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A MATLAB Toolbox for Optimization of Deep Borehole Heat Exchanger Arrays

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ABSTRACT

Due to their slow thermal response arrays of borehole heat exchangers (BHE) represent suitable storage systems for seasonally fluctuating sources like solar energy. Excess heat is fed in during summer and extracted in winter. Certain requirements have to be met by such a system: the stored heat must remain in place and the working fluid must maintain an extraction temperature sufficiently high for the specific heating purpose at all times. Since drilling is the most critical cost factor, the required number of BHEs and their respective distance and length need to be optimized. For this purpose a MATLAB toolbox code was developed. It deploys a tetrahedron mesh-based finite element code, a thermal resistance and capacity model for BHE interaction and mathematical optimization techniques. It can effectively simulate and optimize a BHE heat storage system.

1. INTRODUCTION

In Germany, heating accounts for approximately 65% of the total end energy consumption in private households (AGEB, 2013). Consequently, there is a high potential for energy conservation in this sector. Renewable energy sources like solar collectors are increasingly used to cover this heat demand, to reduce the consumption of fossil fuels and to mitigate the CO_2 emissions. In summer, solar thermal collector panels provide excess heat whereas the heating demand is low. Yet, during winter time, a secondary system has to provide the heat when the situation is reversed. Thus, a seasonal storage can enhance the efficiency of a solar collector system by extending the phase of solar heating to winter time. Borehole heat exchanger arrays can tap into large volumes of subsurface rock, which can serve as a heat storage system. Additionally, geothermal heat is feeding such a system. This combination of solar heat usage, seasonal storage and geothermal heat has already demonstrated in practice (Bauer et al., 2010) to be highly efficient. It makes a secondary heating system backing up the solar collector dispensable.

Shallow aquifers are often in contact with the surface water and are usually used for the extraction of drinking water. German legal regulations require heating-induced microbial growth to be prevented in these aquifers. Since solar collectors can provide a temperature output of up to 100 °C and above (Kalogirou, 2004), their excess heat needs to be stored in greater depth where the surrounding temperature is already elevated due to the geothermal gradient. Hence, a high-temperature BHE heat storage system should ideally reach a couple of hundred meters deep and needs to be thermally insulated at the topmost part. On the other hand, the stored heat will not dissipate as fast as in a shallow storage and therefore the extraction temperatures will be higher. This increases the coefficient of performance (COP) of the heat pump and possibly allows for the use with conventional radiator heating systems, which require a higher supply temperature.

Drilling is the critical cost factor in the development of a geothermal reservoir. Deeper BHEs raise the costs for a high-temperature underground storage system significantly. It is imperative to simulate the performance of a planned system prior to the investment of building a storage. The amount, length and respective distance between the BHEs have to be optimized for the storage purpose to avoid an oversized and therefore overpriced system, which is the purpose of the work presented.

2. SIMULATION MODEL

The storage system is simulated with a set of MATLAB functions, which calculate the codependent thermal interaction between the BHEs and the embedding rock on the one hand and the conductive subsurface heat transport on the other hand. For that purpose a one-dimensional model for the thermal BHE behavior is coupled with a three-dimensional finite element model. The simulation model's results were compared with the commercial software FEFLOW (Diersch, 2014), which uses a similar approach, and showed good agreement.

2.1 Finite Element Code and Tetrahedron Mesh

The core of the simulation model is a simple MATLAB implementation for finite elements (Galerkin method of weighted residuals, Zienkiewics et al., 2005) developed by Alberty et al. (1999). It calculates the transient heat diffusion within the subsurface (Fig. 1) by solving Fourier's Law of heat conduction for the model domain:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot \lambda \nabla T + Q \tag{1}$$

With ρ: density, c: heat capacity, T: Temperature, t: time, λ: thermal conductivity and Q: heat sources and sinks. The heat input of the BHEs is calculated using an analytical solution and is added to the 3D solution as sources/sinks.

The weak formulation of Fourier's Law of heat conduction for the domain Ω with temperature interpolation and weight functions ψ can be expressed in a short matrix equation (Eq. 2).

$$\mathbf{N}^{e}\dot{\mathbf{T}}^{e} + \mathbf{L}^{e}\mathbf{T}^{e} = \mathbf{G}^{e} \tag{2}$$

Where $\mathbf{N}_e$ is the heat storage matrix, $\mathbf{L}_e$ is the thermal conductivity matrix and $\mathbf{G}_e$ is the right-hand side containing heat source terms for each element e of Ω.

$$\mathbf{N}^{e} = \rho c \iiint_{\Omega} \psi\psi^{T}\, dx\, dy\, dz \tag{3}$$

$$\mathbf{L}^{e} = \lambda \iiint_{\Omega} \left(\frac{\partial\psi}{\partial x}\frac{\partial\psi^{T}}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial\psi^{T}}{\partial y} + \frac{\partial\psi}{\partial z}\frac{\partial\psi^{T}}{\partial z} \right) dx\, dy\, dz \tag{4}$$

$$\mathbf{G}^{e} = \iiint_{\Omega} \dot{q}_{Q}\psi\, dx\, dy\, dz + \int_{\Gamma^{e}} q_{n}\psi\, ds \tag{5}$$

The computation time benefits from the simplification that convective heat transport is disregarded in the simulation, since a dominantly conductive regime is required and therefore assumed for the model. Furthermore, the finite element solution is calculated on an unstructured tetrahedron mesh generated with TetGen (Si, 2010), which speeds up the computation: The mesh is coarse where possible and is only refined around the BHEs for computational precision. Additionally, the unstructured tetrahedron mesh also allows deviated non vertical BHE orientations.

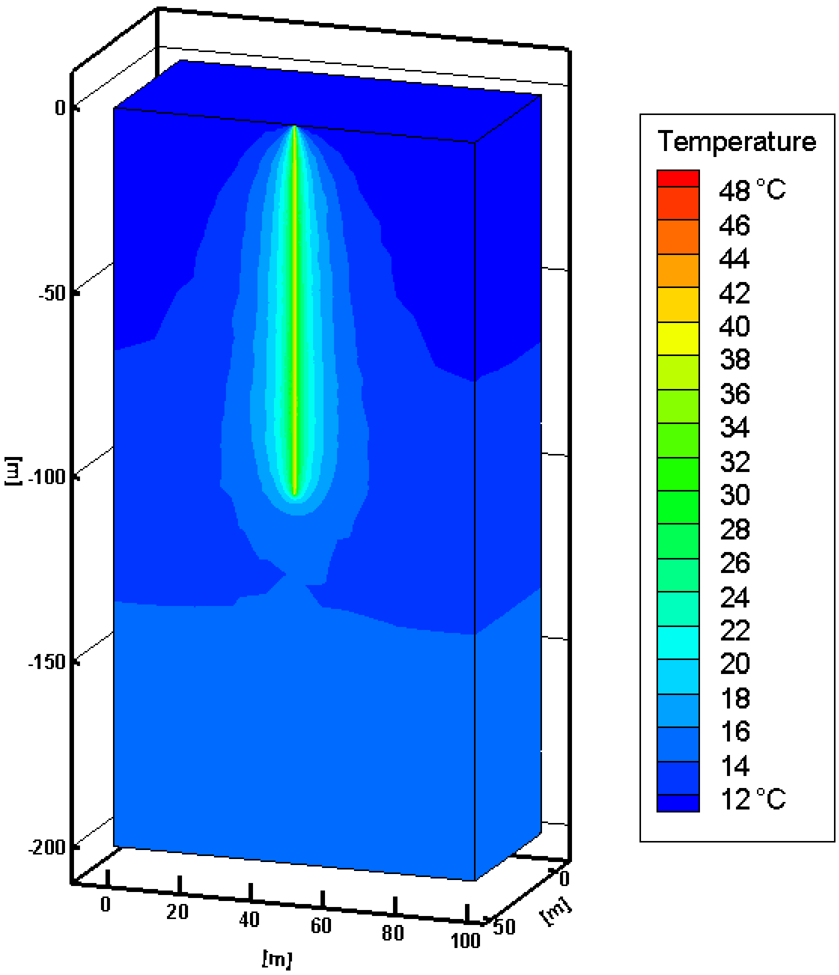

Figure 1: Subsurface temperature distribution of a 100 m coaxial BHE after 5 years of constant loading with 50 °C.

2.2 Analytical BHE Solution

The thermal interaction of the BHEs is calculated by a one-dimensional thermal resistance and capacity model (Bauer et al., 2011) based on the simulation approach of Eskilson and Claesson (1988). Fed with inlet temperature and flow rate data, it provides the temperature distribution in the inlet- and outlet-pipes in predefined depth levels (Fig. 2). The solution takes into account all thermal and hydraulic parameters of the BHE materials and the borehole wall temperature and is very fast compared to the finite element code. However, the solution is stationary.

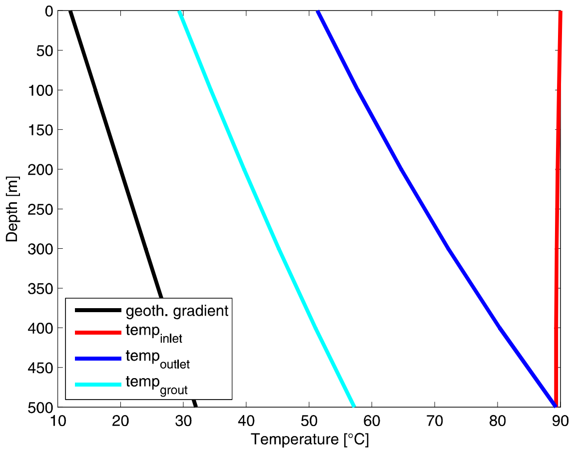

Figure 2: Temperature distribution of a 500 m coaxial BHE (central inlet) in grout, inlet- and outlet pipe; inlet temperature is 90 °C.

2.3 Model Coupling

Both models are linked to take into account the transient subsurface heat transport: while the finite element code calls the analytical solution as a function, the operational parameters are read from a separate file for each time step and passed to the analytical solution. The one-dimensional discretization of the BHE is implemented in the tetrahedron mesh as a line of nodes, which determines the resolution of depth levels for the analytical solution. The temperature field defined by these nodes sets the borehole wall temperature of the BHE function. In return, the analytical solution's results are passed to the BHE nodes as singular point heat sources. As a result, additional terms appear on the right-hand side of the matrix system, which depend on the analytical BHE solution (Diersch, 2014):

$$\mathbf{N}\dot{\mathbf{T}} + \mathbf{L}\mathbf{T} = \mathbf{G} - \mathbf{R}^{\mathbf{Soil-BHE}}\left(T^{BHE}\right) + \mathbf{R}^{\mathbf{BHE}} \cdot \mathbf{T} \tag{6}$$

With

$$\mathbf{R}^{\mathbf{Soil-BHE}}\left(T^{BHE}\right) = \int_z \left(\frac{T_{inlet}(z)}{R_1(z)} + \frac{T_{outlet}(z)}{R_2(z)} \right) dz \tag{7}$$

and

$$\mathbf{R}^{\mathbf{BHE}} = \delta \int_z \left(\frac{1}{R_1(z)} \frac{1}{R_2(z)} \right) dz \tag{8}$$

being output parameters of the analytical solution (T_{inlet}: temperature distribution in inlet tube, T_{outlet}: temperature distribution in outlet tube, R_1/R_2: combined thermal resistances, δ: Dirac delta function). The resulting matrix system with time discretization is a non-linear equation:

$$\left[\frac{\mathbf{N}}{\Delta t} + \mathbf{L} - \delta \int_z \left(\frac{1}{R_1(z)} \frac{1}{R_2(z)} \right) dz \right] \cdot \mathbf{T}_{\mathbf{n+1}} = \mathbf{N} \cdot \frac{\mathbf{T}_{\mathbf{n}}}{\Delta t} + \mathbf{G} - \int_z \left(\frac{T_{inlet}(z)}{R_1(z)} + \frac{T_{outlet}(z)}{R_2(z)} \right) dz \tag{9}$$

MATHEMATICAL OPTIMIZATION

In finding the ideal setup for a BHE heat storage, the layout of the array can be evaluated: the number of boreholes, their respective distance and their length. Despite the simplified design of the simulation model, a single simulation run is still costly in terms of computational time. Thus, a technique is required, which finds an optimal solution with only a few iterations. The program applies mathematical optimization algorithms included in MATLAB's optimization toolbox to solve these problems in an efficient way: genetic algorithms (Goldberg, 1989), which mimic natural selection in the process of developing an optimal solution, simulated annealing (Kirkpatrick et al., 1983), which iteratively improves a candidate solution with respect to a quality threshold, as well as single-variable nonlinear minimization (Brent, 1973). The Optimization results can be compared with implemented Monte-Carlo simulations as a comparably simple approach (Sabelfeld, 1991). The objective function, i.e. the variable(s) to be optimized with respect to a certain parameter, are chosen along with the optimization algorithm by the user. Our toolbox addresses the cost critical optimization objective for BHE heat storage systems: minimize the number and length of the boreholes with respect to a specific amount of heat to be recovered from the storage, which is defined by the building's heat demand. The setup of the program structure is illustrated in Figure 3.

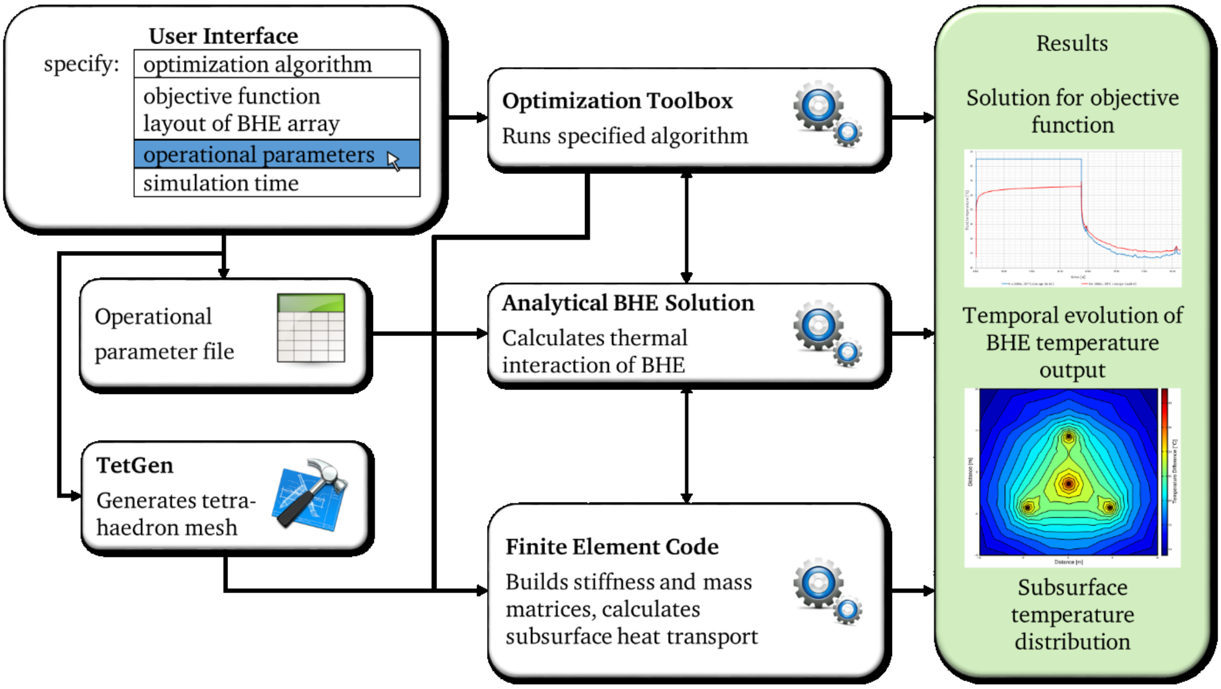

Figure 3: Workflow chart of the program structure.

RESULTS

A model of seven BHEs was tested with the presented toolbox: The depth is set to 100 m while the layout was chosen in a way that the axial distance is the same for all BHEs. In the model, heat is stored with 90 °C injection temperature at a flow rate of 2.5 l/s for each BHE for 182 days and extracted with an inlet temperature of 30 °C at the same flow rate for 183 days. The performance of the storage depends on the axial distance between the BHEs: Heat storage becomes more inefficient with larger spacing, while a small axial distance leads to faster depletion of the storage. Consequently, the aim of the optimization is to maximize the heat extraction by finding the ideal axial distance between the BHEs.

The BHE array layout was optimized with simulated annealing (Kirkpatrick et al., 1983) and a single-variable nonlinear minimization with bounds (Brent, 1973). Additionally, the optimum was estimated with a manual iteration for comparison. For both algorithms, the lower and upper boundary for the search range were set to 1 m and 10 m respectively. The simulated annealing algorithm started the search at 3 m to find an optimum within 10 iterations with a function value tolerance of 0.1 MWh. An optimum was found after 9 iterations at 5.3 m and 468.6 MWh while the single-variable nonlinear minimization with bounds found an optimum after 10 iterations at 5.4 m and 472.2 MWh. The results are in good agreement with the manual estimation, which localizes the optimum between 5 m and 6 m (Fig. 4)

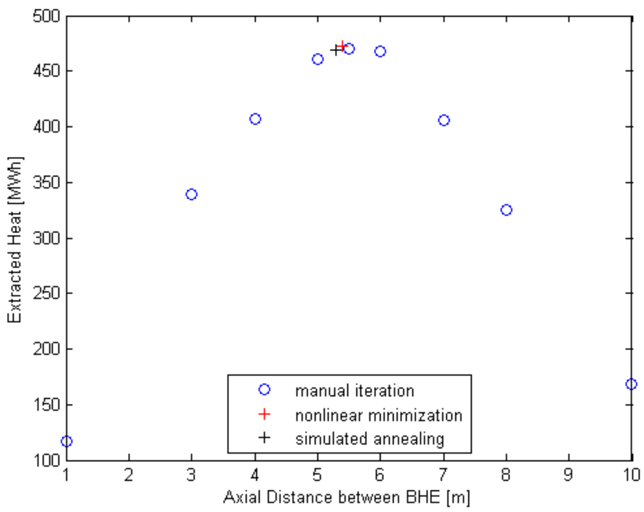

Figure 4: Results of mathematical optimization compared to manually iterated solution

CONCLUSION

The presented toolbox can effectively simulate a BHE heat storage system. In order to avoid an oversized storage, this program can minimize the total drilling length and optimize the axial distance. Consequently, it is a useful tool to design layouts of deep BHE arrays for seasonal heat storage. However, there is still potential for further improvement of the code, like automated time stepping, a strong coupling of the finite element code and the analytical solution or optimization of additional objectives (e.g. operational parameters).

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Graduate School of Energy Science and Engineering

REFERENCES

AGEB - Arbeitsgemeinschaft Energiebilanzen e.V., Ed.: Anwendungsbilanzen für die Endenergiesektoren in Deutschland in den Jahren 2011 und 2012 mit Zeitreihen von 2008 bis 2012. Berlin, (2013).

Alberty J., Carstensen C. and Funken S.A., Remarks around 50 lines of Matlab: short finite element implementation, *Numerical Algorithms*, 20(2-3), (1999), 117-137.

Bauer D., Marx R., Nußbicker-Lux J., Ochs F., Heidemann W. and Müller-Steinhagen H., German central solar heating plants with seasonal heat storage, *Solar Energy*, 84, (2010), 612-623.

Bauer D., Heidemann W., Müller-Steinhagen H. and Diersch H.-J.G., Thermal resistance and capacity models for borehole heat exchangers, *International Journal of Energy Research*, 35(4), (2011), 312-320.

Brent R.B., *Algorithms for Minimization without Derivatives*, Prentice-Hall, Englewood Cliffs, New Jersey, (1973), 195 p.

Diersch H.-J.G., *FEFLOW Finite Element Modeling of Flow, Mass and Heat Transport on Porous and Fractured Media*, Springer-Verlag Berlin Heidelberg, (2014), 996 p.

Eskilson P. and Claesson J., Simulation model for thermally interacting heat extraction boreholes, *Numerical Heat Transfer*, 13, (1988), 149–165.

Goldberg D.E., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley Longman Publishing Co., Inc. Boston, (1989), 372 p.

Kalogirou S.A., Solar thermal collectors and applications, *Progress in Energy and Combustion Science*, 30, (2004), 231-295.

Kirkpatrick S., Gelatt C.D. Jr. and Vecchi M.P., Optimization by Simulated Annealing, *Science, New Series*, Vol. 220, No. 4598, (1983), 671-680.

Sabelfeld K.K.: Monte Carlo Methods in Boundary Value Problems, Springer Series in Computational Physics: Springer-Verlag Berlin Heidelberg, (1991), 281 p.

Si H., Constrained Delaunay tetrahedral mesh generation and refinement. *Finite elements in Analysis and Design*, 46 (1-2), (2010), 33-46.

Zienkiewics O.C., Taylor R.L. and Zhu J.Z., *The Finite Element Method: Its Basis and Fundamentals*, 6th edition, Butterworth-Heinemann Oxford, (2005), 752 p.