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A Simple Engineering Analysis Method to Evaluate the Geothermal Potential from a Hot Water Aquifer

Xiaoyu Lu[1], Kenichi Soga[1], Xiaohui Cheng[2] and Santo Bains[3]

1 University of Cambridge, Department of Engineering, Trumpington Street, Cambridge, UK, CB2 1PZ

2 Department of Civil Engineering, Tsinghua University, Beijing

3 BP plc, Chertsey Road, Sunbury-on-Thames, Middlesex, TW16 7LN, UK

xl340@cam.ac.uk, chengxh@mail.tsinghua.edu.cn, santo.bains@uk.bp.com

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ABSTRACT

Axisymmetric analytical solutions for transient heat transport in geological media are generally derived using Lauwerier's model. In this study, an axisymmetric solution for a water injection well based on Lauwerier's model was derived with constant well temperature conditions. Using the convective heat transfer boundary condition, the model considers both the conduction in the aquifer and the heat exchange at the boundary. This solution intuitively illustrates the temperature distribution within the aquifer and therefore allows us to develop a simple engineering method that can determine a suitable location for an extraction well. It can also evaluate the amount of geothermal energy that can be extracted given different positions of the extraction well in relation to the injection well. In order to utilize the analytical solution, one of the input parameters is the heat transfer coefficient at the convective heat transfer boundary condition. In this study, a series of finite element analysis was conducted to determine an equivalent heat transfer coefficient from a given geometry, water injection rate and thermal properties of the geothermal aquifer and aquitard. Using the analytical solution coupled with the numerical results, it is possible to conduct a simple and fast estimation of geothermal potential of a particular site.

1. INTRODUCTION

For deep geothermal energy, the ground temperature is influenced by the geothermal heat flow that is conducted upwards from the Earth's core and mantle. In other words, the temperature gradient along the depth is the source of the deep geothermal energy. Generally, deep geothermal energy examines the heat resources up to 5 km in the ground. At this depth, the heat is sufficient and drilling costs are economically feasible. Temperatures greater than 100 ℃ can be used to generate electricity and greater than 60 ℃ used to heat buildings. With the technology developed in the offshore oil and gas industries, we can use such technology to extract hot water from rocks or aquifers in the ground that can support the heating demands of buildings.

There are two main categories of deep geothermal systems in the UK. One is Hot Sedimentary Aquifer (HSA) systems where the heat is stored in the aquifers. The other is Engineered Geothermal Systems (EGS) where the heat is stored in the rocks so that we need to engineer a reservoir by stimulation. HSA resources are found in sedimentary basins. It is attractive to explore for heat resources in the sedimentary basins because they have not only thermal insulation to hold heat flow but also reservoir formations with storage capacity. Due to these properties, we can get great geothermal potential in the low and medium temperature natural aquifers at economically drillable depths. Besides, as the sedimentary aquifers are permeable, the geothermal energy can be extracted without stimulation or permeability enhancement (SKM, 2012).

In order to better utilize the geothermal potential from a hot water aquifer, we need to know the temperature distribution and thermal breakthrough around the injection well so that we can find the best location for the extraction well and then evaluate the geothermal potential that can be extracted. In this paper, it is proposed that such important information about the geothermal system can be obtained quickly and conveniently using a simple engineering analysis method developed in this study.

2. THE CONCEPTUAL MODEL

The Lauwerier model (Lauwerier 1955) has been used to find an analytical solution to the transient heat transfer problems in geological media (planar symmetric and axisymmetric) as shown in Figure 1. The problem considered in this study is heat conduction and convection in a homogeneous aquifer layer of thickness H [m]. The initial temperatures for the aquifer and overburden layers are denoted by T_0 [K]. At time $t = 0$, water with temperature T_w (lower than T_0) is injected into the aquifer layer with a constant rate through the injection well. The lower boundary of the aquifer layer is impermeable and adiabatic. The upper boundary is impermeable and non-adiabatic. Symmetry can be used if both lower and upper boundaries are adiabatic. The solution gives spatial and temporal temperature variation around the injection well. This in turn allows us to determine the optimal location of the extraction well as shown in the figure. An important assumption of this model is that the temperature of the aquifer layer is uniform in the vertical direction. There are already a couple of analytical solutions for this model with different simplified assumptions and boundary conditions. Lauwerier (1955) proposed a planar solution that only heat convection is considered in the aquifer, assigning the heat conduction boundary at the interface between the aquifer and overburden layer. Ogata and Banks (1961) obtained another planar solution considering both heat convection and conduction in the aquifer but neglecting the heat exchange at the interface. Zeng et al. (2002) gave an axisymmetric solution for heat conduction in the finite line-sourced model under the constant heat flux boundary perpendicular to the line source. Barends (2010) improved the Lauwerier's planar solution by considering both heat convection and conduction in the aquifer. Tan et al. (2012) put forward a planar solution which considered both heat convection and conduction in the aquifer and applied the convective heat transfer boundary at the interface.

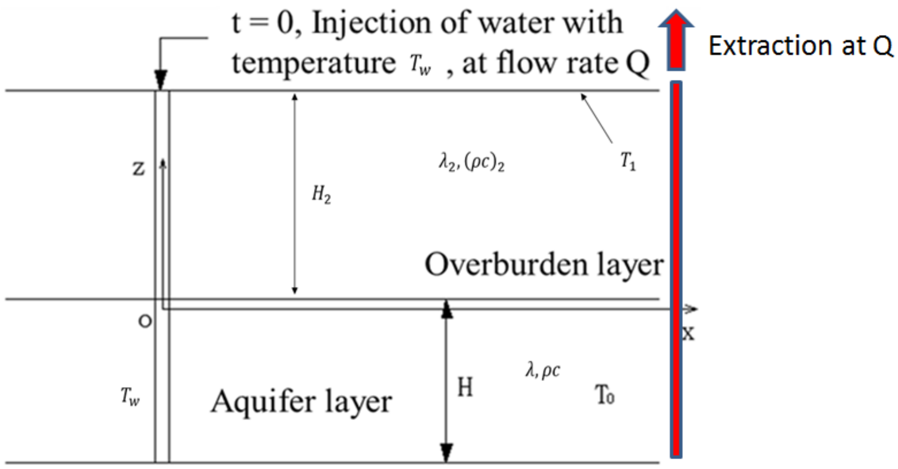

Figure 1: The Lauwerier model

3. NEW ANALYTICAL SOLUTION WITH CONVECTIVE HEAT TRANSPORT BOUNDARY

3.1 Problem

The new solution to the axisymmetric problem considers the interface (boundary between the overburden layer and aquifer layer) to be a convective heat transfer boundary, that is, the heat flux $w_0=h(T-T_0)$; h is the convective heat transfer coefficient. In the boundary, water flows at the interface of the aquifer and overburden layers, so it is convective heat transfer between fluid and solid. Therefore, it is appropriate to apply a convective heat transfer boundary. Another merit of using coefficient h, is that the geometry and properties of the whole overburden layer can be compacted into a single coefficient h so that the influence of the overburden layer to the aquifer is expressed by coefficient h, which will greatly simply the model but still accurately characterize the model. The method to determine the value of h will be discussed in Section 4.

3.2 Analytical Solution

The basic equations are as follow:

$$\frac{1}{r}\frac{\partial}{\partial r} rD \frac{\partial T}{\partial r} - v\frac{\partial T}{\partial r} = \frac{\partial T}{\partial t} + \frac{w_0}{\rho c H}, \quad r > 0, \quad t \geq 0$$

$$v = q\frac{(\rho c)_f}{\rho c}, \quad q = \frac{Q}{2\pi r H} \rightarrow v = \frac{2\alpha D}{r} = \frac{\beta}{r}, \quad \alpha = \frac{Q}{4\pi n H D R}, \quad \beta = \frac{Q}{2\pi H}\cdot\frac{(\rho c)_f}{\rho c}$$

$$\alpha = \frac{\beta}{2D}, \quad R = 1 + \frac{1-n}{n}\frac{(\rho c)_s}{(\rho c)_f}$$

$$D = \frac{\lambda}{\rho c}$$

$$w_0 = h(T - T_0)$$

$$t = 0, \quad T = T_0$$

$$r \rightarrow 0, \quad T = T_w$$

$$r \rightarrow +\infty, \quad T = T_0$$

where ρ is the density, c is the specific heat capacity, D is the diffusivity, λ is heat conductivity, v is the convection rate of the aquifer layer, w_0 is the heat flux at the junction, H is the aquifer layer thickness, q is the Darcy flow velocity in the aquifer layer, Q is the volume velocity, α is a dimensionless coefficient, R is the delay factor, n is the porosity, h is the convective heat transfer coefficient, T_0 is the initial temperature, T_w is the water temperature, subscript f represents the water phase, subscript s represents the soil, and no subscript represents the average properties.

After a series of steps, the solution for the above problem is as follow:

$$T = T_0 + \frac{T_w - T_0}{\Gamma(\alpha)} \int_{\frac{r^2}{4Dt}}^{\infty} e^{-\left(x + \frac{hr^2}{4\rho c D H} \cdot \frac{1}{x}\right)} x^{\alpha - 1} dx \tag{1}$$

In dimensionless form, it reads:

$$T^* = \frac{1}{\Gamma(\alpha)} \int_{\frac{r^{*2}}{t^*}}^{\infty} e^{-\left(x + h^* r^{*2} \cdot \frac{1}{x}\right)} x^{\alpha - 1} dx \tag{2}$$

With $T^* = \frac{T - T_0}{T_w - T_0}$, $r^* = \frac{1}{H} r$, $t^* = \frac{4\lambda}{\rho c H^2} t$, $h^* = \frac{H}{4\lambda} h$, $\alpha = \frac{1}{4\pi n R} \cdot \frac{\rho c}{H \lambda} Q$

h^* is the dimensionless convective heat transfer coefficient. λ is heat conductivity. This analytical solution is complete and accurate considering the conduction and convection in the aquifer and heat transfer at the interface. The curve of this new analytical solution (1) is shown in Figure 2. The parameters used in this paper are shown in Figure 3, unless indicated specifically in the figures.

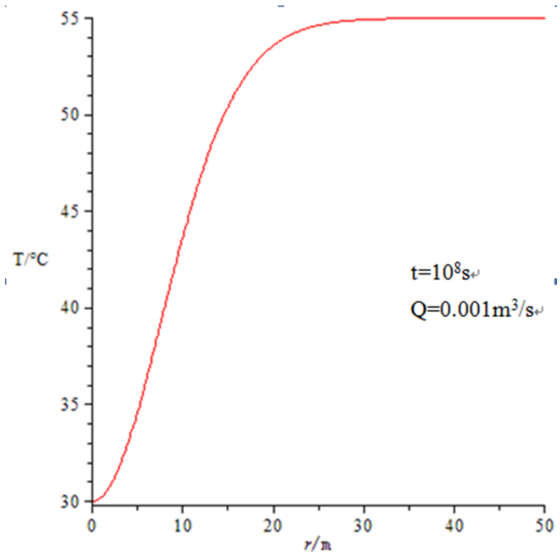

Figure 2: New analytical solution

T_w (°C)	T_0 (°C)	Q (m³/s)	H(m)	λ ($W/m\cdot K$)
30	55	0.001	50	1.125

$\rho_w c_w$ ($J/m^3\cdot K$)	$\rho_s c_s$ ($J/m^3\cdot K$)	n	h ($J/m^2 sK$)
1000×4200	2800×830	0.25	10

Figure 3: The value of parameters used in the figures

The solution was tested against numerical model. The results of the comparison are shown in Figure 4. The analytical solution agrees with the numerical solution.

3.3 Extracted energy

Since the temperature distribution of the aquifer at different times is known from the injection well, it is possible to estimate the time-dependent energy extracted from the extraction well, which is located at a given distance from the injection well. This estimation assumes that the two wells do not interact, which is not true when the two wells are close to each other. However, in such case, the estimated value will be of practical usage as the energy extracted will be small.

$$P = \rho Q \cdot c(T - T_w)$$

where

$$Q = \frac{2\pi H \rho c}{(\rho c)_f} \cdot \beta$$

Substitute the temperature expression, we obtain:

$$P = 2\pi H \rho c \cdot \beta [-(T_w - T_0) + \frac{T_w - T_0}{\Gamma(\frac{\beta}{2D})} \int_{\frac{r^2}{4Dt}}^{\infty} e^{-(x + \frac{hr^2}{4\rho c D H} \cdot \frac{1}{x})} x^{\frac{\beta}{2D} - 1} dx] \tag{3}$$

The above equation is the energy that can be extracted at the extraction well at different times. The energy extracted against the location of the extraction well and against water injected rate (at quasi-steady state condition) are shown in Figure 5.

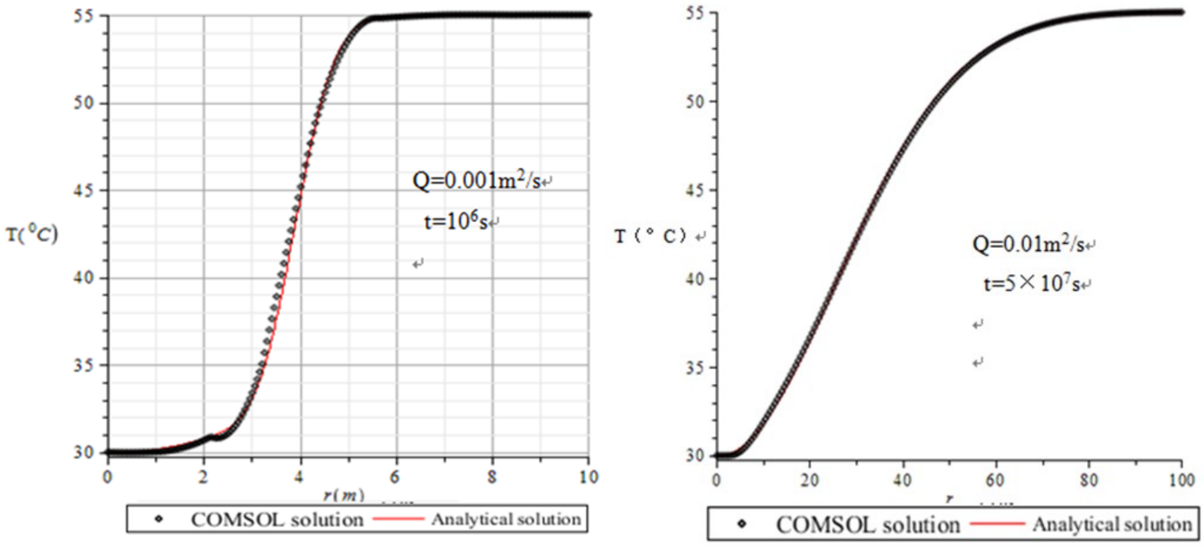

Figure 4: Comparison between the results of the analytical solution and Finite Element Analysis

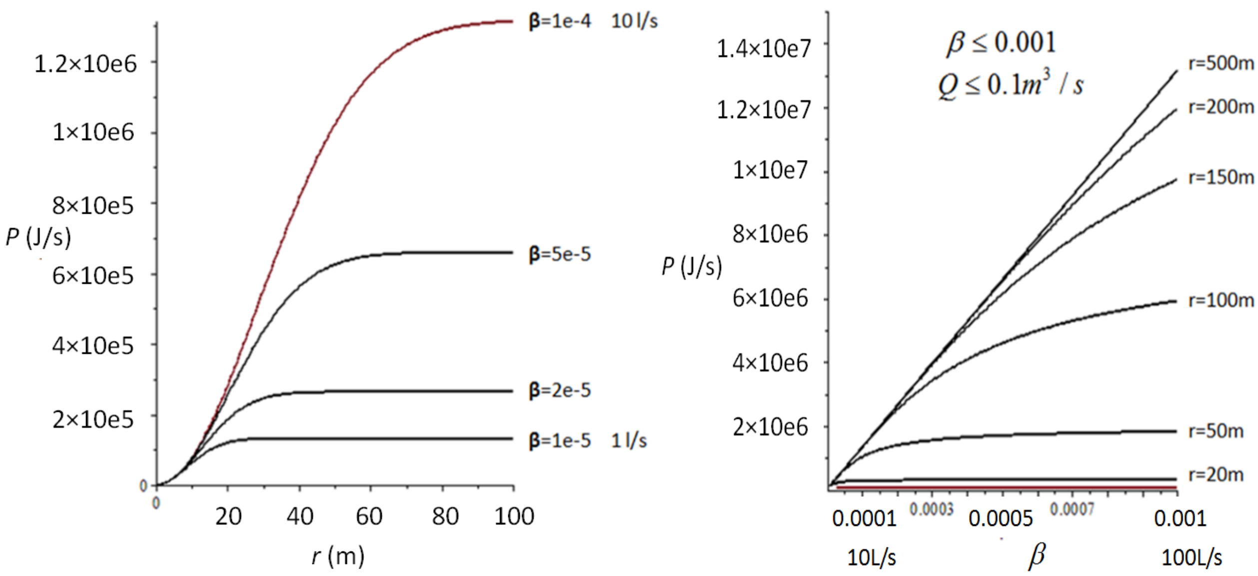

Figure 5: Evaluation of extracted energy

At a given injected rate, the extraction well should be located beyond the area at which the temperature is affected by the injection well. Also, at a given location of the extraction well, there is an upper limit for the energy we can get even if the injected and extracted rate becomes very large. Hence there is a suitable set of rate and location that is economically viable to extract heat from the ground.

4. DETERMINATION OF THE CONVECTIVE HEAT TRANSFER COEFFICIENT

4.1 Methodology

The major issue in using the proposed analytical solution is that the heat transfer coefficient h needs to be given. The value should be a function of the geometry of the reserviour, thermal properties as well as the injection rate. In order to determine the equivalent convective heat transfer coefficient h, a series of finite element analysis was conducted to get the temperature distribution of the aquifer in the model with the overburden layer. The computed temperature distribution was compared to the result given by the analytical solution. By adjusting the value of coefficient h, the analytical solution can match with the temperature distribution in the model with overburden layer. This h is the equivalent convective heat transfer coefficient for the given geometry and properties of the overburden layer.

4.2 Finite Element Analysis of the model with the overburden layer

We assume that the initial temperature in the overburden layer is proportional to the depth. The temperature of the whole model at a given time is shown in Figure 6. The temperature distribution in the aquifer layer for different times is shown in Figure 7.

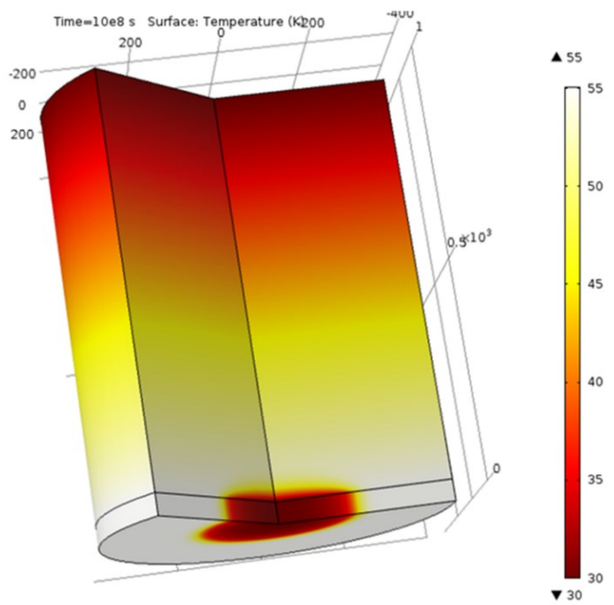

Figure 6: Temperature of the whole model

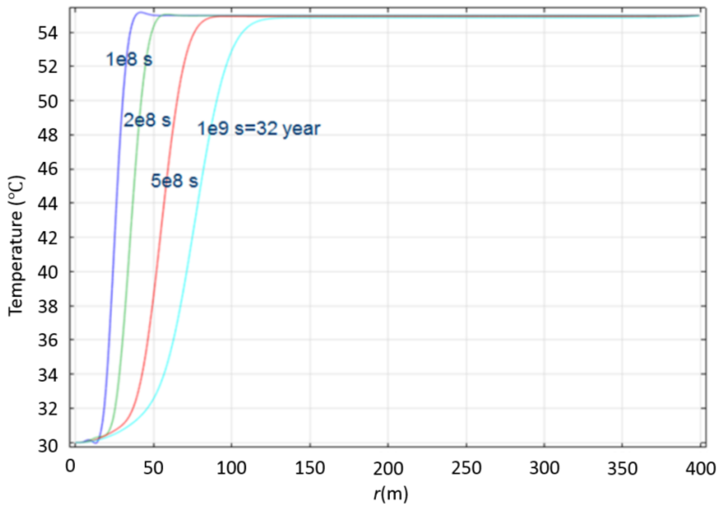

Figure 7: Temperature distribution in the aquifer layer for different times

4.3 The function of dimensionless coefficient *h**

The dimensionless parameters for the overburden layer relative to the aquifer layer are as follow:

height: $H^* = \dfrac{H_2}{H}$; heat capacity: $(\rho c)^* = \dfrac{(\rho c)_2}{\rho c}$; heat conductivity: $\lambda^* = \dfrac{\lambda_2}{\lambda}$; surface temperature: $T_1^* = \dfrac{T_1 - T_0}{T_w - T_0}$

It is considered that non-dimensional h* = ($4Hh/\lambda$) is a function of these parameters; $h^* = f(\alpha, H^*, \lambda^*, \rho c^*, T_1^*)$, α is the dimensionless water injected rate, which is defined as $\alpha = \dfrac{1}{4\pi nR} \cdot \dfrac{\rho c}{H\lambda} Q$.

As shown in Figure 8 and Figure 9, for typical geothermal energy problems in the field, the influence of the dimensionless overburden height and surface temperature on the temperature distribution in the aquifer is small, H^* and T_1^* can be excluded in the expression of h^*. Hence, $h^* = f(\alpha, \lambda^*, \rho c^*)$. In this paper, as a preliminary study, we let $\lambda^*=1$ and $\rho c^*=1$, namely the overburden layer and aquifer layer have the same conductivity and heat capacity. The objective is to find the relation between dimensionless coefficient h^* and dimensionless water injected rate. Further work is currently conducted to examine the effect of other parameters.

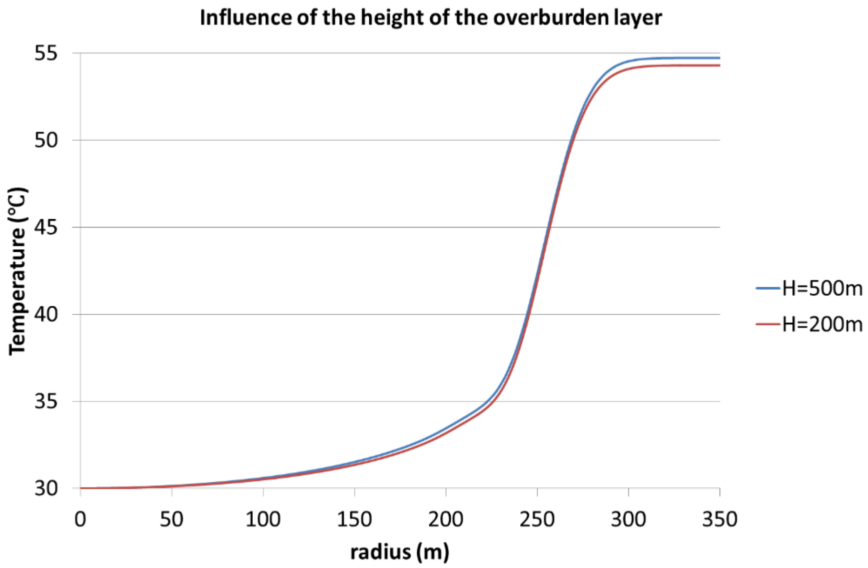

Figure 8: Influence of the height of the overburden layer

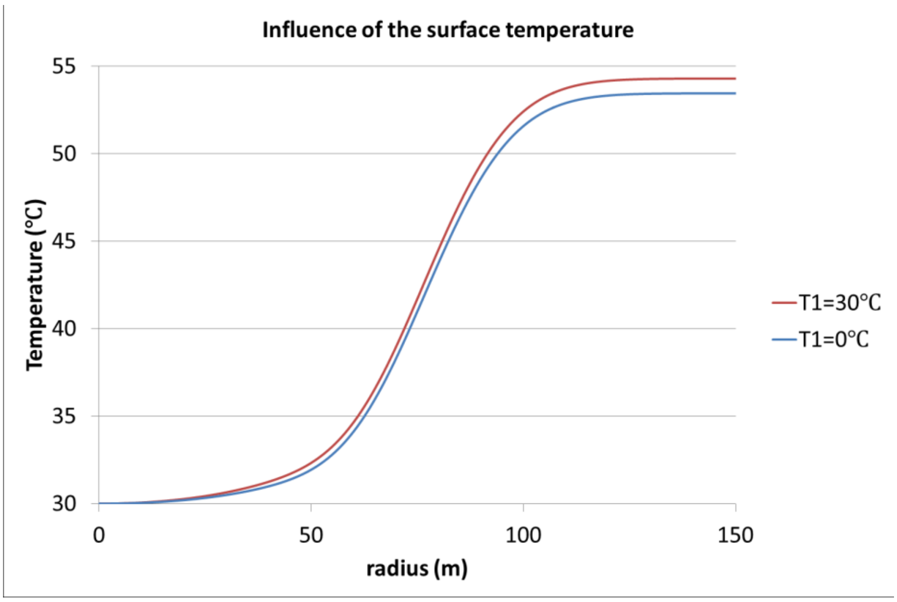

Figure 9: Influence of the surface temperature

4.4 Comparison

As the analytical solution is a transient solution, we must choose a time when we aim to match the two solutions in order to find the optimal equivalent coefficient *h*. The typical life span of a geothermal system is 25 years, therefore the temperature distribution at time=25 years was selected compare the analytical solution with the numerical solution as shown in Figure 10. Although this *h* is obtained for time=25 years, for other time, the two figures can still perfectly match with each other using the same *h* for practical usage of the solution as shown in Figure 11. This proves that the coefficient *h* does not change with time which enables us to assign the same *h* in the analytical solution for any time (up to 25 years).

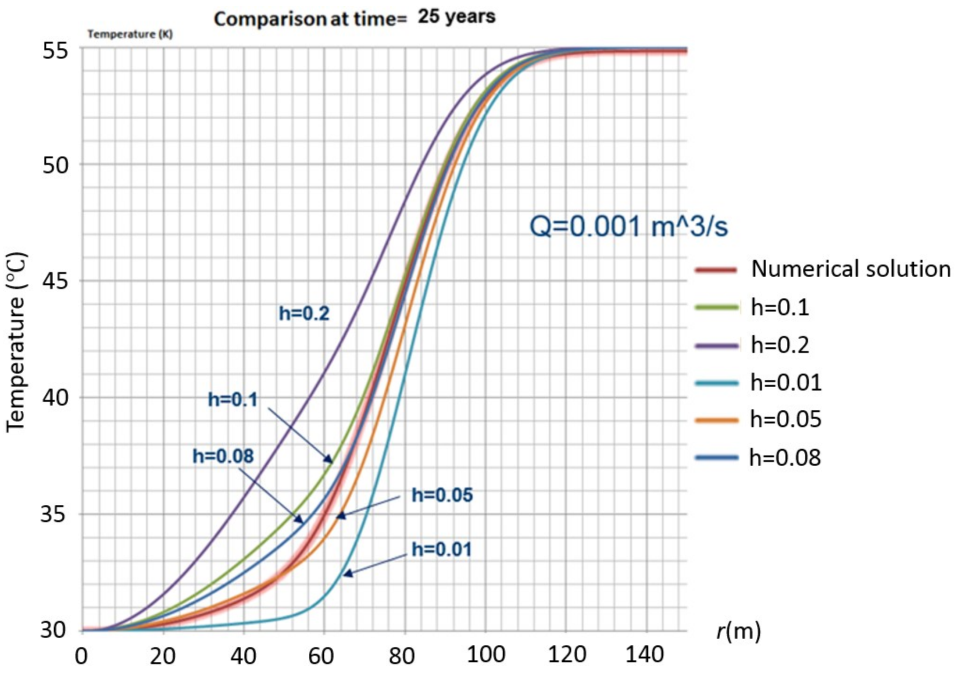

Figure 10: Comparison between numerical solution and analytical solutions with different *h*

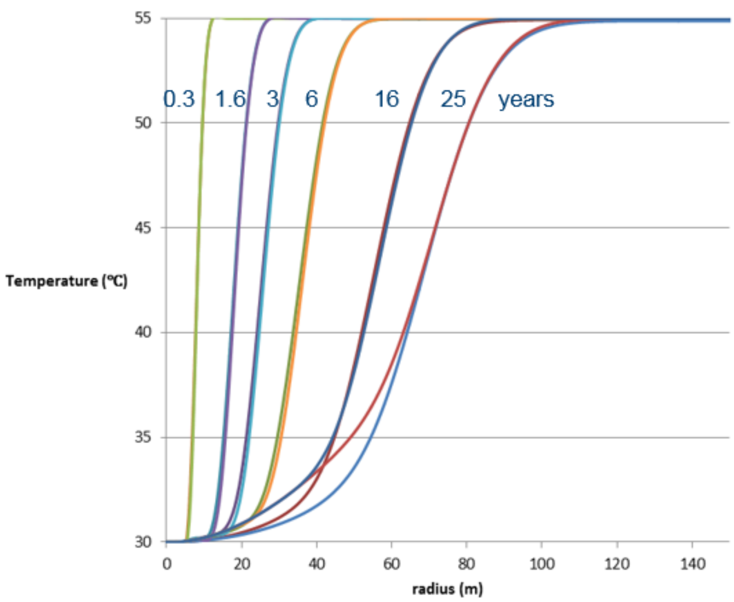

Figure 11: Comparison between the numerical solutions and analytical solutions (*h*=0.11) at different times

In this study, the injection rate was varied to find the equivalent coefficient *h*. The relation between dimensionless coefficient h^* and dimensionless injection rate α found from this series of simulation is shown in Figure 12.

From the figure above, the relation between h^* and α can be approximated as follows.

$$h^* = 0.425\ln(\alpha) + 0.377 \quad (4)$$

This approximation can be used when the overburden layer and aquifer layer have the same conductivity and heat capacity. The effect of other parameters will be reported in future publications.

Equation (4) is very useful in practice when the overburden and aquifer layer have the similar conductivity and heat capacity. For a given injection rate Q, the dimensionless injection rate α can be computed and substitute it into Equation (4) to get the dimensionless coefficient h^* or h. Once coefficient h is known, Equation (1) is used to estimate the temperature distribution in the aquifer with time which in turn can be used to determine the location of the extraction well and evaluate the energy extracted using Equation (3). The calculation can be done for different values of Q to conduct a back-of-the envelope evaluation of geothermal potential of a site.

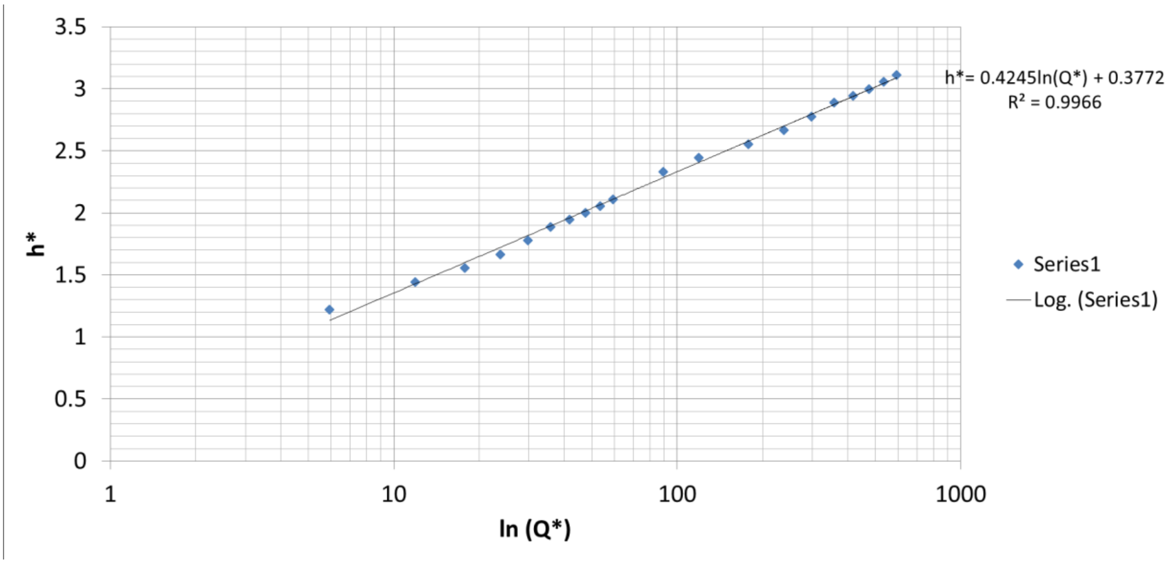

Figure 12: The relation between h^* and α

CONCLUSION

In this paper, a simple engineering analysis method is proposed to obtain the temperature distribution in the aquifer thus to determine the location of the extraction well and evaluate the geothermal potential from the hot water aquifer. Applying the convective heat transfer boundary condition at the interface not only reflect the actual heat transfer process at the interface, but also generalize the whole overburden layer by a coefficient h, which can be estimated the geometry and properties of the aquifer and overburden layers. Using the analytical solution coupled with the h coefficient obtained by a series of numerical simulations, it is possible to conduct a simple and fast estimation of geothermal potential of a particular site.

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