Proceedings World Geothermal Congress 2015
Melbourne, Australia, 19-25 April 2015

Numerical Modeling of 3D Transient Thermal Groundwater Flow and Heat Transport of Geothermal Fields of Low to Moderate Temperature

Zhou Xun[a,b], Zhao Jingbo[a], Li Jingwei[a], Ruan Chuanxia[a,c], Lin Li[c], Zhang Qingxiao[a]

a: School of Water Resources and Environment, China University of Geosciences (Beijing), Xueyuan Road 29, Beijing 100083, P. R. China

zhouxun@cugb.edu.cn

b: Key Laboratory of Groundwater Circulation and Evolution (China University of Geosciences (Beijing)), Ministry of Education, Xueyuan Road 29, Beijing 100083, P. R. China

c: Tianjin Geothermal Exploration and Development-Designing Institute, Weiguo Road 189, Tianjin 300250, P. R. China

Keywords: Thermal groundwater; geothermal resources; mathematical model; numerical modeling; software

ABSTRACT

A thorough understanding of the changes in hydraulic heads and temperatures under the exploitation condition is important for the development and utilization of geothermal resources in a geothermal field. Mathematical models related to a geothermal system of low to moderate temperature are needed to simultaneously describe the 3D transient thermal groundwater flow and heat transport. On the basis of previous work, the governing equation of thermal groundwater flow is established, assuming that the density and dynamic viscosity of water are a function of temperature and pressure. By assuming the geodynamic equilibrium between porous media and water is instantaneous, the energy equation of the geothermal system is developed, and is solved using the standard Galerkin finite element method, the streamline upwind Petrov-Galerkin Method (SUPG) and the Petrov-Galerkin least square (PGLS). Fortran 90 is employed to compile the code FEMFH 1.0 for the numerical implementation of the mathematical models describing the thermal groundwater flow and heat transport. The code is composed of five parts, including grid discretization, input of data, solving for the hydraulic head, solving for temperature, and output of calculation results. The code is run for the numerical simulation of the bedrock carbonate reservoir in the Dongli area of the Tianjin geothermal field in northeastern North China Plain. The results show that the models and code enable description of the changes in hydraulic heads and temperatures with reasonable accuracy under the condition of a 6-month period of exploitation and injection followed by a 6-month period of non-pumping in each year.

1. INTRODUCTION

In a deep-seated geothermal system of low to moderate temperature, the pressure and temperature changes with space and time, and the equilibrium of mass, momentum and energy must be taken into account to establish a mathematical model to simultaneously describe the thermal groundwater flow and heat transport. 3D transient mathematical models related to thermal groundwater flow and heat transport were developed and discussed by Bear (1972); Kipp (1987); Pruess et al. (1999); Zhou et al. (2001); Diersch (2005a, 2005b). The Galerkin finite element method was used to discretize the governing equations. Software such as TOUGH2, PETRASIM and FEFLOW have been compiled and employed in the numerical simulation of geothermal systems. On the basis of the previous work, the governing equation of thermal groundwater flow is established in this paper, and the equation of energy of the geothermal system is developed. The finite element method is used to solve the governing equations. Fortran 90 is employed to compile the code FEMFH 1.0 for the numerical implementation of the mathematical models describing the thermal groundwater flow and heat transport. The code is used to perform the numerical modeling and predict the hydraulic head and temperature of the bedrock aquifers in the Dongli area of the Tianjin geothermal field in China.

2. MATHEMATIC MODELS

2.1 Governing equation for pressure

The equation describing groundwater flow in a porous media is given as (Bear, 1972):

$$\frac{\partial}{\partial t}(\rho\phi)+\nabla\cdot(\rho q_i)=\rho_Q Q \tag{1}$$

where ρ is the density of water, [ML^{-3}]; ϕ is the porosity of the media; q_i is the velocity, [LT^{-1}]; Q stands for source or sink, [T^{-1}]; ρ_Q stands for the density of the source or sink, [ML^{-3}]. Groundwater flow in the media is assumed to obey Darcy's law and the velocity can be expressed as:

$$q_i=-\frac{k_{ij}}{u}(\nabla p+\rho g\nabla z) \tag{2}$$

where k_{ij} is the permeability of the media, [L^2]; u is dynamic viscosity of water; p is pressure, [ML^{-2}T^{-2}]; g is the gravitational acceleration, [LT^{-2}]; $i,j=x,y,z$.

The equivalent hydraulic head under the reference condition can be defined as (Diersch 2005a; Huyakorn et al. 1987; Xue and Xie 2007; Post et al. 2007):

$$H = \frac{p}{\rho_0 g} + z \tag{3}$$

where ρ_0 is the density under the reference condition [ML^{-3}], and Darcy's law in Equation (2) becomes:

$$q_i = -\frac{k_{ij}}{u}(\rho_0 g \nabla H + (\rho - \rho_0) g \nabla z) \tag{4}$$

The reference hydraulic conductivity K_{ij}^0 [LT^{-1}] and the dynamic viscosity difference ratio u_r are introduced:

$$K_{ij}^0 = \frac{\rho_0 g k_{ij}}{u_0} \tag{5}$$

$$u_r = \frac{u}{u_0} \tag{6}$$

where u_0 is the dynamic viscosity under the reference condition.

By substituting Equations (5) and (6) into Equation (4), we obtain the Darcy velocity in the geothermal system as follows:

$$q_i = -\frac{K_{ij}^0}{u_r}(\nabla H + \rho_r \nabla z) \tag{7}$$

where $\rho_r = (\rho - \rho_0)/\rho_0$ is the density difference ratio.

Assuming that the porous media and water in the geothermal system are compressible (Diersch 2005b; Xue 1997), Equation (1) can be rewritten as:

$$\rho[(1-\phi)\alpha + \beta]\frac{\partial p}{\partial t} + \phi \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial t} + \nabla \cdot (\rho q_i) = \rho_Q Q \tag{8}$$

where α represents the coefficient of skeleton compressibility, β is the coefficient of fluid compressibility.

Introduce the reference specific storage coefficient, S_0 [L^{-1}]:

$$S_0 = \rho_0 g[(1-\phi)\alpha + \beta\phi] \tag{9}$$

Substituting Equations (3), (7) and (8) into Equation (1), the governing equation for thermal groundwater flow is obtained:

$$S_0(1+\rho_r)\frac{\partial H}{\partial t} + \frac{\phi}{\rho_0}\frac{\partial \rho}{\partial T}\frac{\partial T}{\partial t} - \nabla \cdot \left[\frac{K_{ij}^0(1+\rho_r)}{u_r}(\nabla H + \rho_r \nabla z)\right] = 0 \tag{10}$$

2.2 Governing equation for temperature

In a geothermal system, the geodynamic equilibrium between porous media and water is considered to be completed instantaneously if the diameter of the particle is small (Houpeurt et al. 1965) and heat exchange between them does not exist (Bear 1972; Xue and Xie 2007). The equation for energy in the system is expressed as (Bear 1972):

$$\frac{\partial}{\partial t}[\phi \rho E_f + (1-\phi)\rho_s E_s] + \frac{\partial}{\partial x_i}(\rho E_f q_i) - \frac{\partial}{\partial x_i}(\lambda_{ij}\frac{\partial T}{\partial x_j}) = Q_T \tag{11}$$

where E_f, E_s are internal energy of the water and the media, [J]; λ_{ij} is the tensor of the hydrodynamic-thermo-dispersion, [$JL^{-1}T^{-1}\Theta^{-1}$].; ρ_s is the density of solid particles, [ML^{-3}]; T is the temperature, [Θ].

The relationship between the specific heat content and temperature is expressed as:

$$C=\frac{\partial E}{\partial T}\Rightarrow\begin{cases}E_f=E_f(T_0)+\int_{T_0}^{T}C_f dT\approx C_f(T-T_0)\\ E_s=E_s(T_0)+\int_{T_0}^{T}C_s dT\approx C_s(T-T_0)\end{cases}\tag{12}$$

where C_f is the specific heat capacity of water, [JM$^{-1}\Theta^{-1}$]; C_s is the specific heat capacity of solid particles, [JM$^{-1}\Theta^{-1}$]; T is the reference temperature, [Θ].

It is assumed that the change of the specific heat content and $\partial q_i/\partial x_i$ can be ignored and there is no heat source in the aquifer, from Equations (11) and (12), the following governing equation for temperature is obtained:

$$[\phi\rho C_f+(1-\phi)\rho_s C_s]\frac{\partial T}{\partial t}+\rho C_f q_i\frac{\partial T}{\partial x_i}-\frac{\partial}{\partial x_i}(\lambda_{ij}\frac{\partial T}{\partial x_j})=0\tag{13}$$

Considering that pumping/reinjection wells exist, Equation (13) can be expressed as:

$$[\phi\rho C_f+(1-\phi)\rho_s C_s]\frac{\partial T}{\partial t}+\rho C_f q_i\frac{\partial T}{\partial x_i}-\frac{\partial}{\partial x_i}(\lambda_{ij}\frac{\partial T}{\partial x_j})+\rho_Q C_Q Q(T-T_Q)=0\tag{14}$$

2.3 Finite element methods

The pressure in a deep-seated geothermal system of low to moderate temperature can be described using the following models:

$$\begin{cases}S_0(1+\rho_r)\frac{\partial H}{\partial t}+\frac{\phi}{\rho_0}\frac{\partial\rho}{\partial T}\frac{\partial T}{\partial t}-\frac{\partial}{\partial x_i}\left[\frac{K_{ij}^0(1+\rho_r)}{u_r}(\nabla H+\rho_r\nabla z)\right]=\frac{\rho_Q}{\rho_0}Q\\ H(x,y,z,t)\big|_{t=0}=H_0\qquad(x,y,z)\in\Omega\\ H(x,y,z,t)\big|_{\Gamma_1}=H_1\qquad t>0,(x,y,z)\in\Gamma_1\\ q_n(x,y,z,t)\big|_{\Gamma_2}=-\frac{K_{ij}^0}{u_r}(\nabla H+\rho_r\nabla z)n_i\qquad t>0,(x,y,z)\in\Gamma_2\end{cases}\tag{15}$$

where Ω is the flow domain; Γ_1 is the boundary condition of the first type; Γ_2 is the boundary condition of the second type; H_0 is the initial hydraulic head (m); H_1 is the equivalent hydraulic head of the boundary condition of the first type (m); q_n is the inflow flux on the boundary Γ_2; n_i stands for the external normal direction of the boundary condition of the second type.

Change in temperature in the geothermal system is described using the following model:

$$\begin{cases}[\phi\rho C_f+(1-\phi)\rho_s C_s]\frac{\partial T}{\partial t}+\rho C_f q_i\frac{\partial T}{\partial x_i}-\frac{\partial}{\partial x_i}(\lambda_{ij}\frac{\partial T}{\partial x_j})+\rho_Q C_Q Q(T-T_Q)=0\\ T(x,y,z,t)\big|_{t=0}=T_0\qquad(x,y,z)\in\Omega\\ T(x,y,z,t)\big|_{\Gamma_4}=T_1\qquad t>0,(x,y,z)\in\Gamma_4\\ q_h(x,y,z,t)\big|_{\Gamma_5}=\lambda_{ij}\frac{\partial T}{\partial x_j}n_i\qquad t>0,(x,y,z)\in\Gamma_5\end{cases}\tag{16}$$

where T_0 is the initial temperature; T_1 is the temperature on the boundary condition of the first type; Γ_4 is the boundary condition of the first type; Γ_5 is the boundary condition of the second type; q_h is the inflow heat flux on the boundary Γ_5.

The mathematical model (15) and (16) can be solved by the standard Galerkin finite element method. However, Equation (14) is an advective-dispersive one, and when the velocity of the groundwater flow is relatively high, the numerical oscillation may occur. The streamline upwind Petrov-Galerkin Method (SUPG) and the Petrov-Galerkin least square (PGLS) are adopted to solve the governing equation for temperature in this case (Zienkiewicz et al. 1977; Brooks and Hughes 1982; Zienkiewicz and Wu 1992; Wang 2008).

3. COMPILAION OF THE CODE

Fortran 90 is used to compile the code of the finite element numerical model of 3D transient thermal groundwater flow and heat transport (FEMFH 1.0). The flowchart for solving the equivalent hydraulic head and temperature of a geothermal system of low to moderate temperature is shown in Figure 1.

The code includes five program blocks. (1) The mesh discretization program block is employed to automatically discretize the flow domain with 3D meshes and outputs the coordinates of the nodes and numbers of the meshes. (2) The input program block is used to input data of boundary conditions and initial conditions, aquifer parameters and information of source and sink. (3) The pressure solution program block is used to solve the pressure/hydraulic head at each node at different times. The Gauss-Seidel point by point iteration method, the successive over relaxation method, and the PCG iteration method are used to solve the linear equations. The successive over relaxation method is used to solve the velocities of thermal groundwater flow in *x, y, z* directions at each node. (4) The temperature solution program block is employed to solve the temperature at each node at different times. The Picard iteration method is used to solve the temperature governing equation (Huang et al. 1996; Putti and Paniconi 1995). (5) The output program block is used to output the calculation results of pressure/hydraulic head and temperature and velocities at each node of different times. Figures are drawn based on the output data.

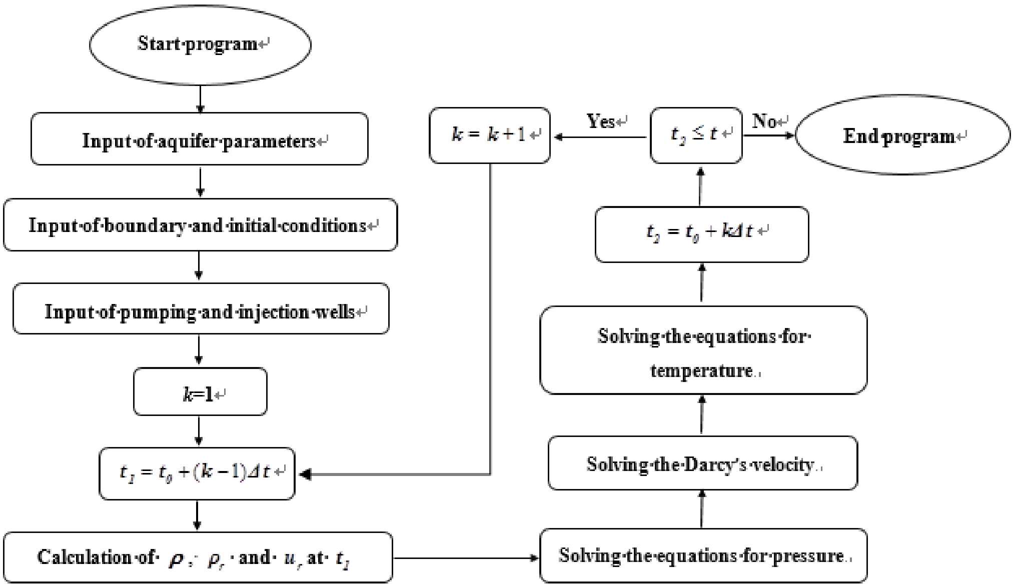


Figure 1. Flowchart showing the solution of hydraulic head and temperature.

4. CASE STUDY

The bedrock geothermal reservoir in the Dongli area to the north of Tianjin is a sedimentary basin-type reservoir in the Tianjin geothermal field in the northeastern part of the North China Plain of China. The modeled domain has an area of 40.124 km^2 and a thickness of 2333 m ranging in elevation from -4000 to -1667 m. The geothermal reservoir consists of Meso-Neo Proterozoic–Cambrian-Ordovician carbonates. For thermal groundwater flow, the southern and northern boundaries are constant head boundaries and the other boundaries are no-flow boundaries. For heat transport, the southern and northern boundaries are prescribed temperature boundaries and the other boundaries are boundaries of the second type. The modeled domain is considered to be homogenous and isotropic. It is discretized with hexahedrons and has four model layers (Figure 2). The distribution of hydraulic head and temperature under national condition, in which pumping wells and reinjection wells are assumed not to exist, is numerically calculated under a steady state to obtain reasonable aquifer parameters. The parameters used in the numerical model are listed in Table 1.

Table 1. Parameters used for simulations.

Parameter	Value
Hydraulic conductivity (K_{ij}^0)	1.0×10^{-6} ms^{-1}
Storativity (S_0)	0.0004
Porosity (ϕ)	0.03
Gravitational acceleration (g)	9.81 ms^{-2}
Thermal conductivity of the water (λ^f)	0.65 Jm^{-1}s^{-1}K^{-1}
Reference pressure (P_0)	1.01325×10^7 Pa
Reference temperature (T_0)	86℃
Thermal conductivity of the solid phase (λ^s)	2.6 Jm^{-1}s^{-1}K^{-1}
Density of solid particles (ρ_s)	2600 kgm^{-3}
Specific heat capacity of solid particles (C_s)	878 Jkg^{-1}K^{-1}
Specific heat capacity of water (C_f)	4200 Jkg^{-1}K^{-1}

In the Tianjin geothermal field, development of geothermal resources using the pattern of one pumping well with one reinjection well is popular in the last few decades. They are operated in the 6-month winter period and stopped in the 6-month period of summer. In this model, one pumping well is set at the coordinate of (42886, 22583) and one reinjection well, at (37670, 14561) (Figure 3). The pumping rate for the pumping well is 43200 m^3/d and the injection rate for the reinjection well is 17280 m^3/d with temperature of 40 °C.The predicted distribution of hydraulic heads and temperatures under the above condition are shown in Figure 4 and the changes in hydraulic head and temperature at the pumping well and the reinjection well are shown in Figure 5.

Significant changes in hydraulic head occur near the pumping well under the pumping condition, with a depression cone of a drawdown of about 4 m near the pumping well. A significant rise in hydraulic head occurs near the reinjection well with a rise of hydraulic head of about 2 m. The hydraulic head declines during the pumping period and rises during the non-pumping period. The temperature does not show any changes at the pumping well but a small decline at the reinjection well over the 5-year period owing to reinjection.

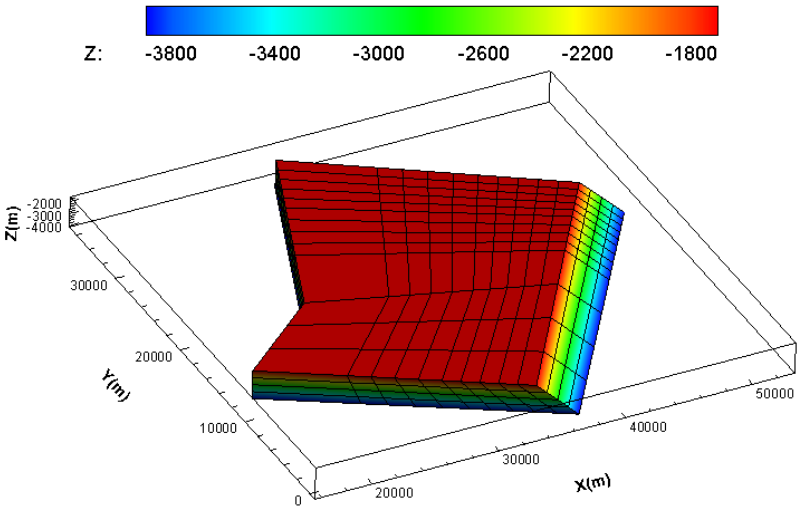


Figure 2. Discretization of the modeled domain.

5. SUMMARY

3D transient mathematical models are established to simultaneously describe the thermal groundwater flow and heat transport in a geothermal system of low to moderate temperature. The change in density of water due to changes in temperature and pressure is considered in the models. The standard Galerkin finite element method is used to solve the governing equation of thermal groundwater flow and the governing equation for temperature is handled by using the standard Galerkin finite element method, the streamline upwind Petrov-Galerkin Method (SUPG) and the Petrov-Galerkin least square (PGLS), respectively. Fortran 90 is employed to compile the code FEMFH 1.0 for the numerical implementation of the mathematical models describing the thermal groundwater flow and heat transport. The code is used to calculate the changes in hydraulic heads and temperatures in the bedrock geothermal reservoir in the Dongli area near Tianjin in China. Spatial and temporal changes in hydraulic heads and temperatures in the modeled area under pumping and reinjection conditions can be obtained.

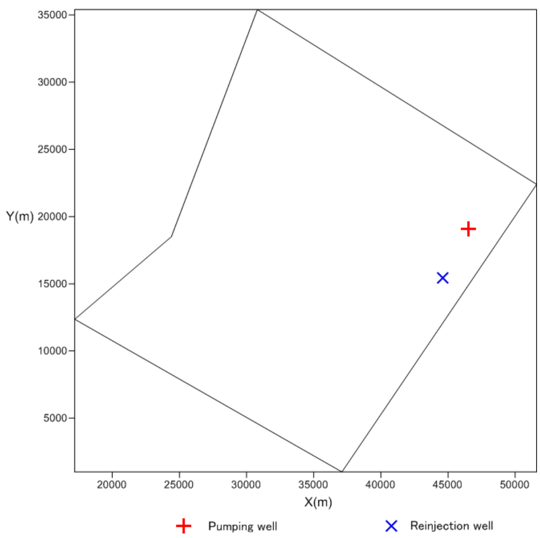
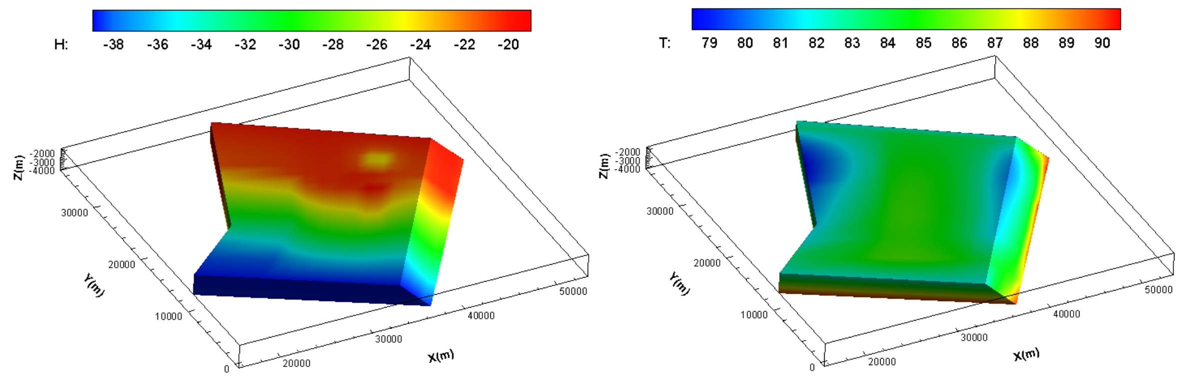
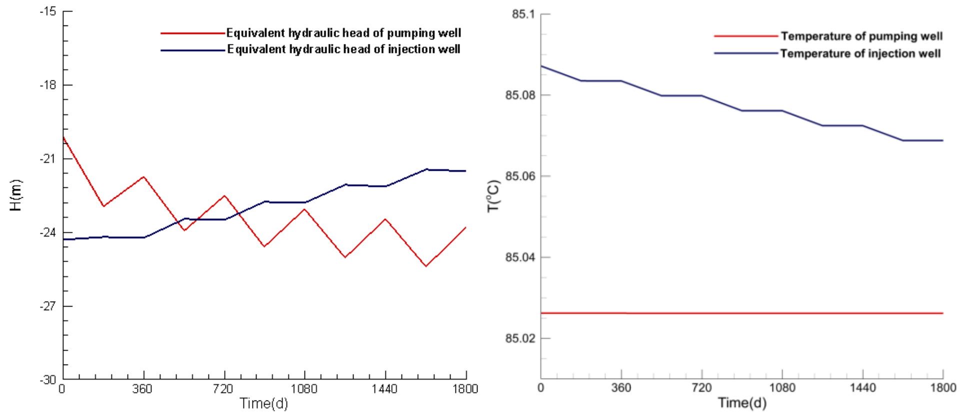


Figure 3. Location of the pumping well and the reinjection well



(a) (b)

Figure 4. Distribution of equivalent hydraulic head (a) and temperature (b) under the condition of one pumping well and one reinjection well.



(a) (b)

Figure 5. Changes in equivalent hydraulic head (a) and temperature (b) at the pumping well and at the reinjection well.

Acknowledgements: This work was supported by the project of China Geological Survey (1212011120064) and the Fundamental Research Funds for the Central Universities of China (2652013085, 2652014089).

REFERENCES

Bear, J.: *Dynamics of Fluids in Porous Media*, American Elsevier, NewYork, (1972).

Brooks, A.N., and Hughes, T.J.: Streamline upwind Petrov-Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equations, *Computer methods in applied mechanics and engineering,* **32**, (1982), 199-259.

Diersch, H.-J.G.: FEFLOW-White papers vol. I, WASY GmbH Institute for water resources planning and systems research Ltd, Berlin, (2005a).

Diersch, H.-J.G.: FEFLOW Reference Manual, WASY GmbH Institute for water resources planning and systems research Ltd, Berlin, (2005b).

Houpeurt, A., Delouvrier, J., and Iffly, R.: Fonctionement d'un doublet hydraulique de refroidissement, *La Houille Blanche*, (1965), 239-246.

Huang, K., Mohanty, B., and Van Genuchten, M.T.: A new convergence criterion for the modified Picard iteration method to solve the variably saturated flow equation, *Journal of Hydrology,* **178**, (1996), 69-91.

Huyakorn, P.S., Andersen, P.F., Mercer, J.W., and White, H.O.: Saltwater intrusion in aquifers: Development and testing of a three-dimensional finite element model, *Water Resources Research,* **23**, (1987), 293-312.

Kipp, K.L.: HST3D: a Computer Code for Simulation of Heat and Solute Transport in Three-dimensional Ground-water Flow Systems, U.S. Geological Survey, Water-Resources Investigations Report, (1987).

Post, V., Kooi, H., and Simmons, C.: Using Hydraulic Head Measurements in Variable-Density Ground Water Flow Analyses, *Ground Water,* **45**, (2007), 664-671.

Pruess, K., Oldenburg, C., and Moridis, G.: TOUGH2 User's Guide Version 2, Earth Sciences Division, Lawrence Berkeley National Laboratory University of California, Berkeley, California, (1999).

Putti, M., and Paniconi, C.: Picard and Newton linearization for the coupled model for saltwater intrusion in aquifers, *Advances in Water Resources,* **18**, (1995), 159-170.

Wang, H.: *Dynamics of Fluid Flow and Contaminant Transport in Porous Media*, Higher Education Press, Beijing (in Chinese), (2008).

Xue, Y.: *Groundwater Dynamics*, Geological Publishing House, Beijing (in Chinese), (1997).

Xue, Y., and Xie, C.: *Numerical Simulation of Groundwater*, Science Press, Beijing (in Chinese), (2007).

Zhou, X., Chen, M., Zhao, W., and Li, M.: Modeling of a Deep-Seated Geothermal System Near Tianjin, China, *Ground Water,* **39**, (2001), 443-448.

Zienkiewicz, O., Heinrich, J., Huyakorn, P., and Mitchel, A.: An upwind finite element scheme for two dimensional convective transport equations, *International Journal for Numerical Methods in Engineering,* **11**, (1977), 131-144.

Zienkiewicz, O., and Wu, J.: A general explicit or semi-explicit algorithm for compressible and incompressible flows, *International Journal for Numerical Methods in Engineering,* **35**, (1992), 457-479.