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Fracture Propagation in Anisotropic Rocks During Hydraulic Stimulation: Experimental Results and Theoretical Predictions

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ABSTRACT

This paper presents the results of about thirty pure Mode I fracture growth tests on the metamorphic Grimsel Granite that were obtained in order to better interpret the results of an in situ hydraulic stimulation experiment. The results show that although a pure Mode I loading is applied, the crack tends to kink towards the foliation that is weaker in strength. The experimental data on the angle of crack growth as well as apparent fracture toughness are compared to the predictions of three crack growth criteria: the maximum tangential stress (MTS), the maximum energy release rate (MERR), and the minimum strain energy density (MSED). All these criteria overestimate the kink angle and underestimate the apparent fracture toughness.

1. INTRODUCTION

Due to the complex micro-structure, many rock types behave anisotropic elastically or in-elastically. In metamorphic rocks the anisotropy is often associated with foliation in. The transversely isotropic constitutive law, with five independent material constants, has been proven to be a suitable model for predicting the elasticity of rocks that exhibit features such as foliation. This is because foliation causes a preferred orientation of the constituent minerals, pores, and micro-cracks, which often yields a geometrical axis-symmetry (Dambly et al., 2019). Strength and fracture toughness of such rocks also exhibit anisotropy since the dominant orientation of micro-cracks provides a preferential path for the failure and fracture growth (Dutler at al., 2018). An example where rock anisotropy is of significant relevance is related to the enhanced geothermal reservoirs, normally placed in the crystalline basement, where anisotropic rocks such as granite or gneiss are likely to be present. Doetsch et al. (2019) and Gischig et al. (2018) have recently demonstrated that rock anisotropy plays a critical role in the in-situ stimulation and circulation experiments in the deep underground laboratory at the Grimsel test site in Switzerland. It was concluded that the anisotropy of mechanical properties must be taken into account to accurately predict the rock mass deformation, stability and failure in those experiments.

Although the theory of fracture growth in anisotropic rocks has advanced over the last few decades, its validation through reliable experimental data has not received much attention. Most investigations considered only the end-member configurations, where the crack is oriented along one of the principal material directions i.e. parallel or normal to the orientation of the foliation. In order to validate the theory of the fracture toughness variation, fracture toughness must be measured at different orientations of crack growth with respect to the principal directions. Moreover, traditional fracture toughness tests must be modified in order to take into account the effects of elastic anisotropy on the stress intensity factor (SIF) solution. We have recently modified the SCB and the TCBD tests to include the elastic anisotropy in the calculation of the fracture toughness (Nejati et al., 2019). We have also investigated the variations of the different measures of Mode I fracture toughness between the material's principal directions.

In this paper, we present the results of about thirty Mode I fracture toughness experiments on the metamorphic Grimsel Granite with a clear foliation plane. We use the modified SCB test to conduct fracture toughness experiments (Nejati et al., 2019). We compare these experimental data with predictions from three crack growth criteria: the maximum tangential stress, the maximum energy release rate, and the maximum strain energy density. The results show that all these criteria overestimate the kink angle and underestimate the apparent fracture toughness. However, the maximum tangential stress and the maximum energy release rate yield predictions that are closer to the experimental data.

2. FRACTURE GROWTH CRITERIA

This section discusses the theory of fracture propagation in anisotropic solids, by describing the main three crack growth criteria. Among the models proposed to predict Mixed-Mode I/II crack propagation in isotropic solids, the maximum tangential stress (MTS) (Erdogan and Sih, 1963), the maximum energy release rate (MERR) (Hussain et al., 1974), and the minimum strain energy density (MSED) (Sih, 1974) are notable. The MTS criterion remains the most widely used criterion for isotropic materials due to its simplicity and good prediction of experimental results. While these criteria give consistent predictions for the Mode I crack growth in isotropic solids, they cease to be consistent with each other in anisotropic materials.

2.1 Maximum tangential stress (MTS)

The MTS criterion was originally proposed by Erdogan and Sih, (1963) for isotropic materials and later extended by Saouma et al. (1987) for anisotropic solids. The MTS is based on the two following hypotheses: 1) The crack extension starts at its tip in a radial direction, and starts in the plane perpendicular to the direction of maximum tension; and 2) The maximum stress theory applies, indicating that crack extends once the tangential stress reaches a critical value. These hypotheses imply that the crack will start to grow from the tip in the direction of the maximum tangential stress. It can be shown that at any Mixed-Mode I/II loading combination, the direction of the maximum tangential stress is along the direction at which the shear stress is zero.

For anisotropic materials, the MTS criterion shall be stated as follows: 1) The crack extension starts at its tip in a radial direction, and starts in the plane perpendicular to which the ratio of the tensile stress to the tensile strength is maximum. 2) The crack extends once the tangential stress exceeds the strength in that plane. Fulfilling this condition implies that, unlike the isotropic case, the shear stress is not necessarily zero along the crack extension plane. Therefore, when the MTS is applied to anisotropic materials, the effects of the shear stress on the material de-cohesion at the fracture extension plane are ignored. Let us assume that the variation of the fracture toughness between two principal planes 1 and 2 is obtained from a sinusoidal fit as

$$K_{\mathrm{Ic},\beta} = K_{\mathrm{Ic},1}\cos^2\beta + K_{\mathrm{Ic},2}\sin^2\beta \tag{1}$$

where β is the angle between the crack extension plane and the principal direction 1, and $K_{Ic,1}$ and $K_{Ic,2}$ are the fracture toughness along the principal directions 1 and 2. The formulation of the MTS criterion gives the condition for crack growth as:

$$\begin{cases} \dfrac{\mathrm{d}f(\theta)}{\mathrm{d}\theta} = 0, \qquad \dfrac{\mathrm{d}^2 f(\theta)}{\mathrm{d}\theta^2} < 0 \\ K^a_{\mathrm{Ic},\beta} = \dfrac{1}{f(\theta)} \end{cases} \tag{2}$$

where

$$f(\theta) = \frac{\hat{\sigma}_\theta}{K_\mathrm{I}} = \frac{\Re\left[\dfrac{\mu_1}{\mu_1-\mu_2}\Big(\cos\theta+\mu_2\sin\theta\Big)^{3/2} - \dfrac{\mu_2}{\mu_1-\mu_2}\Big(\cos\theta+\mu_1\sin\theta\Big)^{3/2}\right]}{K_{\mathrm{Ic},1}\cos^2(\theta-\beta)+K_{\mathrm{Ic},2}\sin^2(\theta-\beta)} \tag{3}$$

Here, μ_1 and μ_2 are the complex parameters in the crack tip coordinate, θ is the polar angle, and $K^a_{\mathrm{Ic},\beta}$ is the apparent fracture toughness at the onset of crack growth. The direction of the crack extension, θ_0, is determined from the first statement of Eq. 2 while the apparent fracture toughness is obtained by evaluating the second statement at $\theta=\theta_0$.

2.2 Minimum strain energy density (MSED)

The MSED criterion was originally proposed by Sih (1974) for isotropic materials. This criterion postulates that crack growth occurs in the radial direction at which the local strain energy density (SED) reaches its stationary. This stationary point is in the form of a local minimum in isotropic solids. The critical value of the intensity of the strain energy is assumed to be a material property and is a measure of fracture toughness. The concept of the critical strain energy density has also been extended to anisotropic solids. The difference is that in anisotropic materials, the direction of the crack growth is defined by the stationary point of the ratio of SED to the critical SED. More importantly, this stationary point often presents in the form of a local maximum Ye (1994). We therefore refer to the MSED as the maximum strain energy density criterion in anisotropic materials.

In order to determine the critical value of the SED, we assume that a crack extension occurs in a self-similar manner under pure Mode I. The measure of fracture toughness, $S_{Ic,,\beta}$ is the critical value of S_I at the onset of fracture growth in a self-similar manner (θ=0), given by

$$S_{\mathrm{Ic},\beta} = \frac{K^2_{\mathrm{Ic},\beta}}{4\pi E}\left[\xi\cos^2\beta+\sin^2\beta+\frac{\xi(\kappa_1+\kappa_2-1-2\kappa_3\nu'\sqrt{\xi})}{\cos^2\beta+\xi\sin^2\beta}\right] \tag{4}$$

Here, E is the Young's modulus along direction 1, while $\xi = E/E'$ is the ratio of the Young's modulus (E' is the Young's modulus along direction 2). Also, $\kappa_1 = \kappa_2 = \kappa_3 = 1$ for the plane-stress condition, and $\kappa_1 = 1-\upsilon^2$, $\kappa_2 = 1-\xi\upsilon'^2$ and $\kappa_3 = 1+\upsilon$ for the plane-strain condition. Here, υ and υ' are the in-plane and out-of-plane Poisson's ratios. According the hypotheses of the MSED for anisotropic solids, we have:

$$\begin{cases} \dfrac{\mathrm{d}h(\theta)}{\mathrm{d}\theta} = 0, \qquad \dfrac{\mathrm{d}^2 h(\theta)}{\mathrm{d}\theta^2} < 0, \\ K^a_{\mathrm{Ic},\beta} = \sqrt{\dfrac{1}{h(\theta)}}, \end{cases} \tag{5}$$

where

$$h(\theta) = \frac{\hat{S}_\mathrm{I}(\theta)}{K^2_\mathrm{I}} = \frac{\tilde{\boldsymbol{\sigma}}^T\tilde{\boldsymbol{\epsilon}}}{\Big(K_{\mathrm{Ic},1}\cos^2(\theta-\beta)+K_{\mathrm{Ic},2}\sin^2(\theta-\beta)\Big)^2\left[\xi\cos^2(\theta-\beta)+\sin^2(\theta-\beta)+\dfrac{\xi(\kappa_1+\kappa_2-1-2\kappa_3\nu'\sqrt{\xi})}{\cos^2(\theta-\beta)+\xi\sin^2(\theta-\beta)}\right]} \tag{6}$$

Here, $\tilde{\sigma}$, and $\tilde{\epsilon}$ are the vectors of stresses and strains, respectively.

2.3 Maximum energy release rate (MERR)

The MERR was originally proposed by Hussain et al. (1974) for isotropic materials, and was then extended for anisotropic solids by Azhdari and Nemat-Nasser (1996). The ERR is defined as the rate of change in the potential energy with crack area for a linear elastic material. The ERR failure criterion states that crack extension occurs when the energy available for an increment of crack extension, G, is sufficient to overcome a material-dependent critical value, G_c, as a measure of the fracture toughness. In pure Mode I of isotropic materials, the crack tends to grow in a self-similar manner, and therefore the MERR implies that G_I exceeds G_{Ic} at the onset of crack extension, where G_{Ic} is a measure of Mode I fracture toughness. In pure Mode I of anisotropic materials, however, the crack tends to kink, and the kink crack undergoes both opening and shearing modes. In this paper we assume a simplified assumption that the critical ERR under combined loading is the same as that of Mode I crack for that material. Therefore, the MERR criterion in anisotropic materials can be stated as follows: the crack extends when the ERR, G, exceeds the critical value G_{Ic}. The relation for the critical value of the ERR, $G_{Ic,\beta}$, in terms of elasticity parameters is complex and lengthy for the generic case. However, an approximate relation is given by

$$\mathcal{G}_{\mathrm{Ic},\beta} = \frac{K^2_{\mathrm{Ic},\beta}}{2E}\left[\kappa_2\left(\xi + \sqrt{\xi}\right)\cos^2\beta + \kappa_1\left(1 + \sqrt{\xi}\right)\sin^2\beta\right] \tag{7}$$

According to the MERR criterion, the fracture growth occurs under the condition:

$$\begin{cases} \dfrac{\mathrm{d}g(\theta)}{\mathrm{d}\theta} = 0, \qquad \dfrac{\mathrm{d}^2 g(\theta)}{\mathrm{d}\theta^2} < 0\,, \\ K^a_{\mathrm{Ic},\beta} = \sqrt{\dfrac{1}{g(\theta)}}\,, \end{cases} \tag{8}$$

where

$$g(\theta) = \frac{\hat{\mathcal{G}}(\theta)}{K_{\mathrm{I}}^2} = \frac{\tilde{S}^*_{11}\,\Im\left[-\tilde{k}_{\mathrm{I}}^2(\mu_1^* + \mu_2^*)\bar{\mu}_1^*\bar{\mu}_2^* + \tilde{k}_{\mathrm{II}}^2(\mu_1^* + \mu_2^*) - 2\tilde{k}_{\mathrm{I}}\tilde{k}_{\mathrm{II}}\bar{\mu}_1^*\bar{\mu}_2^*\right]}{\left(K_{\mathrm{Ic},1}\cos^2(\theta-\beta) + K_{\mathrm{Ic},2}\sin^2(\theta-\beta)\right)^2\left[\kappa_2\left(\xi + \sqrt{\xi}\right)\cos^2\beta + \kappa_1\left(1 + \sqrt{\xi}\right)\sin^2\beta\right]} \tag{9}$$

$$\begin{aligned} \tilde{k}_{\mathrm{I}} &= \Re\left[\frac{\mu_1}{\mu_1-\mu_2}(\cos\theta + \mu_2\sin\theta)^{3/2} - \frac{\mu_2}{\mu_1-\mu_2}(\cos\theta + \mu_1\sin\theta)^{3/2}\right], \\ \tilde{k}_{\mathrm{II}} &= \Re\left[\frac{\mu_1}{\mu_1-\mu_2}(\sin\theta - \mu_2\cos\theta)\sqrt{\cos\theta + \mu_2\sin\theta} - \frac{\mu_2}{\mu_1-\mu_2}(\sin\theta - \mu_1\cos\theta)\sqrt{\cos\theta + \mu_1\sin\theta}\right] \end{aligned} \tag{10}$$

$$\tilde{S}^*_{11} = \kappa_1\cos^4(\theta-\beta) + \kappa_2\xi\sin^4(\theta-\beta) + \left(\frac{1+\xi+2\xi\nu'(1-\kappa_3\eta)}{4\eta}\right)\sin^2 2(\theta-\beta) \tag{11}$$

3. EXPERIMENTAL SETUP

The granitic samples were prepared from rock cores extracted from the Grimsel Test Site (GTS) in the central Alps of Switzerland. This underground laboratory is located in the granitic formations of the Aare massif at a depth of about 450m underneath the Juchlistock. The granitic rocks in this area were metamorphosed during the Alpine deformation, resulting in the transformation of the lamprophyres into meta-basic dykes with enriched biotite content. Additionally, deformation caused a reorientation of sheet silicates, forming a pervasive foliation that is clearly visible in the rock cores. This deformation history explains why mechanical anisotropy is present in this meta-granite. One meter of core from the injection borehole INJ2, with an original diameter of 120mm were used to prepare test specimens. The core material was taken at a depth of 27-28m, and was used to prepare 29 SCB samples for a set of fracture toughness experiments, referred to as Granite-Set-I.

We employed a modified SCB test in order to conduct pure Mode I fracture toughness experiments (Nejati, 2019) (see Figure 1a). Note that the SCB test with symmetrical loading (S_1=S_2), yields a Mixed-Mode I/II crack tip loading condition in materials that do not show symmetry with respect to the point-load direction. To prepare the Granitic SCB samples, the original core was sub-cored in the direction normal to its axis in such a way that the foliation plane is parallel to the axis of the sub-core. A meter of the original core of granite produced eight sub-cores with a diameter of 95mm, each being cut to yield four SCB samples that have identical angles between the SCB base and the foliation. In total, 29 samples from Granite-Set-I with seven different angles, $\beta = 0°, 15°, 30°, 45°, 60°, 75°, 90°$ were prepared. Figure 1b shows an example of the modified SCB test conducted on Grimsel Granite.

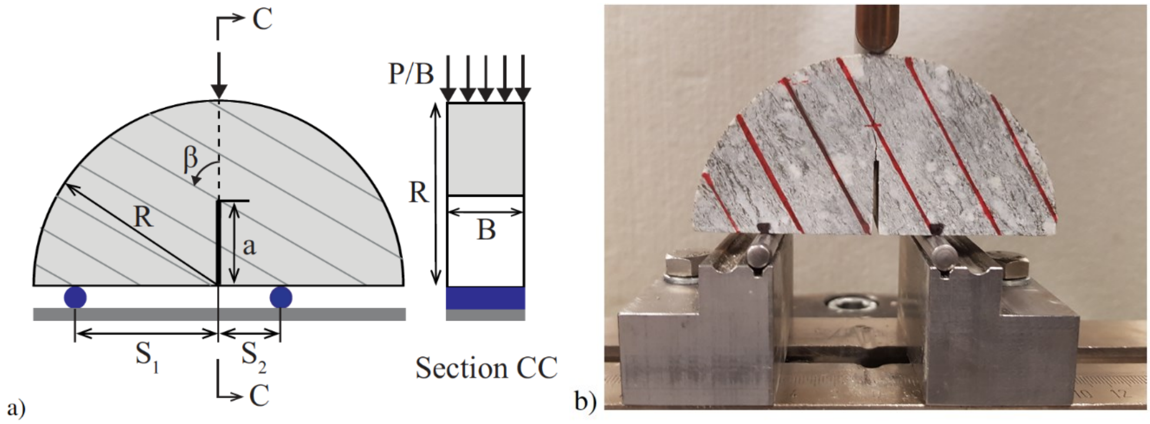

Figure 1: a) Schematics of the modified SCB test; b) Grimsel Granite SCB samples.

3. RESULTS AND DISCUSSION

Figure 2 shows the experimental data on the kink angle, θ_0 in Figure 2, as well as the apparent fracture toughness, $K^{a}_{\mathrm{Ic},\beta}$, for the experiment set. The kink angle is measured based on the average of the angle between the crack path within the process zone and the notch plane. The data shows that the crack generally tends to kink towards the foliation plane that are weaker in strength. The data also demonstrate that while it is straightforward to measure the fracture toughness along the principal direction 1 ($\beta = 0°$), the measurement of the fracture toughness along the principal direction 2 with higher strength ($\beta = 90°$) seems to be more challenging due to the slight crack kinking. Nevertheless, the kink angle for the case $\beta = 90°$ is rather small, with $\theta_0 < 20°$ for most of experiments. This allows to consider the average of the apparent fracture toughness for the cases $\beta = 0°$ and $\beta = 90°$ as the fracture toughness along the principal direction 1 and 2, respectively. These principal values of the fracture toughness together with the elastic constants are substituted in the formulae given in Eqs. 1, 4 and 7 to determine the variations of the three measures of fracture toughness.

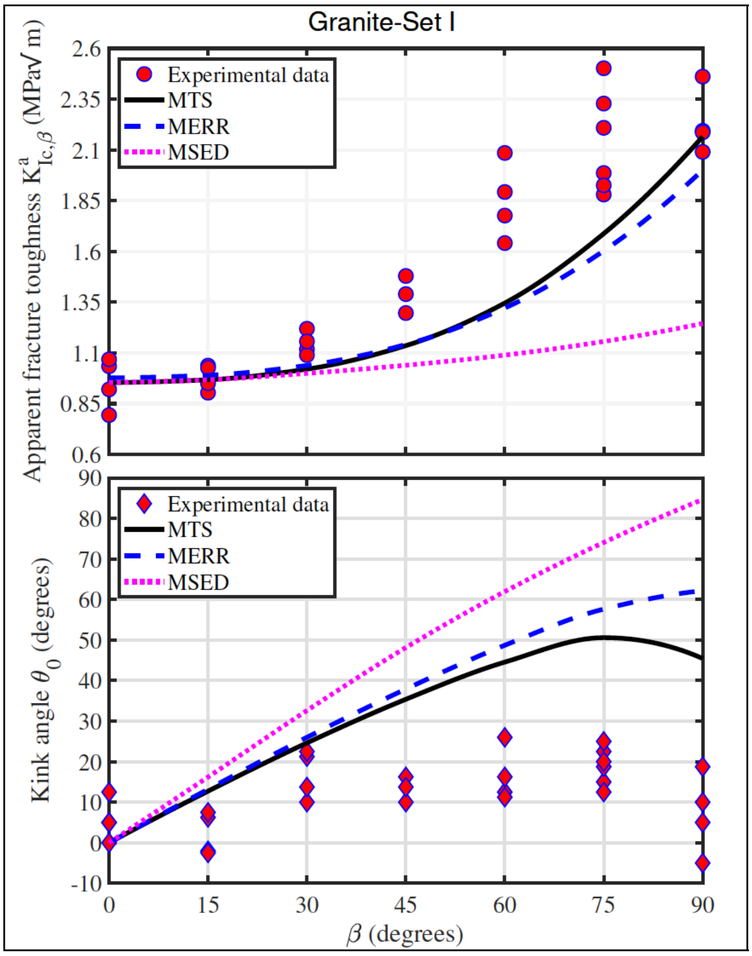

Figure 2: The variations of the apparent Mode I fracture toughness ($K^{a}_{\mathrm{Ic},\beta}$) and the kink angle ($\theta_0$) against β. The predictions of the MTS, MERR and MSED criteria (in the plane-strain condition) are also plotted.

3. CONCLUSIONS

Mode I fracture experiments on Grimsel Granite show that the crack generally tends to kink towards the plane that is weaker in strength (foliation). Unlike isotropic rocks, large kink angles can therefore be expected for Mode I fracture growth in anisotropic rocks. Although it is straightforward to measure the fracture toughness along the principal direction 1 (foliation plane), this measurement along the principal direction 2, that has higher strength, seems to be more challenging due to crack kinking. The significant variations of the kink angle against β implies the competition of the loading and material anisotropy on influencing the fracture growth direction. This competition is mainly visible in the range $60° < \beta < 90°$. The maximum kink angle occurs when the principal direction with weaker strength (direction 1) makes an angle $60° < \beta < 75°$ with the original crack plane. We therefore expect the largest deviation of the crack path when β is in this range. The MSED seems to give predictions that are the furthest from the experimental data, while the MERR and the MTS give better predictions.

REFERENCES

Aminzadeh, A., Fahimifar, A., Nejati, M.: On Brazilian disk test for mixed-mode I/II fracture toughness experiments of anisotropic rocks. Theoretical and Applied Fracture Mechanics 102 (2019), 222–238.

Azhdari, A., Nemat-Nasser, S.: Energy-release rate and crack kinking in anisotropic brittle solids. Journal of the Mechanics and Physics of Solids 44 (1996), 929–951.

Dambly, M., Nejati, M., Vogler, D., Saar, M. O.: On the direct measurement of the shear moduli in transversely isotropic rocks using the uniaxial compression test. International Journal of Rock Mechanics and Mining Sciences, 113 (2019) 220–240.

Doetsch, J., Gischig, V., Villiger, L., Krietsch, H., Nejati, M., Amann, F., Jalali, M., Madonna, C., Maurer, H., Wiemer, S., Driesner, T., Giardini, D.: Subsurface fluid pressure and rock deformation monitoring using seismic velocity observations. Geophysical Research Letters 45 (2018), 10389–10397.

Dutler, N., Nejati, M., Valley, B., Amann, F., Molinari, G.: On the link between fracture toughness, tensile strength, and fracture process zone in anisotropic rocks. Engineering Fracture Mechanics 201 (2018), 56–79.

Erdogan, F., Sih, G. C.: On the crack extension in plane loading and transverse shear. Journal of Basic Engineering 85 (1963), 519–527.

Gischig, V., Doetsch, J., Maurer, H., Krietsch, H., Amann, F., Frederick Evans, K., Nejati, M., Jalali, M., Valley, B., Christine Obermann, A., Wiemer, S., Giardini, D.: On the link between stress field and small-scale hydraulic fracture growth in anisotropic rock derived from microseismicity. Solid Earth 9 (2018), 39–61.

Hussain, M. A., Pu, S. L., Underwood, J.: Strain energy release rate for a crack under combined mode I and mode II. ASTM Special Technical Publication 560 (1974), 2–28.

Nejati, M., Aminzadeh, A., Driesner, T., Saar, M. O.: Semi-circular bend test designed for mode I fracture toughness experiments in anisotropic rocks. Engineering Fracture Mechanics 213 (2019), 153–171.

Saouma, V. E., Ayari, M. L., Leavell, D. A.: Mixed mode crack propagation in homogeneous anisotropic solids. Engineering Fracture

Mechanics 27 (1987), 171–184.

Sih, G. C.: Strain-energy-density factor applied to mixed mode crack problems. International Journal of Fracture 10 (1974), 305–321.

Ye, Z., Ayari, M. L.: Prediction of crack propagation in anisotropic solids. Engineering Fracture Mechanics 49 (1994), 797–808.