $-\frac{1}{\mu_\alpha} f_\alpha(t)$ by $\mathbf{l}^{(\mathrm{dd})}(t)$, a finite power series in $f_\alpha^{(\mathrm{w})}$. By this we gain more flexibility in the modelling of well levels and can, e.g., include nonlinear corrections to the well-drawdown directly in the model. The levels of Eq. 5 thus become

$$\mathbf{l}^{(\mathrm{w})}(t,\mathbf{f}) \quad = \mathbf{l}^{(\mathrm{dd})}(t,\mathbf{f}) + \mathbf{Z}^T\mathbf{l}^{(\mathrm{r})}\left(t,\bar{\mathbf{f}} + \mathbf{Z}\mathbf{f}\right) + \mathbf{Y}\mathbf{l}^{(\mathrm{h})} + \mathbf{l}^{(\mathrm{s})} + l^{(\mathrm{g})}$$

with the diagonal matrix $\mathbf{Y}$ with elements $Y_{\alpha\alpha} = \upsilon_\alpha/\mu_\alpha$ and

$$l_\alpha^{(\mathrm{dd})}(t,\mathbf{f}) = -\mathrm{sgn}\left(f_\alpha^{(\mathrm{w})}(t)\right)\sum_{p=1}^{N_\mathrm{p}} a_{p\alpha}\left|f_\alpha^{(\mathrm{w})}(t)\right|^p \ .$$

Furthermore we introduce yet another shift, $\mathbf{l}^{(\mathrm{j})}$, that is added to the initial levels of each reservoir, i.e., we let $\mathbf{l_0} = \mathbf{l_0^{(\mathrm{i})}} + \mathbf{l}^{(\mathrm{j})}$ in Eq. 4. By this, the expressions for the reservoir levels and well levels become

$$\begin{aligned}
\mathbf{l}^{(\mathrm{r})}(t,\mathbf{f}) \quad &= \mathbf{l}^{(\mathrm{f})}(t,\mathbf{f}) + \mathbf{G}^{(\mathrm{d})}(t,t_0)\mathbf{l_0^{(\mathrm{i})}} + \mathbf{G}^{(\mathrm{d})}(t,t_0)\mathbf{l}^{(\mathrm{j})} + \mathbf{G}^{(\mathrm{c})}(t,t_0)\mathbf{l}^{(\mathrm{b})} \\
\mathbf{l}^{(\mathrm{w})}(t,\mathbf{f}) \quad &= \mathbf{l}^{(\mathrm{dd})}(t,\mathbf{f}) + \mathbf{Z}^T\mathbf{l}^{(\mathrm{f})}(t,\mathbf{Z}\mathbf{f}) \ \ldots \\
&\quad + \left[\mathbf{Z}^T\mathbf{G}^{(\mathrm{d})}(t,t_0)\right]\mathbf{l_0^{(\mathrm{i})}} + \left[\mathbf{Z}^T\mathbf{G}^{(\mathrm{d})}(t,t_0)\right]\mathbf{l}^{(\mathrm{j})} + \left[\mathbf{Z}^T\mathbf{G}^{(\mathrm{c})}(t,t_0)\right]\mathbf{l}^{(\mathrm{b})} \ \ldots \\
&\quad + \mathbf{Z}^T\mathbf{l}^{(\mathrm{f})}\left(t,\bar{\mathbf{f}}\right) + \mathbf{Y}\mathbf{l}^{(\mathrm{h})} + \mathbf{l}^{(\mathrm{s})} + l^{(\mathrm{g})} \ .
\end{aligned}$$

Lastly, we combine the terms that are independent of the well flow by means of matrix functions and find our final expressions as

$$\mathbf{l}^{(\mathrm{i})}(t) \quad = \mathbf{P}^{(\mathrm{r})}(t,t_0)\mathbf{l_0^{(\mathrm{i})}} + \mathbf{Q}^{(\mathrm{r})}(t,t_0)\mathbf{c}^{(\mathrm{r})}$$

and

$$\mathbf{l}^{(\mathrm{w})}(t,\mathbf{f}) \quad = \mathbf{l}^{(\mathrm{dd})}(t,\mathbf{f}) + \mathbf{Z}^T\mathbf{l}^{(\mathrm{f})}(t,\mathbf{Z}\mathbf{f}) + \mathbf{P}^{(\mathrm{w})}(t,t_0)\mathbf{l_0^{(\mathrm{i})}} + \mathbf{Q}^{(\mathrm{w})}(t,t_0)\mathbf{c}^{(\mathrm{w})}$$

with

$$\begin{aligned}
\mathbf{P}^{(\mathrm{r})}(t,t_0) &= \mathbf{G}^{(\mathrm{d})}(t,t_0) \\
\mathbf{Q}^{(\mathrm{r})}(t,t_0) &= \left[-\mathbf{G}^{(\mathrm{f})}(t,t_0), \mathbf{G}^{(\mathrm{c})}(t,t_0), \mathbf{G}^{(\mathrm{d})}(t,t_0)\right] \\
\mathbf{c}^{(\mathrm{r})} &= \left[\bar{\mathbf{f}}^T, \mathbf{l}^{(\mathrm{b})^T}, \mathbf{l}^{(\mathrm{j})^T}\right]^T \\
\mathbf{P}^{(\mathrm{w})}(t,t_0) &= \mathbf{Z}^T\mathbf{G}^{(\mathrm{d})}(t,t_0) \\
\mathbf{Q}^{(\mathrm{w})}(t,t_0) &= \left[-\mathbf{Z}^T\mathbf{G}^{(\mathrm{f})}(t,t_0), \mathbf{Z}^T\mathbf{G}^{(\mathrm{c})}(t,t_0), \mathbf{Z}^T\mathbf{G}^{(\mathrm{d})}(t,t_0), \mathbf{Y}, \mathbf{J}_{N_\mathrm{W}}, \mathbf{j}_{N_\mathrm{W}}\right] \\
\mathbf{c}^{(\mathrm{w})} &= \left[\bar{\mathbf{f}}^T, \mathbf{l}^{(\mathrm{b})^T}, \mathbf{l}^{(\mathrm{j})^T}, \mathbf{l}^{(\mathrm{h})^T}, \mathbf{l}^{(\mathrm{s})^T}, l^{(\mathrm{g})}\right]^T
\end{aligned}$$

where $\mathbf{J}_N$ denotes an $(N \times N)$-unit matrix and $\mathbf{j}_N$ a column-vector of all ones and size N.

2.2 Eras

The lumped parameter reservoir model derived up to this point is still static, i.e., it cannot incorporate changes in the model parameters over time. Of particular interest is an abrupt change in all or some of the parameters, as can happen frequently after a strong earthquake in tectonically active areas. Other probable causes for such spontaneous changes are, e.g., beginning of reinjection or the drilling of wells in the area.

We can quite easily incorporate such changes in our model by introducing a sequence of N_E "eras" in which a different set of parameters describes the model during the respective time period. To distinguish parameters referring to the different eras, we introduce an era index as a subscript in square brackets. The e-th era begins at time t_e and, if not the last era, ends at time t_{e+1}. We let $t_1 = t_0$, i.e., the reference time t_0 coincides with the beginning of the first era. A parameter X from the e-th era will be denoted as $X_{[e]}$.

The expression for the flow component is

$$\mathbf{l}_{[e]}^{(\mathrm{f})}(t,\mathbf{f}) = \mathbf{G}_{[e]}^{(\mathrm{d})}(t,t_a)\mathbf{l}_{[e]}^{(\mathrm{f})}(t_a,\mathbf{f}) - \mathbf{G}_{[e]}^{(\mathrm{f})}(t,t_a)\mathbf{f}(t_a) \ ,$$

where, as before, the first era starts with $\mathbf{l}_{[\mathbf{1}]}^{(\mathrm{f})}(t_1, \mathbf{f}) = 0$. At the transition from one era to the next, the levels are evaluated and serve as the initial levels for the next era, the function now evaluated with the new set of parameters, i.e., $\mathbf{l}_{[\boldsymbol{e}+\mathbf{1}]}^{(\mathrm{f})}(t_{e+1}, \mathbf{f}) = \mathbf{l}_{[e]}^{(\mathrm{f})}(t_{e+1}, \mathbf{f})$.

To simplify the notation hereafter, we use $\mathbf{X}_{[e]}(t)$ as the shorthand expression for $\mathbf{X}_{[e]}(t, t_e)$. We thus find the intrinsic component of the reservoir levels for era e, i.e., for $t \geq t_e$, as

$$\mathbf{l}_{[e]}^{(\mathrm{i})}(t) \quad = \mathbf{P}_{[e]}^{(\mathrm{r})}(t)\mathbf{l}_{\mathbf{0}[e]}^{(\mathrm{i})} + \mathbf{Q}_{[e]}^{(\mathrm{r})}(t)\mathbf{c}_{[e]}^{(\mathrm{r})}$$

and from that the initial levels of the following era as

$$\mathbf{l}_{\mathbf{0}[e+\mathbf{1}]}^{(\mathrm{i})} = \mathbf{l}_{[e]}^{(\mathrm{i})}(t_{e+1}) \quad = \boldsymbol{\mathcal{P}}_{[e]}^{(\mathrm{r})}\mathbf{l}_{\mathbf{0}[e]}^{(\mathrm{i})} + \boldsymbol{\mathcal{Q}}_{[e]}^{(\mathrm{r})}\mathbf{c}_{[e]}^{(\mathrm{r})}$$

where we introduced another shorthand notation $\boldsymbol{\mathcal{P}}_{[e]}^{(\mathrm{r})} = \mathbf{P}_{[e]}^{(\mathrm{r})}(t_{e+1})$ and $\boldsymbol{\mathcal{Q}}_{[e]}^{(\mathrm{r})} = \mathbf{Q}_{[e]}^{(\mathrm{r})}(t_{e+1})$.

We see that the levels of each era are dependent on all previous eras through its initial values. As the initial levels, $\mathbf{l}_{\mathbf{0}[\mathbf{1}]}^{(\mathrm{i})}$, have to be defined, we choose those as the equilibrium levels $\bar{\mathbf{l}}$ of a system having the first era's parameters $\mathbf{c}_{[\mathbf{1}]}^{(\mathrm{r})}$. These constant levels describe the reservoir up to time t_1 and can be calculated in general as

$$\bar{\mathbf{l}} = \overline{\mathbf{H}}\, \mathbf{c}_{[\mathbf{1}]}^{(\mathrm{r})}$$

with $\overline{\mathbf{H}}$ to be defined later. We can thus express the intrinsic levels through a matrix equation where we combine all $\mathbf{c}_{[e]}^{(\mathrm{r})}$ into one vector $\mathbf{c}_{\mathrm{E}}^{(\mathrm{r})}$ to give

$$\mathbf{l}_{[e]}^{(\mathrm{i})}(t) \quad = \mathbf{R}_{[e]}^{(\mathrm{r})}(t)\mathbf{c}_{\mathrm{E}}^{(\mathrm{r})}$$

with

$$\begin{aligned} \mathbf{c}_{\mathrm{E}}^{(\mathrm{r})} \quad &= \left[\mathbf{c}_{[\mathbf{1}]}^{(\mathrm{r})^T}, \ldots, \mathbf{c}_{[N_{\mathrm{E}}]}^{(\mathrm{r})^T}\right]^T \\ \mathbf{R}_{[e]}^{(\mathrm{r})}(t) \quad &= \left[\mathbf{H}_{[\mathbf{1}][e]}(t), \ldots, \mathbf{H}_{[N_{\mathrm{E}}][e]}(t)\right]. \end{aligned}$$

We find the function $\mathbf{R}_{[e]}^{(\mathrm{r})}(t)$ for the time before the first era as

$$\mathbf{R}_{[\mathbf{0}]}^{(\mathrm{r})}(t) = \quad \left[\overline{\mathbf{H}}, \mathbf{0}, \ldots, \mathbf{0}\right],$$

for the first era as

$$\mathbf{R}_{[\mathbf{1}]}^{(\mathrm{r})}(t) = \quad \left[\left[\mathbf{P}_{[\mathbf{1}]}^{(\mathrm{r})}(t)\overline{\mathbf{H}} + \mathbf{Q}_{[\mathbf{1}]}^{(\mathrm{r})}(t)\right], \mathbf{0}, \ldots, \mathbf{0}\right],$$

and for all later eras from the relationship

$$\begin{aligned} \mathbf{R}_{[\boldsymbol{e}+\mathbf{1}]}^{(\mathrm{r})}(t) = \quad & \left[\mathbf{P}_{[\boldsymbol{e}+\mathbf{1}]}^{(\mathrm{r})}(t)\left[\boldsymbol{\mathcal{H}}_{[\mathbf{1}][e]}, \ldots, \boldsymbol{\mathcal{H}}_{[e][e]}\right], \ldots \right. \\ & \left. \mathbf{Q}_{[\boldsymbol{e}+\mathbf{1}]}^{(\mathrm{r})}(t), \mathbf{0}, \ldots, \mathbf{0}\right], \end{aligned}$$

where we used the shorthand notation $\boldsymbol{\mathcal{H}}_{[\boldsymbol{d}][e]} = \mathbf{H}_{[\boldsymbol{d}][e]}(t_{e+1})$, and $\mathbf{0}$ to denote the zero-matrix with the same dimensions as $\boldsymbol{\mathcal{H}}_{[\boldsymbol{d}][e]}$. Along the same lines, we write for the well levels

$$\mathbf{l}_{[e]}^{(\mathrm{w})}(t, \mathbf{f}) \quad = \mathbf{R}_{[e]}^{(\mathrm{w})}(t, \mathbf{f})\mathbf{c}_{\mathrm{E}}^{(\mathrm{w})}$$

with

$$\begin{aligned} \mathbf{c}_{\mathrm{E}}^{(\mathrm{w})} = [\quad & r_{\mathrm{dd}},\ r_{\mathrm{f}},\ \mathbf{c}_{[\mathbf{1}]}^{(\mathrm{w})^T}, \quad \ldots\ , \mathbf{c}_{[N_{\mathrm{E}}]}^{(\mathrm{w})^T}]^T \\ \mathbf{R}_{[e]}^{(\mathrm{w})}(t, \mathbf{f}) = [\quad & \mathbf{l}^{(\mathrm{dd})}(t, \mathbf{f}),\ \mathbf{Z}^T\mathbf{l}^{(\mathrm{f})}(t, \mathbf{Z}\mathbf{f}), \quad \ldots \\ & \mathbf{Z}_{[e]}^T\mathbf{H}_{[\mathbf{1}][e]}(t),\ \delta_{1,e}\left[\mathbf{Y}_{[e]}^T, \mathbf{J}_{N_{\mathrm{W}}}, \mathbf{j}_{N_{\mathrm{W}}}\right], \quad \ldots \\ & \ldots \\ & \mathbf{Z}_{[e]}^T\mathbf{H}_{[N_{\mathrm{E}}][e]}(t),\ \delta_{N_{\mathrm{E}},e}\left[\mathbf{Y}_{[e]}^T, \mathbf{J}_{N_{\mathrm{W}}}, \mathbf{j}_{N_{\mathrm{W}}}\right]\ ], \end{aligned}$$

where we introduced two factors, r_{dd} and r_{f}, which can be used to scale the flow-dependent contributions but will in most applications simply be set to one.

It remains to derive the expression for $\overline{\mathbf{H}}$ to find $\bar{\mathbf{l}}$. The equilibrium levels are the steady-state levels defined by

$$\mathbf{C}_{[1]}\bar{\mathbf{l}} = \mathbf{\Omega}_{[1]}\mathbf{l}_{[1]}^{(\mathrm{b})} - \bar{\mathbf{f}}_{[1]}$$

from which we find

$$\overline{\mathbf{H}} = \left[-{\mathbf{C}_{[1]}}^{-1}, {\mathbf{C}_{[1]}}^{-1}\mathbf{\Omega}_{[1]}, \mathbf{0}_{N_\mathrm{R}}\right].$$

For a "closed" system, i.e., when all ω_i (or all those of any isolated subset of reservoirs) are zero, we cannot find a unique solution to this equation due to the singularity of $\mathbf{C}_{[1]}$. We can, however, evaluate well defined equilibrium levels indirectly. First, we shall construct the pseudoinverse

$${\mathbf{C}_{[1]}}^{+} = \mathbf{U}\mathbf{\Lambda}^{+}\mathbf{U}$$

with

$$\mathbf{C}_{[1]}\mathbf{U} = \mathbf{U}\mathbf{\Lambda} \quad \text{and} \quad \Lambda_{ij}^{+} = \begin{cases} \delta_{i,j}/\lambda_i & \text{if } \lambda_i \neq 0 \\ 0 & \text{if } \lambda_i = 0 \end{cases}.$$

and we find the relative equilibrium levels as $\tilde{\mathbf{l}}^{(\mathrm{f})} = -{\mathbf{C}_{[1]}}^{+}\bar{\mathbf{f}}_{[1]}$. To find the absolute equilibrium levels, we construct an open reference system with $\mathbf{\Omega}_{[0]}$, where the conductance ω of one of the reservoirs (in each of the isolated subsets) is set to unity. Thus, we find the level offset as

$$\tilde{\mathbf{l}}^{(\mathrm{b})} = \left[\mathbf{C}_{[1]} + \mathbf{\Omega}_{[0]}\right]^{-1}\mathbf{\Omega}_{[0]}\mathbf{l}_{[1]}^{(\mathrm{b})}$$

and from $\bar{\mathbf{l}} = \tilde{\mathbf{l}}^{(\mathrm{f})} + \tilde{\mathbf{l}}^{(\mathrm{b})}$ we find eventually

$$\overline{\mathbf{H}} = \left[-{\mathbf{C}_{[1]}}^{+}, \left[\mathbf{C}_{[1]} + \mathbf{\Omega}_{[0]}\right]^{-1}\mathbf{\Omega}_{[0]}, \mathbf{0}_{N_\mathrm{R}}\right].$$

In this formalism, the initial equilibrium levels are defined parametrically through $\bar{\mathbf{f}}_{[1]}$ and $\mathbf{l}_{[1]}^{(\mathrm{b})}$, both for the open and the closed initial system. It should be noted, though, that the accumulated flow for each closed set of reservoirs must be zero to describe a physically meaningful system.

3. INVERSE MODELLING

By comparing calculated and measured observables during a period of operation of the field it is possible to find those model parameters that yield the best agreement of theory and "experiment". What is to be considered *best* depends on the reservoir's properties of interest and often, to complicate things even further, on personal preference of the analyst. In this section we will be focusing on a generalized least squares approach to optimize agreement of calculated and measured well levels. Nevertheless, the general optimization procedure illustrated here will apply for any qualifier of a model's performance, if it can be expressed in an objective function of equivalent form.

We seek to optimize the agreement of N_o observed and calculated properties at one of N_s sites. These are for time t described by the vectors $\mathbf{o}(t)$ and $\mathbf{l}(t,\{p\})$, respectively, where the latter function depends both on time and the set of model parameters, $\{p\}$. Our set of weighted observations, $\{\mathcal{O}^W\}$ consists of $N_\mathbf{o}$ tuples of time t_k, site-index s_k, value $o_k = \mathbf{o}_{s_k}(t_k)$ and weight w_k. We now expand the calculated function as the sum of a function of t alone and a linear combination of N_b basis functions depending on both t and $\{p\}$. The expansion coefficients are built up as a linear transform of N_v variable coefficients.

$$\mathbf{l}(t,\{p\}) = \mathbf{l}_0(t) + \mathbf{B}(t,\{p\})\mathbf{c}$$

with

$$\mathbf{c} = \mathbf{c}_0 + \mathbf{T}\mathbf{v}$$

The generalized least-squared-deviation (GLSD) object function

$$S(\{p\},\{\mathcal{O}^W\}) = \sum_{k=1}^{N_\mathrm{o}} w_k \left(l_{s_k}(t_k,\{p\}) - o_{s_k}(t_k)\right)^2$$

can in thus be rewritten in matrix form as

$$S(\{p\},\{\mathcal{O}^W\}) = \mathbf{v}^T\mathbf{A}\mathbf{v} + 2\,\mathbf{v}^T\mathbf{b} + s_o \tag{7}$$

with

$$\begin{aligned}\mathbf{A} &= \mathbf{T}^T\left[\sum_{k=1}^{N_o} w_k \mathbf{B}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{B}(t_k)\right]\mathbf{T} \\ \mathbf{b} &= \mathbf{T}^T\left[\sum_{k=1}^{N_o} w_k \mathbf{B}(t_k)^T \mathbf{E}^{(s_k)} (\mathbf{d}_0(t_k)+\mathbf{B}(t_k)\mathbf{c}_0)\right] \\ s_0 &= \sum_{k=1}^{N_o} w_k (\mathbf{d}_0(t_k)+\mathbf{B}(t_k)\mathbf{c}_0)^T \mathbf{E}^{(s_k)} (\mathbf{d}_0(t_k)+\mathbf{B}(t_k)\mathbf{c}_0)\end{aligned}$$

where $\mathbf{d}_0 = \mathbf{l}_0 - \mathbf{o}$, the $(N_\mathrm{s} \times N_\mathrm{s})$ matrix $\mathbf{E}^{(\mathrm{i})}$ has a single non-zero element, $E_{ii}^{(\mathrm{i})} = 1$ and we dropped denoting the dependency on $\{p\}$ explicitly for the sake of readability. For completeness, we arbitrarily set unmeasured values in $\mathbf{o}(t_k)$ to zero, as they anyway do not appear in the expanded form of the above expressions.

The set of coefficients $\mathbf{v}_\mathrm{opt}$ that minimizes S for a given $\{p\}$ and $\{\mathcal{O}^W\}$ can be found analytically from Eq. 7 as the solution to

$$\mathbf{A}\mathbf{v}_\mathrm{opt} + \mathbf{b} = \mathbf{0}\,,$$

yielding the minimum

$$S_\mathrm{opt} = \mathbf{v}_\mathrm{opt}^T \mathbf{b} + s_o \tag{8}$$

If derivatives with respect to the model parameters are desired, e.g. for more efficient minimization of the object function, these can be derived directly from Eq. 8: For an infinitesimal variation of $\mathbf{B}(t) \to \mathbf{B}(t) + \varepsilon\tilde{\mathbf{B}}(t)$, the minimum value becomes $S + \varepsilon\tilde{S}$ with

$$\tilde{S}(\{p\},\{\mathcal{O}^W\}) = \mathbf{v}^T\tilde{\mathbf{A}}\mathbf{v} + 2\,\mathbf{v}^T\tilde{\mathbf{b}} + \tilde{s}_o$$

where

$$\begin{aligned}\tilde{\mathbf{A}} &= \mathbf{T}^T\left[\sum_{k=1}^{N_\mathrm{o}} w_k \left(\tilde{\mathbf{B}}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{B}(t_k) + \mathbf{B}(t_k)^T \mathbf{E}^{(s_k)} \tilde{\mathbf{B}}(t_k)\right)\right]\mathbf{T} \\ \tilde{\mathbf{b}} &= \mathbf{T}^T\left[\sum_{k=1}^{N_\mathrm{o}} w_k \left(\tilde{\mathbf{B}}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{B}(t_k) + \mathbf{B}(t_k)^T \mathbf{E}^{(s_k)} \tilde{\mathbf{B}}(t_k)\right)\right]\mathbf{c}_0 \\ &\quad + \mathbf{T}^T\left[\sum_{k=1}^{N_\mathrm{o}} w_k \left(\tilde{\mathbf{B}}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{d}_0(t_k)\right)\right] \\ \tilde{s}_0 &= 2\mathbf{c}_0^T\left[\sum_{k=1}^{N_\mathrm{o}} w_k \left(\tilde{\mathbf{B}}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{B}(t_k)\right)\right]\mathbf{c}_0 \\ &\quad + 2\mathbf{c}_0^T\left[\sum_{k=1}^{N_\mathrm{o}} w_k \left(\tilde{\mathbf{B}}(t_k)^T \mathbf{E}^{(s_k)} \mathbf{d}_0(t_k)\right)\right].\end{aligned}$$

The derivative of S_opt with respect to the of the parameters p_a can thus be evaluated analytically if $\tilde{\mathbf{B}}(t) = \frac{\partial}{\partial p_a}\mathbf{B}\,(t)$ is known.

4. SOFTWARE IMPLEMENTATION

4.1 Implementation Status

The generalized LPRM derived in the previous sections has been implemented in the computer program GLUMP. The developmental version of the program has been written in modern Fortran. To promote ease of use and reusability it is currently re-implemented in Python. It is planned to publish the code as free, libre and open source software (FLOSS) in the hope that it might be of use also outside of our organization.

4.1 Optimization Algorithm

The inverse modelling in GLUMP is based on the GLSD object function as specified in the previous section. While analytical gradients of this object function are obtainable analytically, they are tedious to implement in a robust way due to the possibility of degenerate eigenvalues. So far it was not a priority to implement these and thus the gradient-free Melder-Nead minimization (Melder and Nead, 1965) was used in the inverse modelling and provided sufficient performance for the following test cases.

The form of the derived LPRM works very well with our minimizer: Firstly, all reservoir levels and the initial flow are contained as extrinsic parameters in the $\mathbf{c}_\mathrm{E}^{(\mathrm{w})}$ vector and their optimal values are thus found analytically. Secondly, the remaining intrinsic parameters are all non-negative by definition. This allows to manage their widely different orders of magnitude efficiently by using their logarithm as the unknowns in the numerical optimization.

4.2 Numerical Issues

Numerical issues hamper a solid implementation of the LPRM fit for use in self-consistent inverse modelling.

The solution to the integral given in Eq. 3 is an analytic function but is numerically instable if λ_i is very small or zero. This occurs for "closed" (sub-)systems, i.e., if the reservoirs (or an isolated subset of reservoirs) are only connected to each other and not to any background reservoir. Often the numerical issues were tried to overcome by classifying a system either as "open" or "closed" and using a different analytical expression for the integral in the latter case. However, during the inverse modelling process an open system would "close up" if such a system would yield a better model, resulting in numerical issues or floating-point exceptions.

A way around this is to reformulate Eq. 3 as $\mu(\delta,\lambda) = \delta\nu(\lambda\delta)$ using

$$\nu(x) = \frac{1 - e^{-x}}{x},$$

which we then approximate by the finite power series

$$\nu(x) \approx \tilde{\nu}^{(N)}(x) = \sum_{n=1}^{N} \frac{(-x)^{n-1}}{n!}.$$

Using an appropriate weight function $w(x)$ with $w(x) = 1$ for $x > \varepsilon_1$ and $w(x) = 0$ for $0 \le x \le \varepsilon_0$ we can patch the problematic formula as

$$\mu(\delta,\lambda) \approx \delta\left[\nu(\lambda\delta)[w(\lambda\delta)] + \tilde{\nu}^{(N)}(\lambda\delta)[1 - w(\lambda\delta)]\right].$$

By this the integral can be evaluated for all values of λ_i and by careful selection of $w(x)$, ε_0, and ε_1, control over the magnitude of errors and, if desired, continuity of the function and its derivatives is regained.

Another numerical issue occurs in the reverse process: A closed system can possibly "open up" by increasing the base area of a reservoir to infinity, by which it then effectively acts as a background reservoir. Reviewing the expression for the reservoir levels, we notice that the base areas $\mathbf{A}$ enter these only indirectly through their effect on the eigenvectors $\mathbf{T}$ and eigenvalues $\mathbf{\Lambda}$. As the matrix $\mathbf{A}$ is by definition diagonal and positive definite, the generalized eigenvalue problem can be transformed to a simpler problem with identical eigenvalues: By left-multiplication of Eq. 2 with $\mathbf{A}^{-½}$ and insertion of the identity $\mathbf{A}^{-½}\mathbf{A}^{½}$, we find

$$\left(\mathbf{A}^{-½}\mathbf{C}\mathbf{A}^{-½}\right)\left(\mathbf{A}^{½}\mathbf{T}\right) = \left(\mathbf{A}^{-½}\mathbf{T}\right)\mathbf{\Lambda}$$

so that by letting $\mathbf{B} = \mathbf{A}^{-½}\mathbf{C}\mathbf{A}^{-½}$ the eigenvalues can be found from

$$\mathbf{B}\mathbf{U} = \mathbf{U}\mathbf{\Lambda}$$

and the general eigenvectors are found as

$$\mathbf{T} = \mathbf{A}^{-½}\mathbf{U}\,.$$

As only $\mathbf{A}^{-½}$ enters these matrix equations, we can use the inverse of the base areas, A_i^{-1}, as the primary parameters in the model specification. In this convention, infinitely large reservoirs are conveniently realized by letting $A_i^{-1} = 0$, avoiding these numerical issues.

At least two further numerical issues are possible: The base areas could approach zero, corresponding to a reservoir with vanishing volume. As we saw while introducing wells, a system containing a reservoir of zero volume can be reformulated as an equivalent system of one less reservoir. One could thus calculate the levels in the remaining reservoirs by constructing such a system, as a workaround.

Another possible numerical instability will be caused by any conductance approaching infinity. This corresponds to an immediate equilibration of levels in the connected reservoirs. Again, as a workaround, an equivalent model with a reduced number of reservoirs can be constructed, by replacing the reservoirs of this connection by a single reservoir with a base area equal to their combined base areas.

5. SELECTED APPLICATIONS

5.1 Seismic Disruption

Earthquakes can result in significant changes to the properties of a geothermal field. Fractures can be both opened or closed, by which permeability changes and a new pressure equilibrium is realized. The porous aquifers can furthermore be compressed or decompressed, by which both their storage capacity changes, and their water levels shift before finding a new equilibrium. This was observed, e.g., in well NJ-21 in the Nesjavellir field for the second South-Iceland earthquake of 2000.

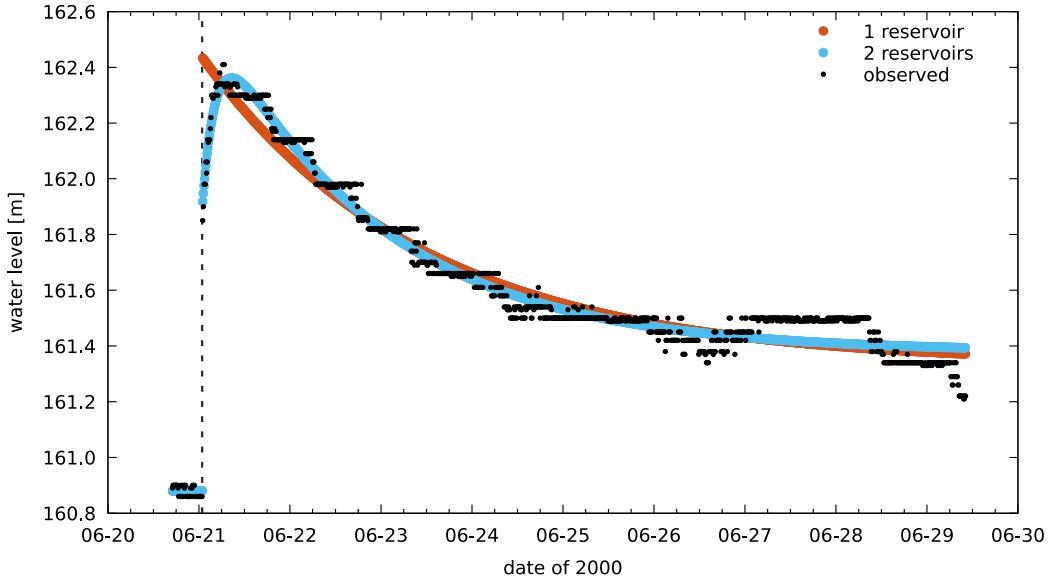

Figure 1: Water levels in NJ-21. The dashed line marks the time of the earthquake.

We assume the levels to be in equilibrium before the quake and neglect any production in neighboring wells. Two models are compared, both with a second era starting at 00:51 on June 21, 2000. In the first case the well is connected to a single reservoir, which itself is connected to a background reservoir with a level of 160.88 m. In the other case, this reservoir is connected to another, single reservoir. In the second era, the background level, background conductance, and reservoir level jump are optimized. For the second case, we furthermore optimize the ratio of reservoir areas as well as the reservoir conductance between them. While the simpler model can simulate the jump and decline, it fails to model the intermediate increase of water levels. The still very simple two-reservoir model predicts the jump, rise and decline of the water levels remarkably well.

5.2 Field Data Salvage

It may happen that field data readings are known to be erroneous, but it may not be possible to replace or recalibrate the equipment right away. As an example, we shall consider a failure in the water level measurements, which are done by measuring the hydrostatic pressure of the water column in the well through a tube. The tube begins to leak at some point down the well. This corresponds to shifting the reference of the calculated water level. By comparing readings before and after the failure one may find by hand a sensible shift to use as a correction of the water levels. We shall now attempt this using our reservoir model.

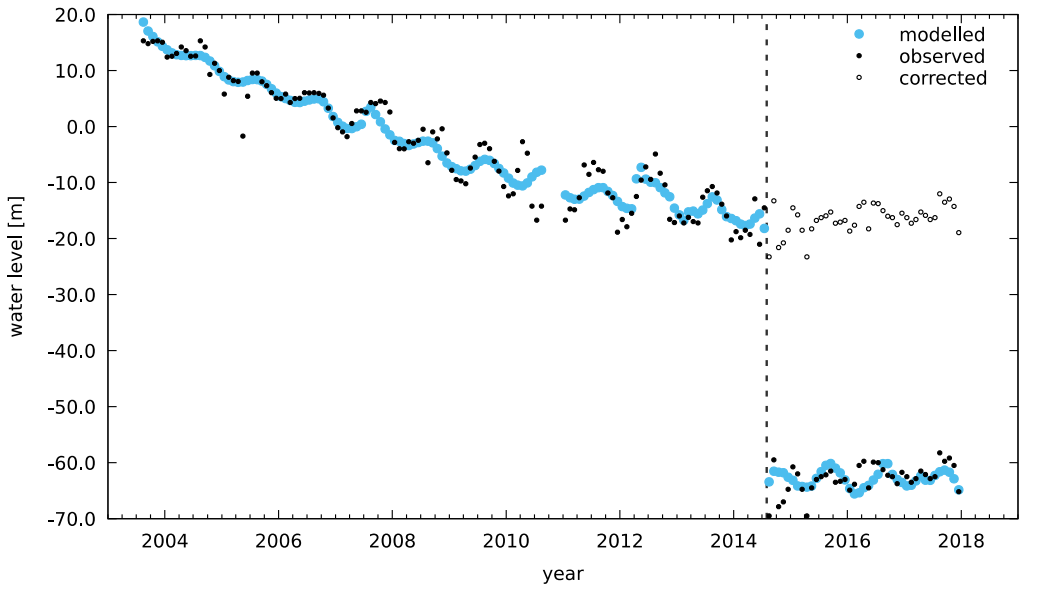

Figure 2: Water levels in ÖN-29. The dashed line marks the assumed time of the leak.

The example is taken from well ÖN-29 in West-Iceland, which shows an unreasonable drop of about 50 m in 2014. In a very simplified model, we neglect the effect of other wells in the area, use available monthly averages for flow and water level, and approximate this

system by a "two reservoir open" model. We add a second era on August 1st, 2014, where we merely change the $\mathbf{l}^{(s)}$ of the well. This does not in any way affect the reservoir levels, but merely shifts the well levels by a constant.

We find the optimal shift around -46.25 m. The corrected levels are visualized by open circles in the diagram. The results appear to be quite reasonable, having in mind the simplicity of the model, how coarse the data is, and that no further production from the area was considered.

5.3 Well Testing

Rating curves for well-drawdown are useful for the operation of geothermal fields, e.g., to estimate safe production limits. Using our reservoir model, we take into account both the well's properties as well as the response of the reservoir during a well test and find the parameters describing the well-drawdown directly from the optimized model.

A recent well test of AS-01 in West-Iceland was analyzed in this way. The raw water levels were corrected for tidal effects. As a model we chose a well connected to two reservoirs. The first one with a background reservoir, the second one connected to another reservoir without a background connection. As the weights of our observations we choose a simple time-average, i.e., the weight for each datum is proportional to the time-distance between the two neighboring data. Other production from the area was neglected.

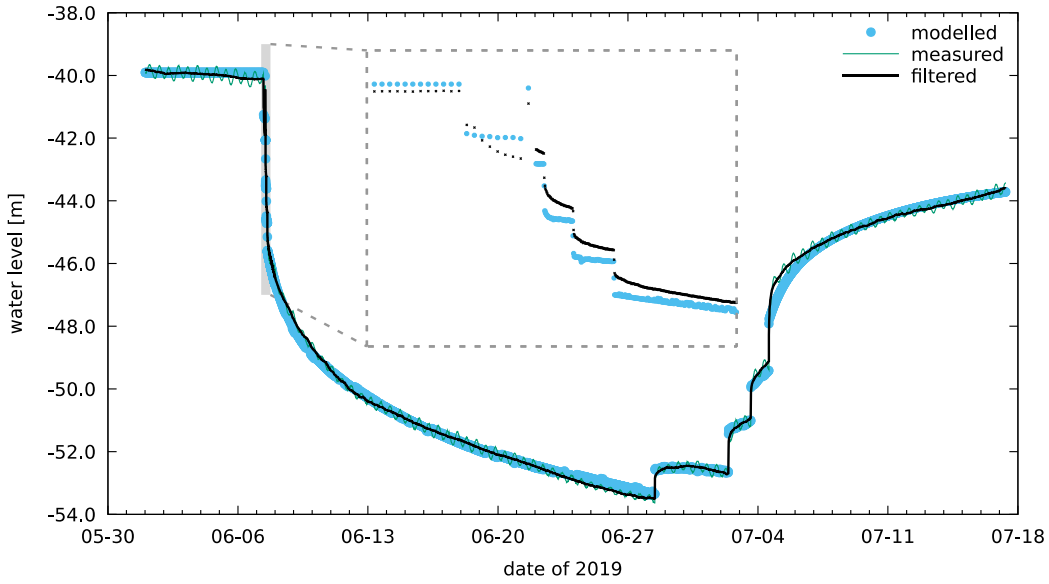

Figure 3: Water levels in AS-01. The inset shows the initial phase of the well test.

After optimization we find the well drawdown as

$$l^{(w)} = l^{(r)} - 0.20\frac{\mathrm{m}}{\mathrm{L/s}} \cdot f - 0.027\frac{\mathrm{m}}{(\mathrm{L/s})^2} \cdot f^2 \,.$$

While the agreement with the well test data is not too impressive, in particular during the short initial phase, it may be expected that a better overall agreement could be achieved by using a different model setup and, in particular, by adjusting the weights of the data points according to their significance. But even for this simple model, the drawdown parameters lie in a reasonable range.

5.4 Long Term Production History

As a last example we will model a field heavily affected by the South Iceland earthquake from 2000. While average flow values are recorded from the 1982, water levels in well LL-04 are currently only available from 1988. In the beginning of 2000, water from another field was injected in one of the wells of this field. Only flow data on the net uptake are available for this time period. After the earthquake on June 17, 2000 the water level has increased significantly.

We will use a model with four reservoirs connected in series where one terminal reservoir has a background connection. The well is connected to the other terminal reservoir. We compare this model to the one with two eras added, one at the beginning of reinjection, and the other one after the earthquake. We find the GLSD values of the inverse modeling as 74.7 m^2 and 39.5 m^2, respectively.

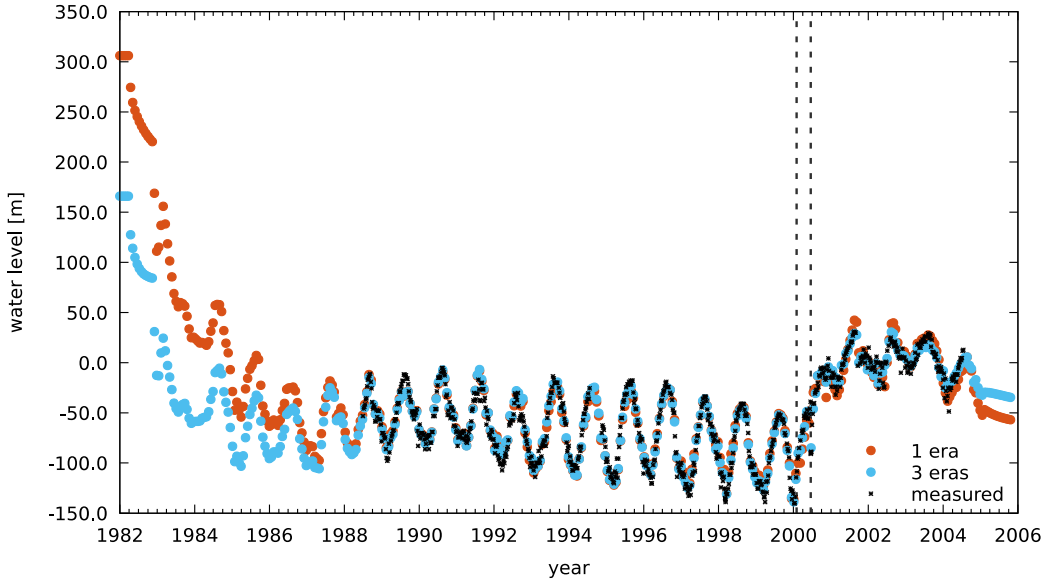

Figure 4: Water levels in LL-04. Observed and calculated values are shown.

It shall be pointed out here, that though these results are an optimal solution of the inverse modelling procedure, the optimal background levels are off the scale and unrealistic. This is a result of the missing measured levels in the first six years of operation, which leaves the model parameters too much freedom, resulting in overfitting.

To get closer to a realistic model, we now fix the background level at 75 m, the surface level of the well. While this is of course only a guess, it will at least force the levels into a more realistic range. Here, we find the difference of the two models to be much more pronounced for the time after the earthquake.

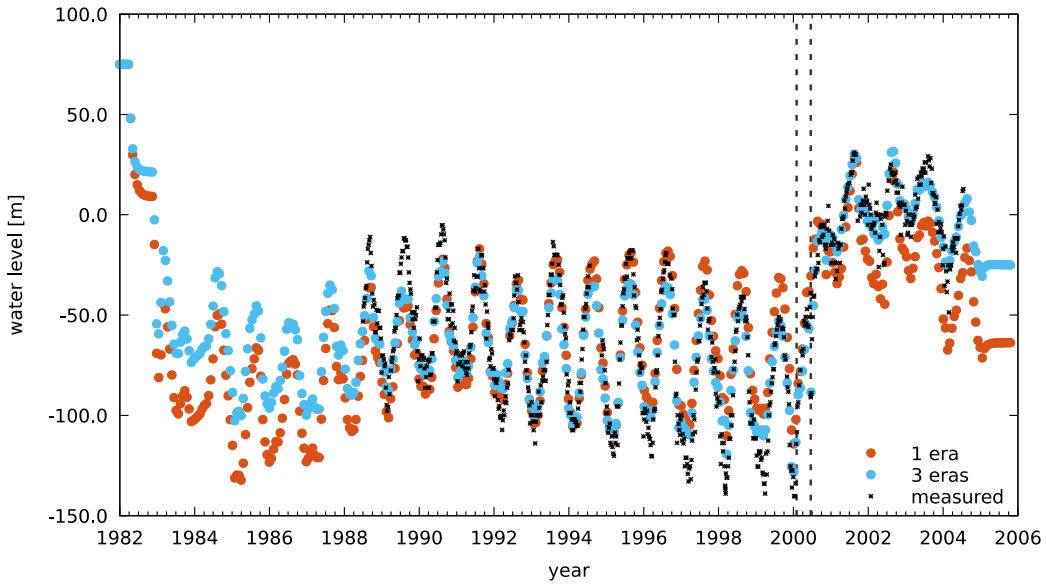

Figure 5: Water levels in LL-04. Background levels were fixed at 75 m.

6. CONCLUSIONS

We have derived a flexible form of a lumped parameter reservoir model and illustrated its features on a few selected test cases. While it is still, by its nature, a very crude approximation to any real geothermal system, it can model some of the systems quite well.

The model is not an alternative to the hitherto often used lumped models but builds upon them and should thus be seen as an extension to them. By choosing the right set of parameters, the conventional models can still be realized within our new framework.

By using the semi-analytical optimization approach, the number of parameters to be optimized numerically is reduced significantly. The numerical optimization is in general not bound to any particular algorithm. Analytical gradients of the objective function are possible to evaluate.

The last example showed us, that the quality of the model strongly depends on the observables it is fitted to. The GLSD qualifier should not be used blindly to select the "best" model, but all of its properties should be viewed together, to avoid overfitting or physically nonsense solutions.

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