Mathematical Modelling Approach for Evaluation of Thermal Water Aquifers: A Case Study of Paipa Geothermal System

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ABSTRACT

The Paipa geothermal aquifer system, located in the axial zone of the eastern mountain range of Colombia, is the municipality's tourism industry base. Sedimentary rocks characterize the geology of the region, with some magmatic intrusions. This study presents a mathematical modelling approach of the system, considering information from secondary sources. The model shows that the information available is not sufficient to prepare a complete model. Data on the volumes exploited from the aquifer by the tourism industry is not sufficient. Furthermore, there is no information on pumping tests to establish the hydraulic conductivity of materials in the area.

The results of the mathematical modelling approach show the relevance of alluvial deposits for the interaction between the geothermal system, the Chicamocha River, and Sochagota Lake. Also, it shows the relevance of the interaction between intrusions of magmatic origin and the groundwater flow within the aquifer, and the possibility of secondary permeability of this material. The simulation highlights the importance of the Tilatá formation, and the clay rocks of the Guaduas formation, in the behavior of the aquifer.

1. INTRODUCTION

The main objective of this study is to mathematically describe the geophysical behavior of a geothermal system at Paipa, a municipality in the department of Boyacá, Colombia. The mentioned system is the base of the tourism and therapeutic industry, the primary industries of Paipa, Mayor Municipality of Paipa (n.d.). Besides, it verifies whether existing data is sufficient to make this model and whether it is available as open data; and to find an appropriate methodology for the geophysical analysis of an aquifer that allows the proper management of groundwater. Then, because of the high demand of the hydrothermal resource and its importance for the municipality's economy, it is necessary to understand the physical behavior of the system and the medium in which the springs originate.

Based on a bibliographic review, a methodology to make a mathematical model for geothermal aquifer systems was achieved. Then, from second source data obtained from the geological service of Colombia (SGC), the Autonomous Corporation of Boyacá (CORPOBOYACA), the NASA, the Institute of Environmental and Meteorological Studies of Colombia (IDEAM) and the National Administrative Department of Statistics of Colombia (DANE), a model using open-source software, MODELMUSE is developed.

Based on the work carried out in the Paipa geothermal aquifer, it is expected to mathematically describe the geophysical behavior of a geothermal aquifer, to verify if the existing data is enough to carry out this model, and if it is available as open data to accomplish the prosses.

2. METHODOLOGY

To address the issue of geothermal aquifers used in the tourism industry, different sources, including academic databases such as Scopus and Google Scholar were surveyed. In each of these databases, articles with geothermal, model, hydraulic, hydrogeology, ground water, and soil as key words were searched, and they were filtered by looking at those in English, and those published after 2016.

From the surveyed sources we found an analysis of the characteristics of groundwater of the Chicú river basin, Velandia (2018), and a fault characterization and heat-transfer model, Moreno (2018), that give a guide about the methods used locally on the matter. Also, we found a study on Cataluña that uses an isotopic analysis, Albert (1979); a study in Turkey that uses a trace of the elements found in a geothermal aquifer, Öztekin Okan (2018); and a study in Japan that uses pH mapping in the geothermal system, Susuki (2017).

Based on the bibliographic review, we establish the methodology to make the mathematical model of a geothermal aquifer system. The methodology considers secondary source data and open-source software.

To model Paipa's aquifer, the topography was taken from satellite images from NASA with resolution 30x30, considering those after January of 2016 that got less than 5% of cloudiness. From the images taken, a raster of the terrain was generated. Then, the hydrology of the region was modeled using the software HEC-HMS to determine precipitation, evapotranspiration, and runoff in the area. All the data was prepared on the software QGIS, to fit the coordinate system UTM zone 18N, and then imported to MODELMUSE software to prepare the mathematical model.

Using the MODELMUSE software, a set of four layers was set to represent the Paipa's aquifer system geology. The top layer represents the quaternary deposits with 50 meter of thickness. In the second layer, there are groups of quartz-sand intercalated by siltstones and claystones and some black laminated clays, reddish and green limonites, clayey and feldspathic sands, and some conglomerate levels at the north of the region, this layer is represented with a thickness of 300 meters. The third layer, where a group of friable clays, reddish to yellowish clays and some intercalations of fine red and white sandstones, conglomerate sandstones, and reddish to green siltstones at the north; this layer is represented by a 300-meter layer. The bottom layer, represented by a 550 meters thickness, is a compound sequence of gray clays, friable sands, horizontal charcoal, some white quartzsandites, and reddish shales intercalations at the north.

The system geotechnics' model considers a set of 14 faults imported as a set of vector plans on the aquifer. This vector plan has an equation constructed by the fault azimuth, the inclination, and a point of the fault over the terrain.

With the aquifer structure set up, we import the precipitation, the evapotranspiration, and the runoff data over a shape file that contains the sub-basins of the region. Additionally, we set the Chicamocha River as a line type shape file. The Sochagota Lake was considered as a shape file also. Finally, the wells of the region were applied using a shape file of points to assign each one the corresponding pumping data.

The proposed mathematical model is formulated and seeks to propose a model that describes the physical behavior of Paipa's geothermal system, based on secondary source data mentioned above. The mathematical model covers municipalities of Paipa and Duitama, located under the Boyacá fault, and the area of the municipality of Tibasosa. In a conceptual model of Paipa's geothermal system, it is mentioned that the last one could be a possible main recharge area of the system.

3. STUDY LOCATION

To address the geothermal system study used in the tourism industry, we established the study area of Paipa's municipality located in the center of Colombia, in the department of Boyacá. Below is a map of the general location of the Paipa hydrothermal system (See Figure 1).

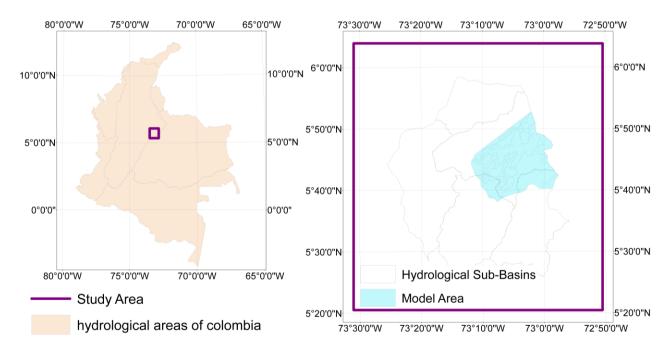


Figure 1 General Location of the Paipa's Geothermal system

3. HYDROLOGICAL MODEL

From NASA satellite images, a set of 6 raster images were selected to build the region topography. From the obtained topography, there is mountainous terrain of the eastern mountain range of the Andes. It is also evident that the municipality of Paipa is located on a high plateau of the mountain range. This topography was processed to export the region basin to the HEC-RAS v5.0 software to determine the hydrology of the zone.

The results obtained were interpolated using a zone statistics analysis to assign the hydrological values to every sub-basin inside the region. These hydrological results will later feed the mathematical model. Table 1 is the hydrological data achieved from the hydrological model for the sub-basins allocated on Paipa, Duitama, and Tibasosa municipalities.

Table 1 Hydrological data for the mathematical model

SUB-BASIN	P (mm)	EVT (mm)	INF (mm)
W620	2265.67	102.86	706.56
W660	1464.68	106.33	605.88
W710	1175.85	91.47	553.20
W720	774.22	91.14	416.88
W730	531.42	104.04	325.68
W740	542.53	91.14	326.64

The first column in Table 1 is the name of the sub-basin, the second is the precipitation on millimeters, the third is the evapotranspiration value on mm, and the last one is the infiltration value on mm. From the hydrological model performed on HEC-RAS v5.0 software the infiltration value for each one of the sub-basins was obtained.

4. GEOLOGICAL MODEL

Considering information obtained from the Geological Service of Colombia, a geological model was built; from 1:100.000 maps, where the distribution of materials in the area was determined. The study determined the geological formation and stratification of materials, considering the reports from each of the maps analyzed.

From the maps obtained, it was determined to make a regional model limited to 3 municipalities, Paipa, Duitama and Tibasosa. This area matched with the study area presented in the Paipa Geothermal System, Boyacá: Review of Exploration Studies and Conceptual Model, Alfaro – Valero et al (2020).

Also, in the geological-geophysical study for the search for underground waters and pre-design of a well, Velandia Nossa (2015), mentions that there are the following formations: Arcabuco, La Russia, Bogotá, Guaduas, Montebel, Tilatá and quaternary deposits.

The geological model proposed for the simulation consists of four layers. The first layer, called quaternary deposits, made up of alluvial deposits and, in general, sedimentary materials with some materials from igneous intrusions of volcanic origin. The second layer is made up of materials present in the Tilatá formation on the southern side of the Boyacá fault, and the Montebel formation to the north of the fault. The third layer formed by the Bogotá formations arranged under the Tilatá formation, and the La Russia formation under the Montebel. Finally, the last layer was formed by the Guaduas formation under the Bogotá formation, and the Arcabuco formation under La Russia. The material distribution achieved is shown in figure 2 (See Figure 2)

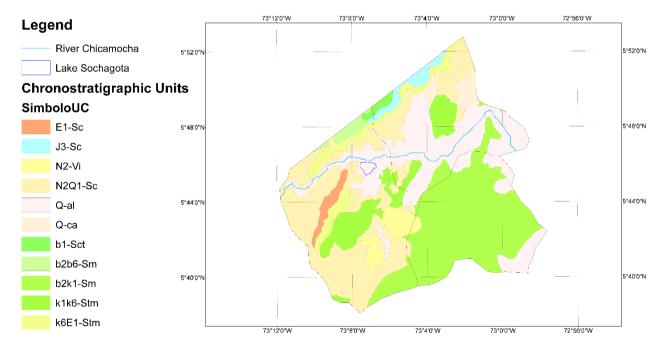


Figure 2 Distribution of materials on the study area

In the previous image the materials Q-ca, Q-al, N2-Vi, k1k6-Stm, b2k1-Sm and b2b6-Sm correspond to the quaternary deposits in the top layer of the aquifer; N2Q1-Sc corresponds to the Tilatá Formation in the second layer; J3-Sc and E1-Sc correspond to the Bogotá and Russia formation in the third layer; k6E1-Stm and b1-Sct correspond to the Arcabuco and Guaduas formation in the bottom layer. This map was realized on QGIS v3.14 software.

The four layers were simulated with a thickness of 50 meters for the first, 300 meters for the second layer, 300 meters for the third, and of 550 meters for the last layer. The layers thickness was selected as Velandia Nossa (2015) mentions the formation material composition, and their thickness, on his geological and geophysical study for the search of underground water and pre-design of a well in Toibita, Paipa Boyacá.

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5. MATHEMATICAL MODEL

The study used free license available software Modflow for the mathematical modeling, MODELMUSE GUI was used for simulation purpose. This program solves the underground flow equation through successive iterations based on the geological and hydraulic information. The equation to solve for the aquifer is:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} + \frac{F}{K} = \frac{S}{T} \frac{\partial h}{\partial t} \tag{1}$$

Where x, y, z is the orthonormal cartesian coordinates, h is the hydraulic potential, F is the flow inlets outside the system, K is the hydraulic conductivity, S is the system storage coefficient, and T is the Transmissivity of the medium.

For this case we consider a statical model so we can simplify the equation as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = -\frac{F}{K} \tag{2}$$

The statical assumption was made to simplify the calculations, and also because there is not enough data on the field to calibrate a more complex model.

The georeferenced information was prepared through QGIS. The program requires geological and geotechnical information prepared in the geological model and, in turn, precipitation and evapotranspiration data taken from the hydraulic model.

The model was calibrated, considering the geothermal aquifer that outcrops freely at points near the Paipa hot springs, and in the area near the Chicamocha river.

The model was run repeatedly until finding parameters of conductivity of the materials that allowed the level observed in the field. Likewise, a low discrepancy in the hydraulic balance of the aquifer, that is, a percentage difference between the inlet and outlet volumes less than 1% or greater than -1%. The values found were compared against those found in the literature; the analysis and conclusions were made.

It is necessary to determine each of the materials' conductivity value, for these reference values were taken from reference literature. These were later modified in the model calibration process, which is described further ahead.

Table 2 Initial values for calibration of the mathematical model

MATERIAL	CONDUCTIVITY (m/day)
B1-SCT	0.01
B2B6-SM	0.0002
B2K1-SM	0.0002
E1-SC	0.001
E6E9-SCT	0.0001
J1J2-VCCT	0.0001
J3-SC	0.01
K1K6-STM	0.0001
K6-STM	0.01
K6E1-STM	0.01
N2-VI	0.000001
N2Q1-SC	0.01
Q-AL	0.003
Q-CA	0.003

For the mathematical model we consider faults in the area, as these geotechnical structures affect the transmissivity of the aquifer.

The following faults are considered for this area: El Hornito, Canoas, El Bizcocho, Cerro Plateado, El Batan, Rancho Grande, Paipa-Iza, Agua Tibia, Buenavista, Santa Rita, Lanceros, Boyacá, El Tuno, Las Peñas. Next, a map is presented with the location of the faults considered. (See Figure 3).

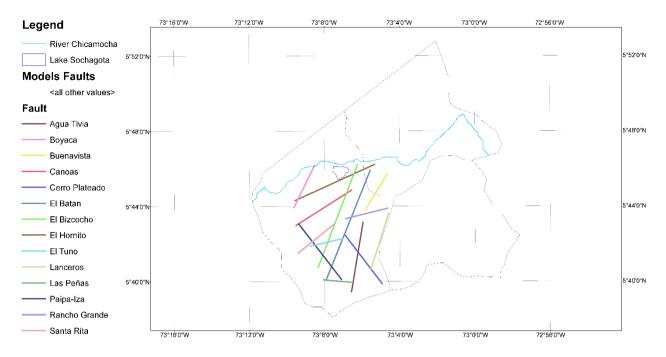


Figure 3 Location of the faults on the study area

In Figure 3, we show the distribution of the faults to be considered on the model of the aquifer of Paipa, it is represented as planes. These planes give the direction of each fault. To represent them and establish fault planes represented by the equation of a plane, each fault plane is assigned to a rectangular element that will represent the fault's hydrological characteristics from the equation of the fault plane. The equation of a plane is:

$$Z = Z_0 - \frac{1}{\sin(B)} \times \left(Sin(A) \times Cos(B) \times (X - X_0) + Cos(A) \times Cos(B) \times (Y - Y_0) \right)$$
(3)

Where (X_0, Y_0, Z_0) are coordinates of a known point on the plane, A is the azimuth of the fault and B is the fault dip.

The chart below presents the values of the coordinates of one known point of each fault, their Azimuth, and fault dip. (see chart 5-2)

Table 3 Faults data for the model

FAULT	Xo (m)	Yo (m)	Zo (m)	Az (°)	B (°)
El Hornito	707711.66	636403.17	2538.00	24.49103266	5
Canoas	706629.08	633922.90	2554.00	32.5159721	5
El Bizcocho	708000.34	633115.81	2675.00	69.10676987	5
Cerro Plateado	710539.75	628866.43	2631.00	127.3761198	5
El Batan	709022.03	632223.17	2556.00	68.12563328	20
Rancho Grande	710842.64	633372.34	2606.00	13.9503428	10
Paipa-Iza	706269.28	629614.20	2815.00	127.7021713	2
Agua Tivia	709917.96	629110.70	2601.00	80.60046767	3
Buenavista	711787.10	635491.14	2542.00	58.70696101	5
Santa Rita	705904.29	630915.49	2828.00	38.81937128	10
Lanceros	712132.52	630658.95	2699.00	72.48235227	5
Boyacá	704714.38	636021.89	2660.00	64.0715424	55
El Tuno	706901.66	630527.65	2833.00	13.43921652	5
Las Peñas	707986.81	626731.35	2740.00	175.1448316	5

The study considered that the conductivity of the cells that intersect a fault plane, have ten times the average conductivity of the layer, or aquifer, that they intersect. Also, it considers the river as a drainage element of the aquifer in the drainage package as a boundary condition of the mathematical model. This package allows the flow to be discharged from the aquifer when the head is greater than the river's elevation.

On the other hand, the model considers the Sochagota Lake as a boundary condition element with the Model Muse Lake package; this package allows simulating the interaction of slow water bodies such as reservoirs or lakes.

Additionally, the model considers the wells with which the aquifer is exploited with the WELL package from Model Muse; this allows for elements that discharge the aquifer with negative flows or recharge it with positive flows. The Table 4 shows the wells considered for the local model.

Table 4 Flows exploited from tourist wells from the Paipa's aquifer system.

ID	WELLS	Q L/S
0	Solapa 1	0
1	Pozo hotel lanceros	0.177
2	Pozo las marismas Solapa 2	0
3	Pozo ojo del diablo	3.59
4	El delfín La playa 2	0.5
5	Santa Rita	0.3
6	Pozo inundado	1
7	Pozo escondido	0.042
8	Baltan La playa 2	1
9	Olitas	6
10	Pozo Blanco	0.8
11	Fondo piscina olímpica	0
12	El Hervidero	0
13	Balneario Publico	0
14	Pozo azul	6.1
15	Erica	0

The first column contains an ID for each well and the second shows the well's name. The last column is the volume exploited in the well per second. The flow was introduced on MODELMUSE in m³/s and with a minus sign to represent the aquifer's extraction.

Below in the Figure 4 is a map with the location of the wells (See Figure 4).

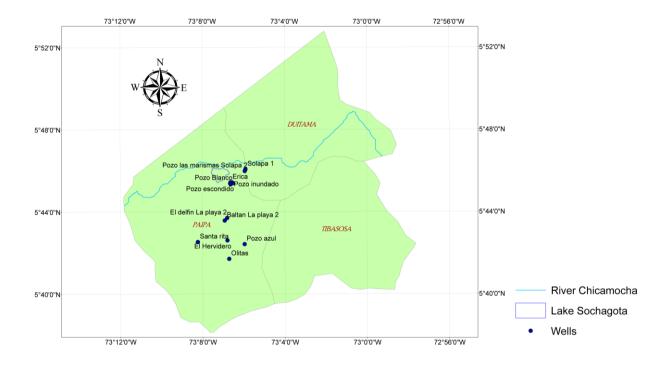


Figure 4 Wells Location

8. RESULTS AND DISCUSSION

There were 143 simulations of the model. Simulation # 138 fits best with the observed state of the aquifer. The next chart presents the conductivity values obtained from simulation 138 of the calibration process. (see Table 5).

Table 5 Hydraulic conductivity of simulation 138

MATERIAL	CONDUCTIVITY (m/day)	
B1-SCT	0.005	
B2B6-SM	0.0003	
B2K1-SM	0.0003	
E1-SC	0.03	
E6E9-SCT	0.0001	
J1J2-VCCT	0.0001	
<i>J3-SC</i>	0.05	
K1K6-STM	0.0003	
K6-STM	0.00001	
K6E1-STM	0.00001	
N2-VI	7.00E-08	
N2Q1-SC	0.001	
Q-AL	0.00003	
Q-CA	0.001	

Figure 3 depicts the model results of the simulation No. 138

Next, we show the simulation # 138 results. In Figure 5, we see the level of the water table in the aquifer. In the image, the simulation for the upper layer is shown with the parameters of simulation # 138. The representation was filtered between the values 2499 and 2500 since for this area of interest no notable variation is observed for the entire range. X and Y axis represent the coordinates of the map. The legend is the water level in meters.

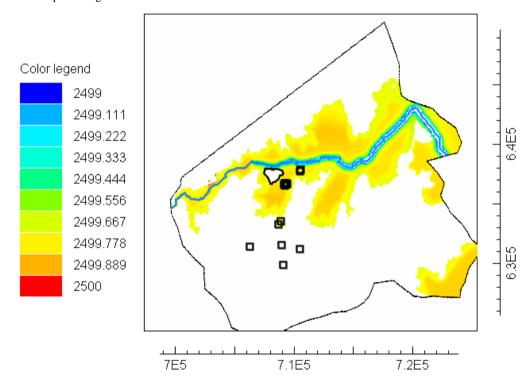


Figure 5 Result of simulation # 138

Figure 5 shows that the water table of the aquifer tends to descend towards the river; this shows that this body of surface water acts as a discharge, controlling the levels of the aquifer.

Figure 6 shows the difference between the head flow and the terrain. Figure 6 shows the points where the water surface is higher than the ground level, therefore it represents the points where outcrops would occur. X and Y axis represent the coordinates of the map. The legend shows the difference between the terrain and the water label in the aquifer in meters.

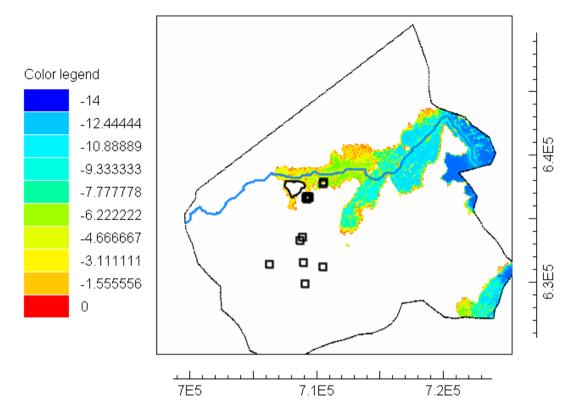


Figure 6 Difference between the simulated water surface and the ground level of the simulation # 138

In Figure 6, it is observed that the level of the water sheet tends to be higher downstream of the river. The results obtained in this simulation highlight alluvial deposits as a material of high incidence in the water layer. Likewise, the importance of the river as a regulatory entity of the system is highlighted, acting as a drainage system.

Now in Figure 7, we show the difference between the water table of the aquifer and the terrain for those points where the water is below the land. X and Y axis represent the coordinates of the map. The legend shows the difference between the terrain and the water label in the aquifer in meters.

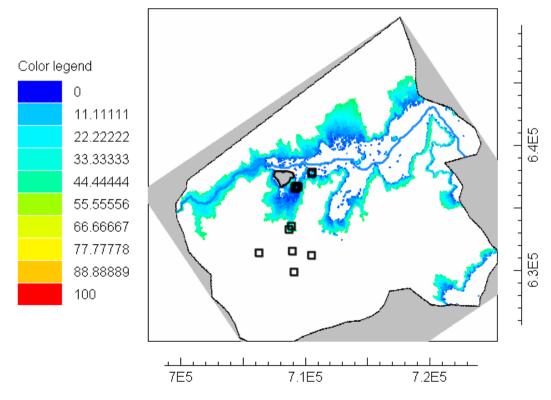


Figure 6 Difference between the simulated water surface and the ground level of the simulation # 138

Also, the results obtained in this simulation and other similar ones, highlight alluvial deposits as a material of high incidence in the water layer. Likewise, the river's importance as a regulator of the system is highlighted, acting as a drainage system.

Other simulations are consistent with this result and are mentioned in an approach to a mathematical model for the evaluation of thermal water aquifers as Chaparro (2020) mentions them.

Figure 8 illustrates the results of simulation # 138 in the second layer, where the Tilatá formation is located. X and Y axis represent the coordinates of the map. The legend shows the water table level on the Tilata's formation aquifer.

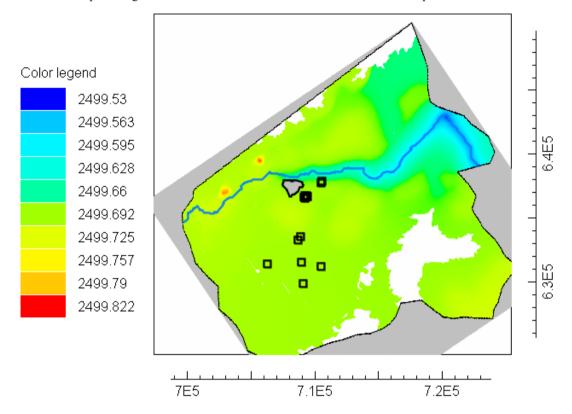


Figure 8 Results of simulation # 138 in the Tilatá formation

Figure 8 show the water head inside the Tilatá formation on the second layer of the simulated aquifer. The N2Q1-SC material of the Tilatá formation has a high permeability, so this material could be responsible in part of the transit of geothermal water into the aquifer.

Also, Figure 8 shows how the Chicamocha River influences the flow pattern of the aquifer in the Tilatá formation. The anomalies observed in the upper left part on blue can be found on the K6E1-Stm layer of the Guaduas formation; this may be due to the low permeability since it is composed of arcillolites and quartz arenites.

7. CONCLUSIONS AND RECOMMENDATIONS

Currently, there are several studies focused on hydrogeological systems with thermal influences. The most used methods for the study and analysis of resources coincide in using geospatial information management software to realize maps focused on the transport of ions and pollutants present in hot springs. Most of the studies carried out regarding this resource are related to the energy or mining industry.

The analysis method for the present study began with obtaining information from secondary sources. In the SGC, geological maps allow us to determine the spatial distribution of the materials in the concerned area. However, not many stratigraphic cuts allow us to determine with some precision the thickness of each aquifer layer. Furthermore, the few studies on the subject are not distributed throughout the area, generating uncertainty. Therefore, the layers chosen for the conceptual geological model were based on the reports that accompanied the maps of the area's geology.

CORPOBOYACA provided a report on monitoring the flow sources in the ITP municipality of the Paipa sector. The report states the values of the exploitation flow of the wells in the area. However, the amount of data is minimal and did not allow for higher precision calibration and model validation. On the other hand, no data was found for wells beyond the ITP Lanceros area. There were no records of pumping tests that allowed determining hydraulic conductivities of the materials in the area, so literature values were used for the simulation. It would be advisable to take data from pumping tests in different seasons of the year to analyze the system's more complex behavior in the face of climatological variations in the area.

From the simulations, the relevance of alluvial deposits where local flows are found, their interaction with direct precipitation, and the Chicamocha River current stands out. All of which is favored by the basin's topography, which favors regional flow towards the lower part of the stream. It would be advisable to establish alluvial deposits' conductivity in the field to reduce the uncertainty of the proposed model.

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It would also be appropriate to estimate the b2k1-sm material's conductivity on the quaternary deposits at the top layer of the aquifer, as it could influence the aquifer recharge process.

Among the simulations carried out, the Tilatá formation stands out; therefore, it would be wise to study the materials' conductivity that makes it up to achieve better detail in the proposed model.

Also, the conductivity of the Chicamocha riverbed, when making different measurements along the channel, should be considered. Likewise, the conductivity of the Sochagota lakebed to determine more precisely the interaction of these surface bodies with the geothermal aquifer should be considered. Moreover, it would be appropriate to carry out isotopic studies campaigns along the Chicamocha river basin that allows a hydrogeochemical analysis to establish connections between aquifers, and monitor the water's quality in the tourism industry.

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