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Mathematical Modeling of Geothermal Sources in the Earth's Crust

Simeon Kostyanev

University of Mining and Geology, Sofia, Bulgaria

simeon44@yahoo.co.uk

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ABSTRACT

In this paper we present some results from the mathematical modeling of particular geothermal fields caused by local volumetric distributions of sources in the earth's crust. Inverse solution of defining the regions occupied by thermal sources when the heat flow on the Earth's surface is known is also treated. The inverse problem is unstable and special methods of regularization of the solution are offered. Mathematical modeling based on the application of regularizing operators of any kind enables the ability to obtain the shape and depth of the anomalous area. The more accurate measurements and more complete a priori information let us obtain more precise results. The technique described is applied to study certain anomalous geothermal areas in the Black Sea.

1. INTRODUCTION

Accurate knowledge of the temperature distributions in the shallow crust is of great practical importance for understanding many geological and drilling phenomena.

A common problem in interpretation of surface heat flow data is to determinate from observations the temperature distributions at depth. This requires knowledge of the variation in thermal conductivity as well as determination of the distribution of heat sources. If the thermal conductivity and the distributions of the heat sources were specified, the temperature distributions in the crust in conductive equilibrium could then be determined. Because of the lack of knowledge of the conductivity and the distribution of the heat sources, determination of the thermal regime of the crust from the heat flow data is an ill-posed problem.

Classical formulations of the downward continuation are generally restricted to the cases of source-free medium with known thermal conductivity, thereby circumventing the difficulty of no uniqueness. A priori information is introduced mainly alleviate the problem of instability. For example, Brott et al. (1981) described a continuation technique utilizing the method of equivalent point sources, and Bodvarsson (1971) presented method of Fourier transform. Mareschal et al. (1985) proposed a variation of Bodvarsson's formalism. More recent works on the downward continuations of heat flow have attempted to address the problem of variations of sources and conductivity.

We may classify these recent approaches into two groups. One group consists of the trial-and-error and Monte Carlo type formulations in which the forward problem is solved a large number times with the model parameters sampled according to their a priori distributions. The boundedness of the parameter space ensures the stability of the solutions. The second group consists of those formulations which solve the inverse problem directly. Huestis (1980) described a method for finding the extreme bounds of the subsurface temperature distributions. Of more practical use, and becoming increasingly more popular in recent years, are the methods in which a priori information is incorporated in the form of soft bounds (e.g., probability distributions) and which yield optimal solutions together with the appropriate error estimates. An example is the constrained linear least squares formulations of Stromeyer (1984) and Tarantola at al.(1982).

2. FORMULATION AND SOLUTION OF THE PROBLEM

2.1. The direct problem

Let's treat the thermal field caused by the local volumetric distribution V of thermal sources q(M), M=(x, y, z), in a gradient media, i.e. media when thermal conductivity K(M) changes in an arbitrary way (Figure 1) . The temperature T(M) on the Earth's surface is considered to be T(z=0)=0 and on the surface (z = H), . Thus we come to the problem:

$$\begin{cases} div\left(K(M)\,gradT(M)\right) = -q(M) \\ T(M)\big|_{z=0} = 0 \\ \dfrac{\partial T(M)}{\partial z}\bigg|_{z=H} = 0. \end{cases} \qquad (1)$$

$$0 < z < H\ ,$$

Problem (1) is a problem about an anomalous field. Our problem is to model the field caused by sources q(M). Knowing Green's function $G(M,M_0)$ (Kostyanev, 1982), the thermal field can be represented in the following way:

$$T\left(M\right)=\iiint\limits_{V} q\left(M_0\right)G\left(M,M_0\right)dV_{M_0}.$$

(2)

If we treat a problem with constant distribution of sources q(M) = q = const and conductivity K(M) = K = const, we shall obtain.

$$T\left(M\right)=\frac{q}{4\pi K}\iiint\limits_{V} G_0\left(M,M_0\right)dV_{M_0}.$$

(3)

where

$$G_0\left(M,M_0\right)=\int\limits_0^{\infty} J_0\left(\lambda\rho\right)\left(e^{-\lambda(z-z_0)}-e^{-\lambda(z+z_0)}+2e^{-\lambda H}\frac{\left(Sh\lambda z.Sh\lambda z_0\right)}{Ch\lambda H}\right)d\lambda.$$

(4)

The measurement on the Earth's surface is the vertical heat flow, i.e.

(Q(x, y)=K $\frac{\partial T}{\partial z}$) at z=0. According to (3) and (4) we have:

$$Q\left(x,y\right)=\frac{q}{2\pi}\iiint\limits_{V} L\left(\rho,z_0\right)dV_{M_0}.$$

(5)

where

$$L\left(\rho,z_0\right)=\int\limits_0^{\infty} J_0\left(\lambda\rho\right)\left(e^{-\lambda z_0}+\frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\right)\lambda d\lambda=\frac{z_0}{\left(\rho^2+z^2\right)^{\frac{3}{2}}}+\int\limits_0^{\infty} J_0\left(\lambda\rho\right)\frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\lambda d\lambda. \quad (6)$$

The expression of $L\left(\rho,z_0\right)$ can be simplified by the substitution

$$\frac{1}{Ch\lambda H}=\frac{2e^{-\lambda H}}{1+e^{-2\lambda H}}=2e^{-\lambda H}\sum_{n=0}^{\infty}\left(-1\right)^n e^{-2\lambda nH}.$$

(7)

Substituting (7) in (6) we obtain:

$$L\left(\rho,z_0\right)=\frac{z_0}{\left(\rho^2+z_0^2\right)^{\frac{3}{2}}}+\sum_{n=0}^{\infty}\left(-1\right)^n\int\limits_0^{\infty} J_0\left(\lambda\rho\right)\left(e^{\lambda z_0}-e^{-\lambda z_0}\right)e^{-2\lambda(n+1)H}\lambda d\lambda.$$

But, knowing that

$$\int\limits_0^{\infty} J_0\left(\lambda p\right)e^{-\lambda h}\lambda d\lambda=\frac{h}{\left(\rho^2+h^2\right)^{\frac{3}{2}}}.$$

we obtain:

$$L(\rho,z_0)=\sum_{n=0}^{\infty}(-1)^n\left(\frac{2nH+z_0}{\left(\rho^2+(2nH+z_0)^2\right)^{\frac{3}{2}}}+\frac{2(n+1)H-z_0}{\left(\rho^2+(2(n+1)H-z_0)^2\right)^{\frac{3}{2}}}\right). \tag{8}$$

If $H >> z_0$ and the dimension of the volume V, which is occupied by geothermal sources, is also much smaller than H, we approximate the value to be considered as

$$L(\rho,z_0)\cong\frac{z_0}{\left((x-x_0)^2+(y-y_0)^2+z_0^2\right)^{\frac{3}{2}}}. \tag{9}$$

Substituting (9) in (5) we obtain:

$$Q(x,y)=\frac{q}{2\pi}\iiint_V\frac{z_0dx_0dy_0dz_0}{\left(\left((x-x_0)^2+(y-y_0)^2+z_0^2\right)^{\frac{3}{2}}\right)}. \tag{10}$$

The expression (10) enables us easily to model the heat flow on the Earth's surface due to a given volume V, which is occupied by geothermal sources.

2.2. The inverse problem

Knowing the heat flow on the Earth's surface, the inverse problem of the defining the region V occupied by geothermal sources can be treated. This problem is unstable and special methods of regularization of the solution are necessary (Tikhonov and Arsenin, 1979); Kostyanev, 1982, 1991). Usually, additional information about the shape of the region V is used.

Let's suppose that the shape of V can be satisfied by the existence of mean plane at the certain depth. Thus means that the plane crosses the region V in such a way that when the shape of V is described by function and when - by the function (Figure 1). Let's symbolize the section of the region V by the mean plane with D. Then formula (10) can be written in the following way:

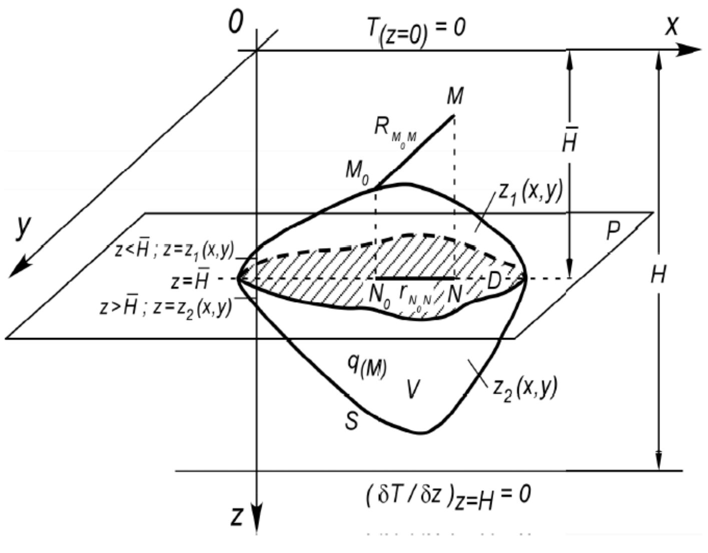

Figure 1. Local volumetric distribution V of thermal sources q (M) in a gradient media

$$Q(x,y)=\frac{q}{2\pi}\iint_D dx_0dy_0\int_{z_1(x_0,y_0)}^{z_2(x_0,y_0)}\frac{\partial}{\partial z_0}\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}dz_0 \quad (11)$$

Formula (11) completely determinates the heat flow on the Earth's surface, if $\bar{H}$, $z_1(x,y)$, $z_2(x,y)$ are known. Let's we write formula (11) in operational form:

$$Q(x,y)=A\left[\bar{H},z_1,z_2\right]. \quad (12)$$

where A – nonlinear operator. The boundary of domain D is determined by the equation $z_1(x,y)=\bar{H}$ or $z_2(x,y)=\bar{H}$.

If the heat flow measurements have errors, we have an approximated $\bar{Q}$, where $\|Q-\bar{Q}\|\leq\delta$, (δ -error of the measurement). Besides, an additional condition for the smoothness of the function must be given, for example, in the form of minimum of the special functional:

$$\Omega(\bar{H},z_1,z_2)=\beta(\bar{H}-H_0)^2+\sum_{i=1}^{2}\iint_D\left(P_i\left(\frac{\partial z_i}{\partial x}\right)^2+R_i\left(\frac{\partial z_i}{\partial y}\right)^2+S_i z_i^2(x,y)\right)d_x d_y \quad (13)$$

where β is a weight coefficient, and P_i (x, y) > 0, R_i (x, y) > 0, S_i (x, y) > 0 - weight functions. The value H_0 is hypothetical a priori value adjacent to the depth of the mean plane.

Thus the inverse problem is reduced either to a problem of conditional extremum:

$$\begin{cases}\min_{\bar{H},z_1,z_2}\Omega(\bar{H},z_1,z_2),\\ \left\|A\left[\bar{H},z_1,z_2\right]-\bar{Q}\right\|\leq\delta,\end{cases} \quad (14)$$

or to a problem of unconditional extremum:

$$\min_{\bar{H},z_1,z_2}\left[\left\|A(\bar{H},z_1,z_2)-\bar{Q}\right\|+\alpha\Omega(\bar{H},z_1,z_2)\right], \quad (15)$$

where α - coefficient of regularization, is determined by the condition

$$\left\|A(\bar{H}_\alpha,z_1^\alpha,z_2^\alpha)-\bar{Q}\right\|=\delta, \quad (16)$$

and $\bar{H}_\alpha,z_1^\alpha,z_2^\alpha$ is the solution of the problem (15) at a given α.

Thus both the shape and the depth of the region V are determined. We have to note that the main supposition – the existence of the mean plane is not applicable to all types of regions V. What shall we do if the region does not have a mean plane? In this case we propose an equivalent region processing mean plane to be found. The same problem arises if the region is not smooth or has a complicated configuration. An equivalent region with the "smooth" shape of the surface, from the class of regions processing a mean plane, has to be found. The described method is applied to the investigation of some geothermal sources in the Black Sea.

3. EXAMPLES OF MODELING OF GEOTHERMAAL FIELDS IN BLACK SEA

The speeds of the seismic waves of the Black Sea's lithosphere were previously studied (Gobarenko, Egorova, 2010) via local seismic tomography, based on the formalism of Backus--Gilbert. As a result, some valuable information of the 3D speeds of the P-waves was obtained up to a depth of 60 km. This allows a conclusion to be made for a different structure of the Black Sea's lithosphere. We used these results (Kostyanev et al, 2015) for a numerical solution to the thermal conductivity equation along three profiles; the Varna – Sukhumi profile and two transverse profiles. Calculations were carried out using real properties of sedimentary rocks and basement and they have shown that the regional variation of temperature along the Moho plane varies from 420 to 754 ^{o}C. The heat flow along the same plane varies from 15-20 to 29-41 mW/m^2. The part of the heat flow that is caused by the radiogenic sources amounts to 17-30 mW/m^2. The modeling results are presented as sections that illustrate the distribution of temperature and heat flow in depth to 50 km. (Fig.2,3,4).

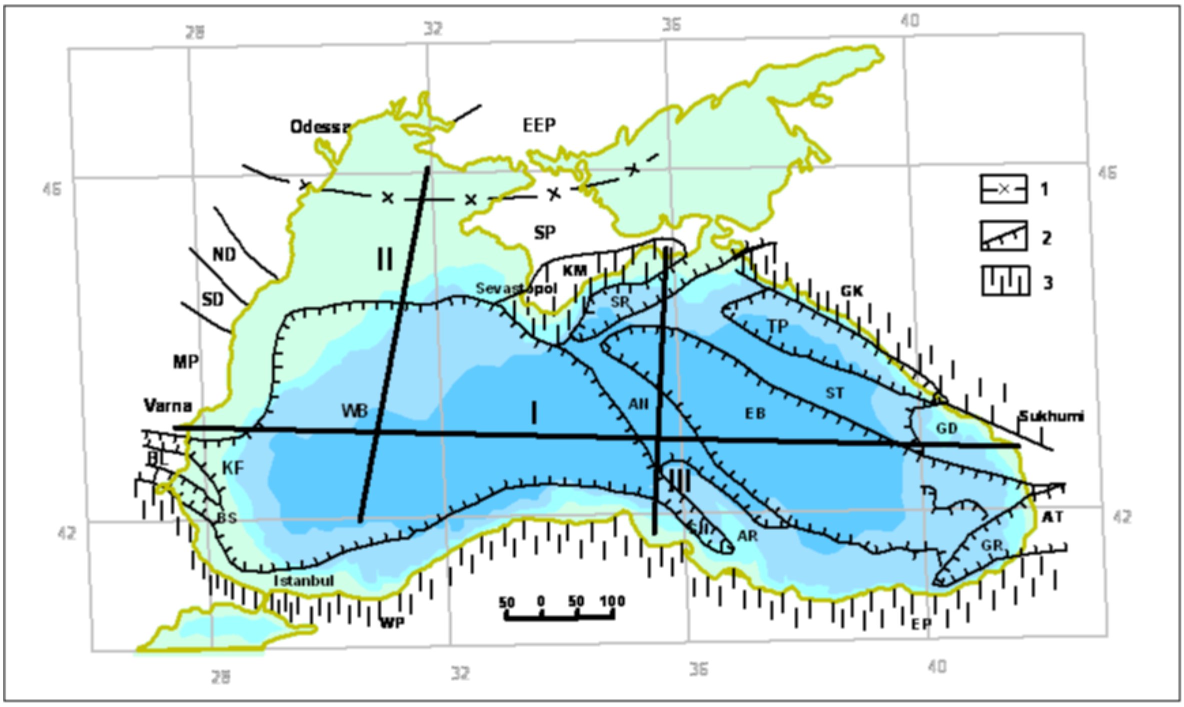

Figure 2: Scheme of the main geological structure of the Black Sea area: 1 = boundary of the East European Platform; 2 = boundary of the West and East Black Sea basins, uplift and trough; 3 = Alpine folded area; WB, EB = West and East basins; AN, AR, ST = Andrusov (Mid-Black Sea), Arkhangelsky and Shatsky ridges, respectively; GD = Gudauta uplift; SR, TR, GR, SN = Sorokin, Tuapse, Curian, Sinop trouth, respectively; KF = Kemchian fore deep; BS = Burgas depression; EEP = East European Platform; SP, MP =Scythian and Moesian plates; ND, SD = North and South Dobrogea; BL = Balkanides; WP, EP = West and East Pontides; AT = Adzaro-Trialet system; GK = Great Caucasus; KM = Crimea Mountains; A-A' = profile Varna-Sukhuni; I, II, III = Geothermal profiles

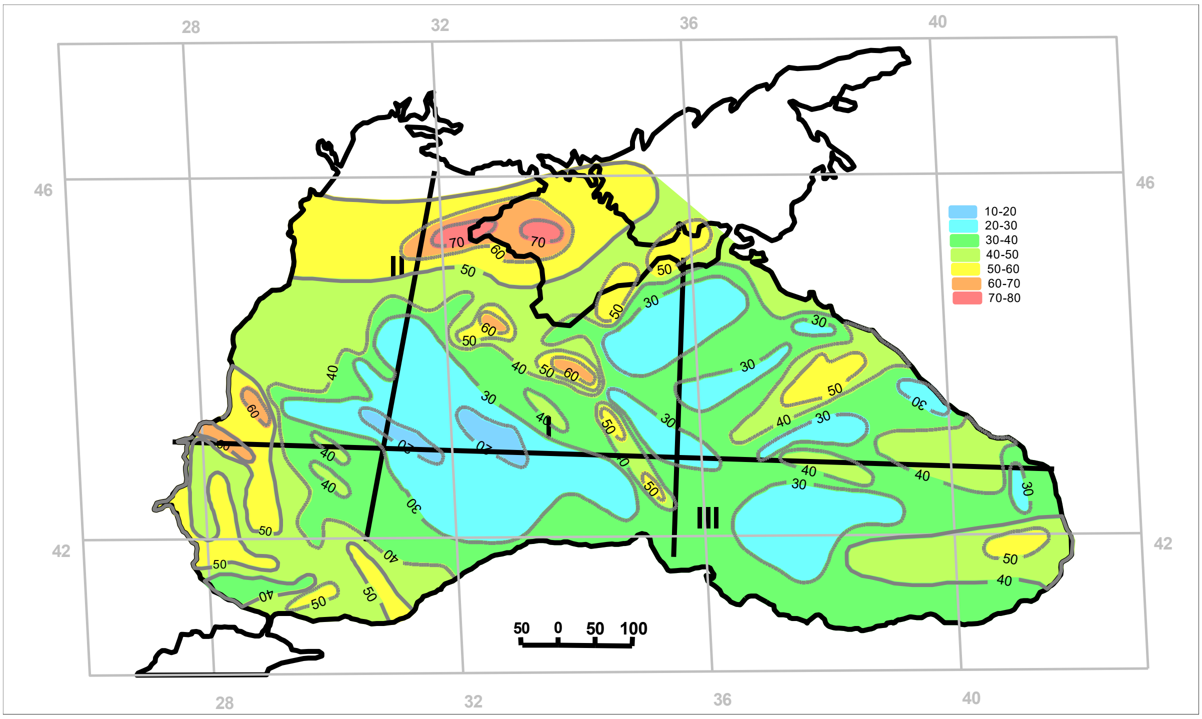

Figure 3: Map of the measured heat flow in seafloor sediments of the Black Sea (in mW/m^2); I, II, III = Geothermal profiles

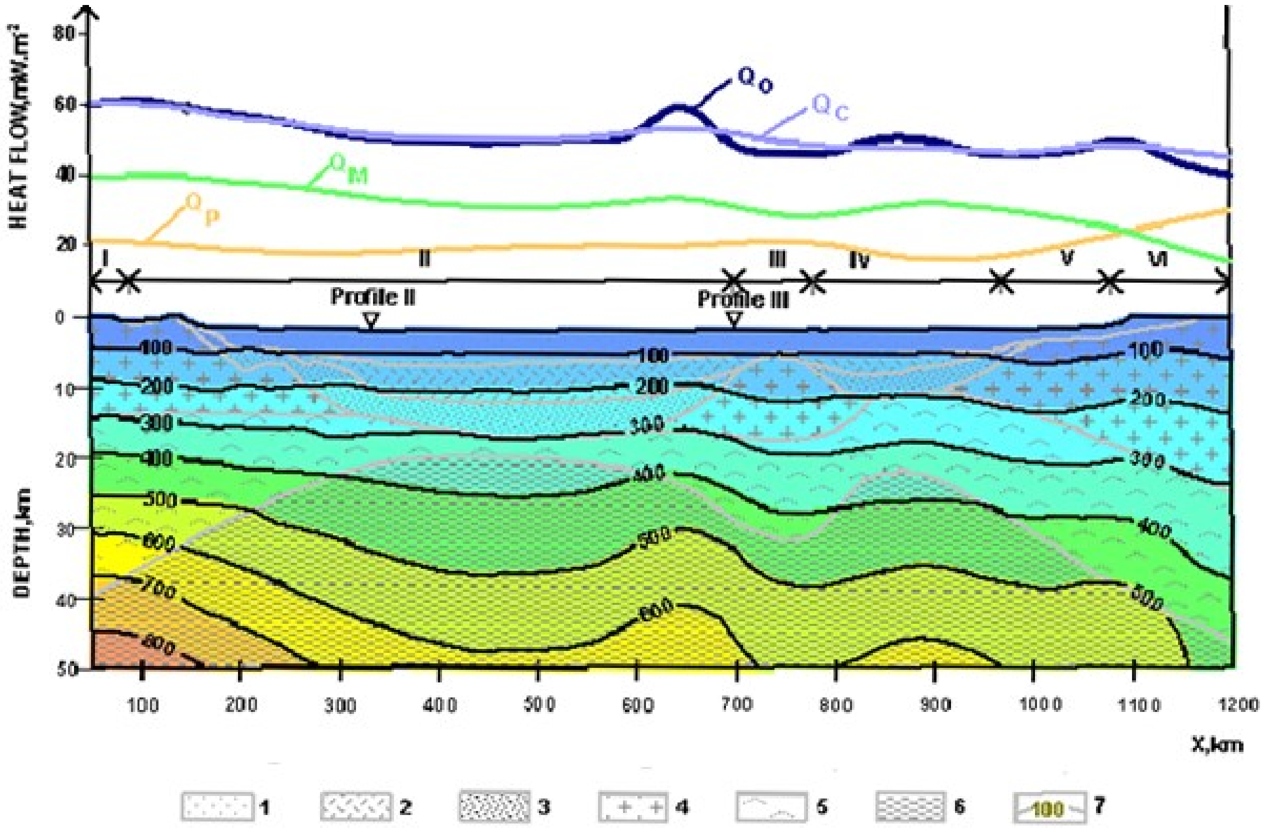

Figure 4: Geothermal model along the Profile I (Varna – Suhumi)

1 – Quaternary-Miocene, Oligocene-Eocene; 2 – Paleocene; 3 – Mesozoic; 4 – Granitic; 5 – Basaltic; 6 – Mantle; 7 – Isotherm Structures: I – Balkanides, II – Western Black Sea depression, III – Central Black Sea dome, IV – Eastern Black Sea depression, V – Shatski dome, VI – Great Caucasus

Q_O – observed sea bottom heat flow, QC – calculated sea bottom heat flow, QM – heat flow from the mantle, QP – internal heat flow from radiogenic sources

The main problem of the mathematical modeling of geothermal fields is finding such distributions of the thermal sources, physical parameters of the rocks, heat flows, which would correspond best to the observed field and the geological-geophysical data.

The interpretation of geothermal anomalies in the Black Sea is based on the assumption that they are linked to export or generate thermal energy in tectonic processes in the lithosphere. The mechanisms of export or generation of energy could be different. This creates a source of anomaly of heat flow in the upper part of the Earth's crust. The amplitude and size of such an anomaly depends on the source's potential, its depth, age and the conditions of heat conductivity.

The thermal anomalies in the western part of Black Sea encompass the entire area where the granite layer is missing. In the eastern part of Black Sea, a whole series of local thermal anomalies is observed. They are mainly linked to the structure of the Earth's crust and various processes of temperature change and the conditions of heat conductivity. (Fig.2,3,4).

One of the applications of the method for mathematical modeling of geothermal sources in the Earth's crust that we suggest is based on the anomalous heat flow observed in the bottom of the Black Sea's and reaches up to 62 mW/m^2. This heat flow is generated by a source of tectonic origin, evidenced by the presence of deep fault. It is located in the south-western end of the Central Black Sea, (so called Andrusov ridge) and it has an almost cylindrical shape. Its depth is from 9 to 23 km and its coordinates are 43º latitude and 35º 40′ longitude (Fig.2,3,4).

CONCLUSIONS

The technique described consists of inverting geothermal field data by regularization method. It is useful to obtain a numerical estimation of the temperature in the Earth's crust directly from subsurface heat flow data. Numerical modeling of a geothermal field based on application of regularizing operators of any kind enables one to obtain the shape and the depth of the anomalous area. The more accurate measurements and more complete a priori information let us obtain more precise results.

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