

Characterization of a Geothermal Reservoir Using a Transdimensional Inversion Method

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ABSTRACT

The Waiwera hydrothermal system is a geothermal reservoir in a fractured sandstone aquifer. Existing models simulate the aquifer using continuum approximations, neglecting the effect of individual geological features. In this paper we present a methodology using discrete fractures to model the thermal anomaly at the site. There are a number of challenges associated with modeling discrete features. For instance, the exact number of stratified layers of the sandstone aquifer and intersecting faults is unknown. This limits the applicability of any discrete fracture network model, as it requires a predefined number of model parameters for inversion. To resolve this issue, we apply a transdimensional fracture network inversion method. The approach relies on a reversible-jump Markov chain Monte Carlo algorithm that allows for variable number of model parameters, thereby simultaneously calibrating the number of fractures in the model. This technique has already successfully been applied on smaller scales, to interpret cross-borehole tracer tomography experiments. We demonstrate that the methodology is also applicable for reservoir scale aquifer characterization, with some modifications. We show how the discrete approach could reconstruct temperature observations and we demonstrate how this approach could reveal the discrete features (faults and layers) of the aquifer. The presented findings will be used for the preparation of new investigations, and to enhance existing 3-D continuum models.

1. INTRODUCTION

In hard rock aquifers fractures provide the fastest pathways for flow and transport, with hydraulic conductivities many of orders of magnitude higher than the rock matrix. Fracture patterns have a huge impact on aquifer heterogeneity and anisotropy. Hydraulic properties react in a non-linear way to fracturing, as small changes in the fracture geometry could lead to completely different flow regimes. The development of characterization methods for fractured media is still in focus, with very different approaches from recent years.

Continuum models substitute the fractured rock with a porous medium of equivalent hydrological properties. These models are most suitable for aquifers where the fracture intensity is very high or very low. The continuum approach however strongly limits the possibility of identifying dominant discrete geological features and could not model fracture flow and transport that well (Day-Lewis et al., 2000; Illman and Neuman, 2003; Sahimi, 2011; Vesselinov et al., 2001). Discrete fracture network models (DFNs) are realistic representations of the fractured media, which can be generated based on the geostatistical observations. Beside the realism aspect, DFN models also have the computational advantage as the flow-transport problem can be separated between the fracture space and the rock matrix. Still DFN models are not commonly used for inverse modelling, due to the challenges originating from their parametrization. In a DFN model every fracture is parametrized separately, by multiple parameters. This leads to models with many hundreds of parameters, and to ill-posed inversion problems with non-unique solutions (Aster et al., 2013). When only limited observation data is available, the parameter numbers have to be reduced. Simplified DFN models were used by (Fischer et al., 2018; Le Goc et al., 2010; Klepikova et al., 2014), to interpret various cross-well experiments. In these studies, the fractured medium was simplified to only a handful of fractures that represent the relevant flow and transport pathways inside the fractured rock.

Tomographic experiments or joint inversion of geophysical and hydraulic data could eliminate the need for simplification, but they lead to another challenge of DFN parametrization, that standard inversion algorithms require a predefined number of fractures in the beginning of the inversion. The exact number is usually not known before any interpretation is done. Some studies cope with this problem by not solving for the DFN itself, but for its statistical parameters, such as fracture density or fracture length distribution (Jang et al., 2008). Others keep the number of fractures fixed by turning fractures on and off during the inversion, or by mathematically transforming the fracture models (Yao et al., 2018).

Transdimensional inversion is capable to change the number of model parameters during the inversion. This method was used by (Somogyvári et al., 2017) to interpret synthetic cross-well tracer tomography. The methodology has been further improved by (Somogyvári and Reich, 2019) and by (Ringel et al., 2019) and adapted for different synthetic tomographic scenarios. Using tomographic data as an input provided enough information for the inversion to calibrate DFN models of hundreds of parameters. This study is the first application of the transdimensional fracture network inversion on field data. Temperature observations from multiple boreholes are available from the Waiwera site and are used for the interpretation. While tomographic aquifer characterization studies use spatially distributed information on aquifer heterogeneity, temperature profiles are a significantly poorer source of data. We compensate for the limited amount of information by simplifying the used DFN model, similarly to Klepikova et al., (2014). We investigate how this limited amount of information affects the quality of the reconstruction, whether it is possible to obtain robust reconstructions.

2. METHODS

2.1 Waiwera geothermal site

The geothermal reservoir of Waiwera, New Zealand, is located in a geologically complex area in close proximity to the sea. Geothermal fluid of approximately 50°C feeds the aquifer from below, via a highly permeable fault system that intersects the bedrock.

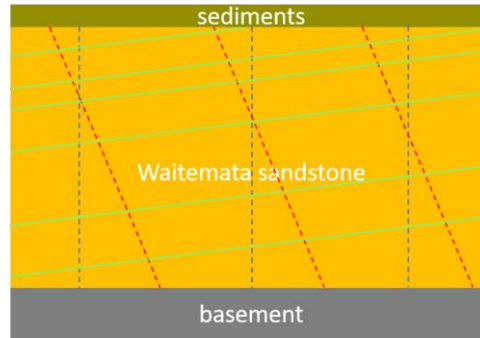


Figure 1. Geologic structures of the Waiwera geothermal reservoir.

The geothermal fluid mixes with marine and fresh water, forming a stable temperature anomaly with a lateral extent of 500 m. The shape of the anomaly is affected by the aquifer flow system, which is governed by the tilted, folded, faulted and stratified sandstone aquifer matrix shown in Figure 1. The average apertures of the two high inclination fault sets are estimated as 0.5 mm, from core samples. The stratification of the tilted sandstone provides another fracture set with an estimated aperture of 0.8 mm. This stratification is the main objective of our study, as neither the exact location, nor the exact number of fractures are known.

2.2 Transdimensional inversion

To find the number of fractures required modeling the Waiwera aquifer a transdimensional inversion method is used. Transdimensional inversion could test models with different parameter numbers, therefore DFN models with different fracture numbers. The chosen transdimensional inversion approach, the reversible-jump Markov chain Monte Carlo algorithm introduced by (Green, 1995), has been successfully applied for DFN inversion recently by several studies (Ringel et al., 2019; Somogyvári et al., 2017). A summary of this algorithm is shown in Figure 2.

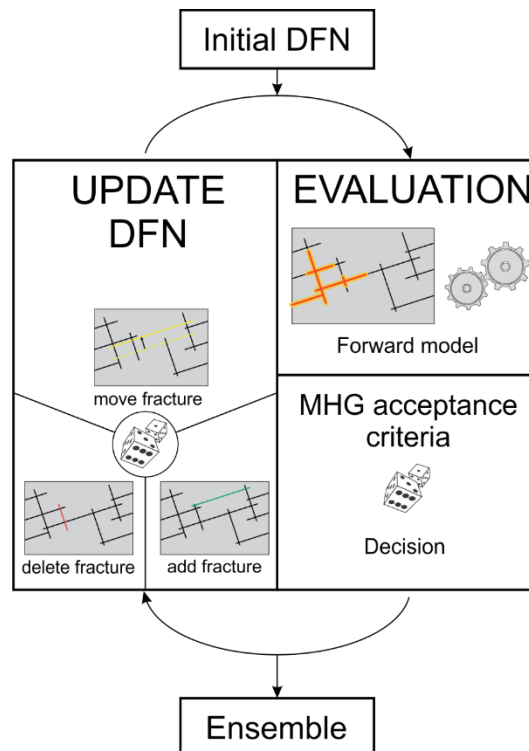


Figure 2. The transdimensional DFN inversion algorithm (modified after Somogyvári et al., 2017).

Transdimensional DFN inversion uses the core concept of random-walk MCMC: it evolves DFN realizations through a Markov chain using random perturbations. The main modification compared to traditional MCMC methods is that these perturbations could include the introduction of a new fracture (parameter birth) or the deletion of an existing fracture (parameter death) (see Figure 3). The perturbations are chosen randomly with a pre-defined probability. For fracture addition we identify possible insertion points the existing DFN using a discretization length. This is important to maintain the minimum allowed fracture spacing (Somogyvári et al.,

2017). The third possible perturbation is fracture movement, where an existing fractures position is modified randomly. Note that the physical properties are not assigned to the fractures, but to their fracture sets (which is given by their inclination) and are kept fixed throughout the inversion.

In the next step the updated DFN realization (θ^*) is evaluated using the Metropolis-Hastings-Green acceptance criteria.

$$\alpha(\theta^*|\theta_n) = \min \left[1, \frac{p(\theta^*)}{p(\theta_n)} \frac{L(\xi|\theta^*)}{L(\xi|\theta_n)} \frac{q(\theta_n|\theta^*)}{q(\theta^*|\theta_n)} |J| \right] \quad (1)$$

where $\frac{p(\theta^*)}{p(\theta_n)}$ is the ratio of the prior distributions, $\frac{L(\xi|\theta^*)}{L(\xi|\theta_n)}$ is the ratio of the likelihood functions, $\frac{q(\theta_n|\theta^*)}{q(\theta^*|\theta_n)}$ is the ratio of the backward and the forward update and $|J|$ is the so called Jacobian (Brooks et al., 2011). Because fracture deletion and addition are discrete updates, the Jacobian term will always equal to 1 (Denison et al., 2002).

The result of the transdimensional DFN inversion is not just one calibrated DFN realization, but a set of realizations called the ensemble. If the chain was run for long enough, the ensemble is a representative sample of the posterior probability distribution of the inverse problem. To eliminate any effect of the randomly chosen initial realization, following the standard MCMC practice the first half of the chain is discarded, and only the second half is used for further statistical analysis.

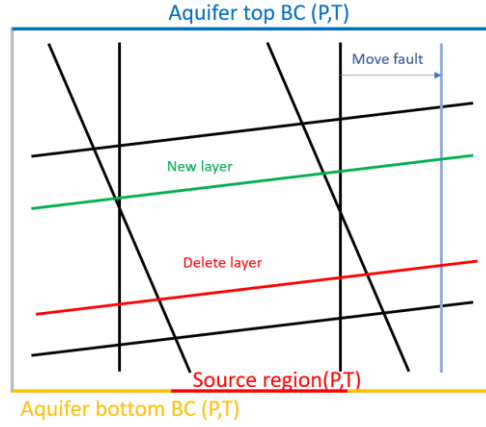


Figure 3. MCMC updates and boundary conditions.

2.3 Implementation

Around 100 boreholes at the site provide a detailed image of the temperature anomaly. For this study an interpolated temperature field from (Kühn and Stöfen, 2005) is used (see Figure 4a), where virtual boreholes are defined to have realistic input.

Using steady state temperature observations as the only input for the transdimensional DFN inversion is a significantly poorer data source compared to previous tomographic studies of the same methodology (Ringel et al., 2019; Somogyvári et al., 2017; Somogyvári and Reich, 2019). To compensate for this, we utilized a simplified DFN model, where transdimensional updates were restricted only to one of the fracture sets (Fig. 3). In this simplified setting fractures completely intersect the domain and fracture apertures and inclinations are fixed. With this simplification the location of each fracture could be described by only one value: for the low inclination sets the intersection with the vertical centerline and for the high inclination sets the intersection with the horizontal centerline. This reduction in parameter numbers ensures the robustness of the inversion.

The boundary conditions of the model are shown in Figure 3. Boundary conditions are defined on the top and bottom edges of the domain, and they get active when fractures are intersecting them. The source of the geothermal water is defined as a high-pressure high temperature area near the bottom center of the domain.

To simulate the temperature field, the DFN simulator developed by (Jalali, 2013) is used. This simulator solves the pressure distribution, flow and heat transport inside a 2-D fracture network. No temperature transport within the rock matrix is simulated only temperature loss at the fracture surface. The temperatures inside the rocks are estimated via interpolation. These simplifications provide a forward simulator that can be run faster than seconds, making it ideal to use in an MCMC framework.

3. RESULTS AND DISCUSSION

The reconstructed fracture probability map is shown in Figure 4b. This map is created by rasterizing all DFN realizations of the ensemble, and then stacking them together. The value of each pixel of the map shows the probability of having a fracture at that location. This map can be interpreted as the mean realization of the ensemble (Bodin and Sambridge, 2009; Jiménez et al., 2016). This result shows high probability of fracturing in the top and the bottom of the domain. Light fracturing is present in the bottom half, with a significant non-fractured zone above. This part of the domain, where no fracture was interpreted for more than 50 meters

Figure 4c shows the mean of the reconstructed temperature anomalies, which can be compared with the observed anomaly in Fig. 4a. The reconstructed temperature anomaly has a similar extent, shows similar temperature gradients and shares a skewed shape with the observed temperature field. Note that the DFN models do not consider the mixing of sea and freshwater, and it only solves the

temperature transport inside the fracture network. Considering these limitations, the reconstruction is remarkably similar. It shows that the geological structures play the key role forming the shape of the temperature anomaly.

An obvious question based on the presented results is that what happens when we increase the number of temperature profiles. Could the quality of the reconstruction be further improved this way as one would normally think? Our tests show that the situation is more complicated than that. Beside the appearance of some artefact features we were not able to get any significant improvement by increasing the number of used virtual boreholes to 5 or 7. The freedom of the simplified DFN model is too small, and could not create more complex temperature anomalies beyond one point. This could be overcome by reducing the level of simplification, with the risk of losing the robustness of the method.

Because of the limited complexity of the used 2-D DFN models, existing 3-D continuum based models provide better reconstructions of the temperature anomaly (Kühn and Stöfen, 2005). While these models excelled in modeling the temperature reconstruction, they fall behind data driven approaches for simulating aquifer management and water level changes over time (Kühn and Altmannsberger, 2016; Kühn and Schöne, 2017). This indicates the limitation of these continuum models where the discrete aquifer features are not considered. The fit quality of temperature profiles using the 2-D DFN approach is significantly lower, which is expected due to the limited model complexity. Still the obtained observations on the structural elements of the aquifer could be valuable in the future to improve the existing 3-D models, using a hybrid continuum-discrete approach.

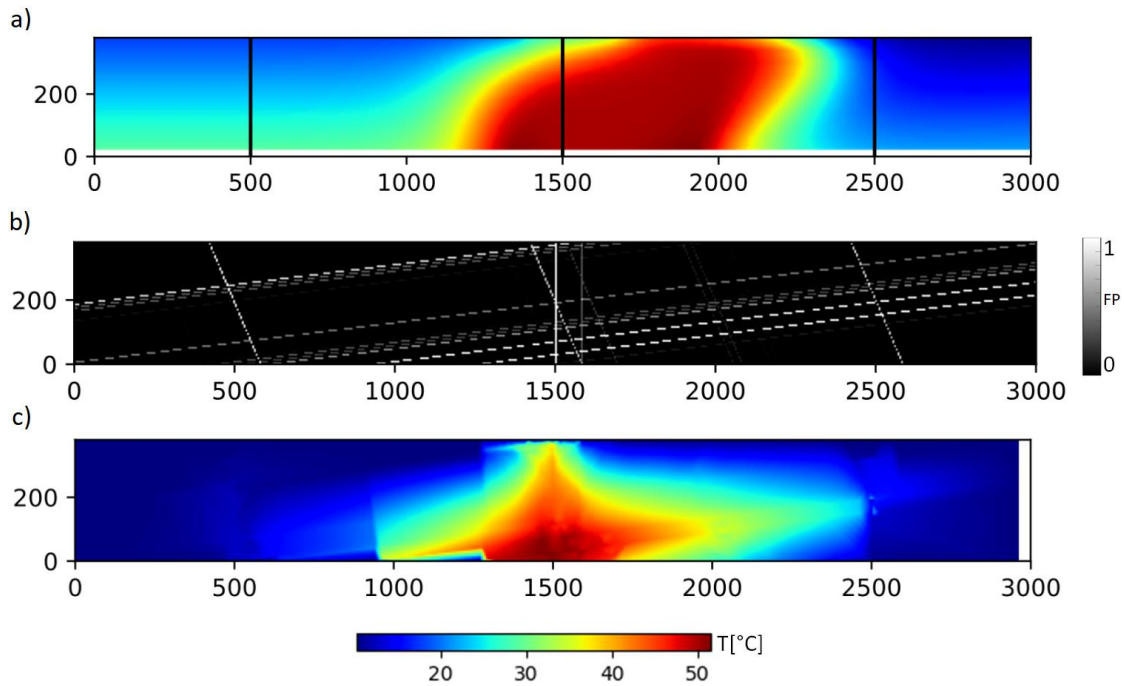


Figure 4. a) Temperature anomaly of the Waiwera geothermal reservoir with virtual boreholes, b) fracture probability map of the reconstruction, c) reconstructed mean temperature anomaly.

4. CONCLUSIONS

Measuring the temperature profiles along boreholes is a cheap and efficient data source for characterizing geothermal groundwater systems. In this paper we have shown that using this information we can reconstruct the main geological structures of fractured geothermal reservoirs. We have shown that transdimensional inversion could be a simple and robust tool, if the models could be constrained with sufficient geological information. The reconstructed DFN realizations are suitable to identify strongly fractured and non-fractured zones of the aquifer. The reconstructed thermal anomalies replicate the asymmetric shape of the observed temperature field. Although the overall reconstructed shape is similar, the method was not capable to provide better local fits of temperature observations than existing porous models of the site.

Still, the presented results showed that the aquifer structure has a strong impact on the temperature anomaly, and it cannot be neglected. For a better understanding of the site, future work will include these discrete features to existing porous models, hoping that a hybrid approach could better predict the evolution of the geothermal site in the future. The identified dominant fractures will also be subject of future investigations at the site, with special focus on the top part of the reservoir. In the future the setup of dynamic reservoir models will use the created numerical models to provide a realistic representation of the aquifer. These models will be used for aquifer management and for a more advanced exploitation of geothermal energy in the area.

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