

Flow Lognormality and Spatial Correlation in Crustal Reservoirs – I: Physical Character & Consequences for Geothermal Energy

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ABSTRACT

‘Effective medium’ uniformity within a formation is the basis for most geological reservoir flow models. With reservoir structure fixed by geological formation boundaries from drilling and/or seismic imaging, model flow properties of constituent formations are assigned values from sparse formation sampling on the assumption that spatial variations in formation flow properties ‘effectively average out’ at important scale lengths. Corollary to the effective medium assumption is that formation properties are independent of other formation properties. The two assumptions imply that spatial variations in crustal rock are effectively *uncorrelated*. The central limit theorem then implies that well flow productivities within a reservoir are normally distributed. In reality, however, *in situ* well flow productivities are conspicuously lognormally distributed in groundwater, hydrocarbon, geothermal and fossil reservoir flow systems worldwide. The statistical fact of lognormality of reservoir flow system well productivity implies that *in situ* spatial variations are spatially *correlated* rather than *uncorrelated*. While statistics do not reveal the physical origin of spatial correlation and lognormality, a great deal of hydrocarbon reservoir well-log and well-core data give strong empirical guidance on the nature of *in situ* spatial correlations; incidental well-log and well-core data extend the hydrocarbon reservoir spatial correlation systematics to hydrogeological, geothermal, and fossil flow systems:

- *In situ* porosity $\phi(x,y,z)$ is spatially correlated with grain-scale fracture damage at all scales from mm to km through Fourier spectral scaling law, $\Phi(k) \sim 1/k$, for five decades of spatial frequency k , $1/\text{cm} < k < 1/\text{km}$;
- *In situ* permeability $\kappa(x,y,z)$ is spatially correlated with *in situ* porosity $\phi(x,y,z)$, $\delta\phi \sim \delta\log(\kappa)$, with typical cross-correlation coefficient 85%.
- *In situ* permeability lognormality arises as $\kappa(x,y,z) \sim \exp(\alpha\phi(x,y,z))$ with values of fracture connectivity parameter α in the range 30-50;
- For empirical value range $30 < \alpha < 50$, the occurrence probability $P(\Lambda)$ of flow systems of characteristic size Λ scales inversely with system size, $P(\Lambda) \sim 1/\Lambda$, plausibly leading to the observed Fourier spectral scaling property of crustal reservoirs, $\Phi(k) \sim 1/k$.

These well-attested spatial correlation phenomena, associated with the ‘geocritical state’ of *in situ* fracture systems keyed to power-law spatial scaling $\Phi(k) \sim 1/k$, have clear implications for a range of geothermal reservoir operations as discussed in this and two accompanying presentations:

- I. Base-load heat extraction rates put excessive demand on the physical scale of EGS bulk thermal conduction volumes; direct-use allows heat extraction at scales that can allow heat extraction by advective fluid percolation through geocritical heat volumes rather than by advective fluid flow along conductive heat exchange surfaces;
- II. Existing natural geothermal flow systems can significantly reduce drilling cost overhead and potentially tap into accessible heat volumes by mapping on-going reservoir acoustic noise associated with long-range spatially correlated fracture systems to precisely locate and assess large-scale permeability pathways within a reservoir volume;
- III. EGS development based on stimulating/controlling well-to-well flow can proceed by emulating natural permeability processes that yield lognormal well-flow spatial distributions in order to artificially generate high values of fracture connectivity parameter α that controls *in situ* high permeability.

1. INTRODUCTION

We begin our discussion of Enhanced/Engineered Geothermal Systems (EGS) by noting a somewhat overlooked functional dichotomy between convectively and conductively recharged crustal heat reservoirs. As summarized by Moeck & Beardsmore (2014), convectively recharged heat reservoirs comprise the vast majority commercial crustal heat energy production, while conductively recharged heat reservoirs contribute virtually nothing. In contradistinction, EGS is, and has been for decades, explicitly identified as a process for extracting heat energy from conductively recharged heat reservoirs in ordinary crust. As the EGS process is conspicuous for lack of fulfillment, it is worth taking a closer look at its tenets.

Convectively recharged crustal heat reservoirs are natural physical systems in which large-scale thermally-driven tectonic forces create large-scale, upper-crustal natural fracture-based fluid flow systems overlying hot, ductilely deforming lower crustal intrusions. The source of the hot ductile intrusives is mantle convective flow on time scales short by comparison to thermal conduction time scales. Mantle convection time scales lead to unusually large thermal gradients in the crust which drive fluid-borne heat transport along the natural fracture-flow pathways generated by rapid large-scale finite-strain deformation. Such convective heat flow greatly exceeds standard conductive heat flow, and has shown the potential for matching wellbore-fluid fluxes needed to run base-load power turbines. For more than 50 years, convectively recharged crustal reservoirs have been (more or less)

successfully exploited worldwide, essentially by drilling enough wells to ultimately intersect naturally-occurring fracture-borne convective fluid flow paths through crust heated by magmatic intrusions (e.g., Grant, Donaldson & Bixley 1983).

Conductively recharged crustal heat reservoirs are natural physical systems in which the conductive flow of heat from earth's mantle is not noticeably influenced by large-scale tectonically-induced crustal flow systems. The empirics of existing geothermal energy production imply that EGS facilities for obtaining base-load heat fluxes from thermally conductive heat stores have fluid-flow/heat-extraction interfaces not present in nature. Tester et al. (2006) describe a long-standing concept of EGS heat extraction:

The heat-transfer system can be thought of as similar to a series of flat plates with gaps (the fractures) between them and a semi-infinite conduction heat source surrounding each fracture. Heat is transferred by conduction through the rock, perpendicular to the surfaces of the fractures.

This EGS concept requires heat fluxes from thermally conductive heat stores to be spread out over large areas in order to reduce the thermal gradients feeding heat to the advective-fluid flow surfaces or interfaces. The scale of the EGS conductive heat recharge constraint is apparent when comparing advective versus conductive heat flow *re* base-load power demands. To provide 10MW_e of electrical energy, a wellbore has to provide $J \sim 100\text{MW}_{\text{th}}$ of heat energy to the turbine. This provision entails wellbore fluid transport rates $\sim 150\text{kg/s}$ of $\sim 150^\circ\text{C}$ fluid containing $\sim 4280 \cdot 150$ Joules of heat energy per kg of fluid. EGS implies thermal conduction heat extraction J of order mean heat flow density j over area A , $J = jA$. Heat flow density $j = K\Delta T/\ell$ is fixed by thermal conductivity $K = 3 \text{ W/m}^\circ\text{C}$, thermal contrast $\Delta T \sim 150^\circ\text{C}$ between low and high system temperatures, and thermal gradient scale length ℓ . The thermal gradient scale length is limited by the duration of expected heat production t_0 . The cooling rate of thermal body of dimension ℓ is given by $\exp(-\tau/t)$, with $\tau = \ell^2/4D$ and $D \sim 1.5 \cdot 10^{-6}\text{m}^2/\text{s}$ the thermal diffusivity of rock. For rock to stay significantly conductively hot for time duration t_0 , $\tau \sim 3t_0$. Taking $t_0 \sim 50$ years, $\tau \sim 150$ years, fixing $\ell \sim 150\text{m}$. For $\Delta T = 150^\circ\text{C}$ and $\ell = 150\text{m}$, the conductive heat flow density is $j \sim 3\text{W/m}^2$. (By comparison, standard crustal heat flow is 60mW/m^2 (Davies & Davies 2010).) The heat exchange area required by standard base-load EGS can be expected to be of order $A = J/j \sim 10^8\text{W} / 3\text{W/m}^2 \sim 33 \cdot 10^6\text{m}^2 = 33 \text{ km}^2$.

In face of these indicative large scale EGS heat exchange areas, two order of magnitude aspects can be noted. First, direct-use power requires an order of magnitude smaller thermal conduction area. Second, if advective fluid flow heat transport is added to EGS scenarios, heat transport efficiency gains can reduce the scale of EGS heat exchange volumes. While conductive heat flow $j_c = K\Delta T/\ell$ is compatible with advective heat flow $j_a = \rho C v T$ for advective flow velocity $v \sim 10^{-9} \text{ m/s}$, feasible values of *in situ* fluid flow $\sim 10^{-7}\text{m/s}$ can replace (i.e., ‘short circuit’) thermal conduction paths to reduce thermal resistance of purely conductive EGS heat flow. Reduced thermal resistance equates to greater thermal efficiency and hence smaller EGS volumes. Table 1 specifies representative physical property values for conductive and advective heat transport and their ratio, Peclet number $Pe \equiv j_a/j_c$.

Table 1. Representative physical constants for conductive and advective crustal heat transport j_c and j_a .

Variable	Symbol	Nominal Value
Rock thermal conductivity	K	$3 \text{ W/m}^\circ\text{C}$
Density of fluid	ρ	1000 kg/m^3
Heat capacity of fluid	C	$4280 \text{ J/kg}^\circ\text{C}$
Ambient temperature	T	300°C
Temperature contrast	ΔT	150°C
Fluid velocity (conduction)	v_c	10^{-9} m/s
Fluid velocity (advection)	v_a	10^{-7} m/s
Characteristic dimension	ℓ	150 m
Peclet number $\equiv j_a/j_c = \rho C v T / K \Delta T / \ell$	Pe	$0.5 - 50$

It is also cautionary to note that some 75%-90% of wells drilled in naturally-occurring convective geothermal systems do not provide enough fluid through-put to run turbines. Even in most bounteous natural crustal heat and fluid flow conditions, most drilled wells do not intersect fluid flow paths that connect to major heat transport conduits. Little has been done to understand or develop the large number of underperforming or non-performing geothermal wells. The well intervention picture has, however, changed dramatically with the advent of drilling and systematic massive hydrofracturing of tight gas sand wells and, later horizontal wellbores in shale bodies. Recent advances in horizontal drilling and hydrofracturing associated with shale oil and gas production provide a large-scale *in situ* laboratory in which to reconsider EGS scenarios based on cylindrical wellbore-centric geometries flow rather than planar flow.

- Horizontal drilling is mature technology giving a logical basis for heat extraction on essentially arbitrary axial scales.
- Physical properties of rock are likely to be laterally consistent on essentially arbitrary scales.
- Horizontal wellbore hydrofracture technology allows realistic assessment of scales of radial intervention from a wellbore.

- Horizontal wellbores allow consideration of essentially 1D radial fluid flow in 2D wellbore-centric sections rather than 2D planar fluid flow 3D rectilinear sections.

We argue there is a clear physical basis for the convective/conductive heat reservoir exploitation dichotomy: *in situ* fluids percolate through crustal rock at all scale lengths, and in doing so efficiently access and transport crustal heat on multiple scales in a manner that is accessible to currently standard horizontal drilling. In the following two sections we focus on EGS matters from the perspective heat extraction via extended reach horizontal wellbores. We first revisit the fundamental power-provision limitation imposed on EGS by pure thermal conductivity. We then indicate the means by which wellbore-centric heat extraction can exploit fluid advective heat transport to increase EGS efficiency and reduce EGS scales. By the simplicity of essentially 1D radial flow in 2D wellbore-centric crustal sections, the discussion can be conducted using standard mathematics and simple/accessible numerical procedures.

Two accompanying papers discuss more thoroughly the *in situ* ‘geocriticality’ aspects lying behind our wellbore-centric approach to EGS. Paper II (Malin et al. 2015) uses shale reservoir production seismic monitoring data to look at fluid flow in crustal rock from the perspective of mapping large-scale permeability structures in geocritical reservoir volumes. Paper III (Pogacnik et al. 2015) considers the physical character of *in situ* fluid flow with regard to enhancing the natural fracture-connectivity pathways through which crustal fluids would percolate in wellbore-centric EGS volumes.

2. SCALE MATTERS: THERMAL GRADIENTS FOR BASE-LOAD & DIRECT-USE GEOTHERMAL POWER

Constraints on EGS power production follow from Fourier’s empirical law for conductive heat flow (Carslaw & Jaeger 1959),

$$\mathbf{j} = K \nabla T. \quad (1)$$

By (1) heat energy flow \mathbf{j} (units of energy per second per unit area) is proportional to the thermal gradient ∇T (units of temperature per unit length). In scientific literature, heat flow \mathbf{j} is measured in watts per meter-squared [W/m^2] and temperature gradient ∇T in degrees-Centigrade per meter [$^\circ\text{C}/\text{m}$]. The proportionality constant K is measured in watts/meter/degrees-Centigrade. For rock, thermal conductivity is limited to small variations around its mean value $\sim 3 \pm 0.5 \text{ W}/\text{m}^\circ\text{C}$ (Jessop 1990; Clauser & Huenges 1995; Beardsmore & Cull 2001).

Heat flow variation over time t is related to temporal changes in temperature T through conservation of energy,

$$\rho C \partial_t T = \nabla \cdot \mathbf{j}. \quad (2)$$

By (2) the net flow of heat in and out of an elementary volume given by the spatial divergence of heat flow $\nabla \cdot \mathbf{j}$ in watts per meter-cubed [W/m^3] is proportional to the time rate of change in temperature $\partial_t T$ in degrees-Centigrade per second [$^\circ\text{C}/\text{s}$]. The proportionality constant ρC in units of Joules per meter-cubed [J/m^3] is determined by material properties mass density ρ and heat capacity per unit mass C .

Fourier’s law (1) combined with conservation of energy (2) describes spatiotemporal heat flow in uniform media,

$$\partial_t T(\mathbf{x}, t) = D \nabla \cdot \nabla T(\mathbf{x}, t) = D \nabla^2 T(\mathbf{x}, t), \quad (3)$$

where $D = K/\rho C$ is the thermal diffusivity in units of meter-square per second [m^2/s]. For rock of mass density $\rho \sim 2400 \text{ kg}/\text{m}^3$ and heat capacity per unit mass $C \sim 840 \text{ Joule}/\text{kg}^\circ\text{C}$, thermal diffusivity is of order $D \sim 1.5 \cdot 10^{-6} \text{ m}^2/\text{s}$ (Robertson 1988).

Generic time-evolution for (3) for a system of characteristic spatial dimension ℓ proceeds according to

$$T(\ell, t) \sim \exp(-\ell^2/4Dt). \quad (4)$$

Characteristic times of heat flow (4) are $\tau \sim \ell^2/4D$; for $\ell \sim 1\text{m}$, $\tau \sim 161000\text{s} \sim 2 \text{ days}$; for $\ell \sim 1\text{km}$, $\tau \sim 5000\text{yr}$; for $\ell \sim 100\text{m}$, $\tau \sim 50\text{yr}$.

From the numbers derived from (4) for rock diffusivity $D \sim 1.5 \cdot 10^{-6} \text{ m}^2/\text{s}$, a crustal heat conduction system of characteristic time constant $\tau \sim 50\text{yr}$ has characteristic spatial dimension $\ell \sim 150\text{m}$. Heat extracted from a crustal volume of dimension $\ell \sim 150\text{m}$ over a time period $\tau \sim 50\text{yr}$ for a characteristic temperature drop across the system of order $\Delta T \sim 150^\circ\text{C}$ is characterized by heat flow $j \sim K \Delta T / \ell \sim 3 \text{ W}/\text{m}^\circ\text{C} \times 150^\circ\text{C}/150\text{m} \sim 3 \text{ W}/\text{m}^2$.

While this rate of heat flow is some 50 times the mean rate of conductive heat flow $\sim 60 \text{ mW}/\text{m}^2$ observed in ordinary crust (Davies & Davies 2010), the main point lies in the other direction: is generic heat flow rate $j \sim 3 \text{ W}/\text{m}^2$ compatible with the demands of commercial electric power generation?

Commercial electrical power generation from geothermal fluids nominally requires wellbore heat transport of order tens of MW_{th} . Well flow figures for two New Zealand geothermal fields indicate production well flow rates of order 50-100 kg/s (Grant 2009). Thermal heat transport for wellbore fluid mass flow 50 kg/s at assumed 100°C temperature drop during turbine transit translates to $50 \text{ kg/s} \times 4200 \text{ J}/\text{kg}^\circ\text{C} \times 100^\circ\text{C} \sim 20 \text{ MW}_{\text{th}}$ of wellbore thermal energy transport supposed for EGS to be ultimately tied to crustal conductive heat recharge. If wellbore crustal heat extraction is accomplished over a 10 km wellbore interval, crustal heat needs to reach the wellbore from the crustal heat volume at rates of order 2 KW per meter of wellbore. From (1)-(4), radial heat flow $j \sim 3 \text{ W}/\text{m}^2$ across a 1 m thick section of crust of characteristic circumference $2\pi\ell \sim 660\text{m}$ can supply the needed $\sim 2 \text{ KW}$ per meter of wellbore section.

Such constructions characterize the EGS thermal conduction constraint: ability to supply $\sim 20\text{MW}$ heat transport per wellbore needed to run turbines only if the wellbore is $\sim 10\text{km}$ long. To further define EGS systems, conductive supply of 2KW per meter of wellbore section is available only at a wellbore annular radius $\ell \sim 100\text{m}$. As it is inefficient for thermal conduction to bring the heat energy from the radius $\ell \sim 100\text{m}$ to the central wellbore, heat transfer to the wellbore must occur through engineered advection systems induced in the $\ell \sim 100\text{m}$ radius wellbore-centric crustal sections.

The 2D cylindrical geometry allows straightforward analytic solutions to conductive and advective flow boundary conditions for uniform media. The analytic solutions can be expanded to numerical solutions of heat transport for arbitrary 2D conductivity structures that effectively simulate advective structures in the context of conductive flow. These numerical solutions conveniently allow direct comparison of advective heat extraction for the two extremes of heat recharge: purely conductive and powerfully convective.

If the enclosing crustal volume is at ambient temperature T_0 and fluid circulating in the wellbore is at temperature $T_1 < T_0$, heat will be removed by the fluid from the crustal volume, and rock temperature at the wellbore will fall in response to the heat removal. At the same time, heat from the enclosing crustal volume will flow towards the wellbore to make up for the heat removed by the wellbore fluid. In principle the wellbore fluid can remove heat at an arbitrary rate – the colder it is or the faster it flows, the more heat it removes. The corresponding conductive thermal recharge cannot, however, change except in proportion to the thermal gradient between the wellbore and the surrounding rock. It follows that the thermal gradient cannot be arbitrarily large without severely reducing the rock temperature at the wellbore, thus degrading the amount of heat the wellbore fluid can remove.

Assuming that the radial heat flow dominates the axial heat flow along the wellbore, heat flow equation (3) can be approximated as 1D radial flow to simply compute the trade-off between rate of heat removed by the wellbore fluid and the heat restored by the surrounding crustal volume. Conductive heat flow in a cylindrically symmetric and axially uniform system is described by (Carslaw & Jaeger 1959, §§7.1, 7.9; Figs 11, 24, and 29 show that the cylindrical systems of characteristic gradient scale length ℓ are compatible in heat flow properties with similarly-scaled thermal flow structures of planar and spherical geometry). The cylindrical temperature gradient expression of Fourier's law is

$$d/dr(rdT/dr) = 0, \quad (5a)$$

with generic solution

$$T(r) = a \log(r) + b. \quad (5b)$$

Setting the dimension and temperature of the overall EGS volume at radial dimension r_o and temperature T_o , heat is extracted at rate j_i by an interior advective heat exchanger of radius r_i . The implied heat flow boundary condition is $-j_i$ at radius r_i with temperature boundary condition T_o at radius r_o :

$$\begin{aligned} a &= -j_i r_i / K \\ b &= T_o + j_i r_i / K \log(r_o) \\ T(r) &= -j_i r_i / K \log(r_o/r) + T_o. \end{aligned} \quad (5c)$$

System heat extraction rate $J = j_i r_i = 1\text{KW/m}$ gives a commercial-grade direct-use resource of 1MW operating off a km -long wellbore. As heat cannot be extracted at this rate from a small wellbore without immediately cooling the rock, we posit a wellbore advective heat exchange system radius $r_i \sim 100\text{m}$ to give a low enough thermal gradient to keep heat flowing conductively without temperature loss through high temperature gradients. Heat flow in a 1m -thick crustal slice of circumference $\sim 2\pi 100\text{m}$ is $j_i \sim 1.6\text{W/m}^2$, ~ 25 times the naturally occurring high crustal heat flow values of order 60mW/m^2 .

At ambient temperature $T_o \sim 300^\circ\text{C}$ (e.g., Kawerau geothermal field in New Zealand; Bignal & Milicich 2012) and outer boundary temperature of the wellbore advective heat exchange system as $T(r_i) = T_i \sim 200^\circ\text{C}$, the cylindrical cross-section of the overall EGS thermal conductivity recharge system is:

$$\begin{aligned} T_o - T_i &= T_s \log(r_o/r_i) \\ T_s &= j_i r_i / K = 1000[\text{W/m}]/3[\text{W/m}\cdot^\circ\text{C}] \sim 333^\circ\text{C}. \\ \log(r_o/r_i) &= (T_o - T_i)/T_s = 100/333 = 0.3 \\ r_o &= r_i \exp(.3) \sim 1.35 r_i \sim 135\text{m}. \end{aligned} \quad (6)$$

The diffusion time constant τ for the annular heat volume is set by the thermal diffusivity of rock $D = K/\rho C = 3[\text{W/m}\cdot^\circ\text{C}]/2400[\text{kg/m}^3]/840[\text{J/kg}\cdot^\circ\text{C}] \sim 1.5 \cdot 10^{-6} [\text{m}^2/\text{s}]$ and annular radius $\Delta r = r_o - r_i$. Approximating the radial flow as planar flow (Carslaw & Jaeger 1959, §3.5), time constant $\tau \sim (2\Delta r/\pi)^2/D \sim (70/\pi)^2 \cdot 0.6 \cdot 10^6 \text{ s} \sim 10 \times 3 \cdot 10^7 \text{ s} \sim 10\text{yr}$.

At 500W/m extraction rate,

$$\begin{aligned} T_s &= j_i r_i / K = 500[\text{W/m}]/3[\text{W/m}\cdot^\circ\text{C}] \sim 165^\circ\text{C} \\ \log(r_o/r_i) &= (T_o - T_i)/T_s = 100/165 = 0.6 \end{aligned}$$

$$r_o = r_i \exp(.6) \sim 1.8 r_i \sim 180\text{m}, \quad (6a)$$

the time constant rises to $\tau \sim 50\text{yr}$.

To extract an order of magnitude more heat from the wellbore, $J_s \sim 10\text{KW/m}$, the order of magnitude higher value of T_s enforces an order of magnitude higher gradient implied by the condition $\log(r_o/r_i) = (T_o - T_i)/T_s \rightarrow 0$, i.e., $r_o/r_i \rightarrow 1$. Where heat extraction and heat recharge scenarios (5)-(6) are approximately consistent, higher heat extraction forces a higher temperature gradient condition $r_o/r_i \sim 1$ that is inconsistent with adequate recharge volume condition $r_o/r_i \sim 1.8$.

The generic counter-relation between conductive recharge and conductive transport is illustrated in Fig 1 for $r_i = 50\text{m}$, maximum recharge time-constant 100 yr, and maximum heat extraction rate 750W/m/wellbore. Open circles indicate a range of viable heat extraction and reservoir duration consistent with direct use rather than base-load electrical generation applications of EGS heat extraction. Base-load power provision requires up-scaling the EGS facility radius to $\sim 500\text{m}$.

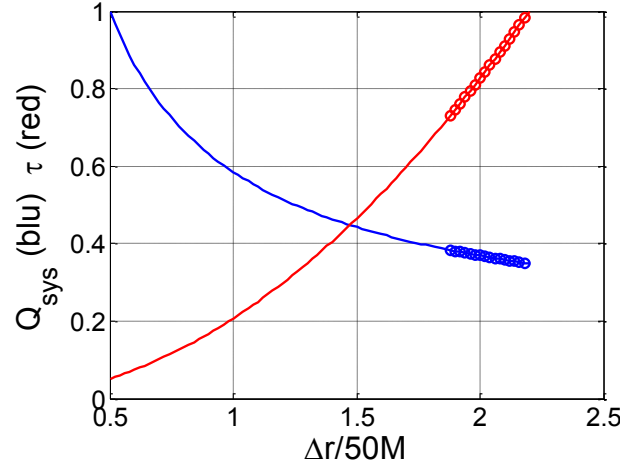


Figure 1. Representative trade-off curves for wellbore-centric conductive heat extraction from crustal rock normalized to 50m radius and 750W heat withdrawal per meter of wellbore. Decreasing extraction rate (blue) increases extraction duration (red). Open circles indicate extraction rates/durations relevant to commercial direct-use activity; for heat extraction system of $\sim 50\text{m}$ effective radius a nominal $0.4 \cdot 750\text{W/m} = 300\text{W/m}$ has $\sim 80\text{yr}$ extraction duration. The equivalent radius for base-load heat extraction is 500m.

3. WELLBORE-CENTRIC HEAT EXTRACTION ON 100M SCALES -- ADVECTION-BUFFERED CONDUCTION

Wellbore-centric heat extraction encounters a critical limitation due to strong thermal gradients at narrow wellbores that reduce the temperature of the crustal heat in contact with wellbore fluids. The defining EGS tactic is to condition crustal rock to permit higher rates of heat extraction. For wellbore-centric EGS the requisite conditioning is to replace lossy conductive flow with more efficient advective flow near wellbores by using *in situ* permeability stimulation to effectively enlarge wellbore radii (cf. Paper III for discussion of permeability stimulation).

To look at wellbore-centric EGS tactics in the realm of practicality, Figs 2-5 consider thermal conductivity without and with advective heat flow component $\mathbf{j}_a = \rho C \mathbf{v} T$ operating in parallel with the conductive flow component $\mathbf{j}_c = K \nabla T$ in crustal volumes of characteristic radius $\sim 100\text{m}$. The steady-state condition for combined conductive and advective heat transport in a 2D medium of thermal conductivity K via fluid of mass density ρ and heat capacity C with temperature field $T(x,y)$ is defined by heat-energy conservation condition

$$K \nabla^2 T(x,y) + \rho C \nabla \cdot (\mathbf{v}(x,y) T(x,y)) = 0 \quad (7)$$

for a prescribed Darcy fluid flow velocity field $\mathbf{v}(x,y)$ and appropriate temperature/heat-flow boundary conditions. Temperature field (7) can be computed with the Matlab PDE finite-element solver for spatially varying values of K , ρ , C and permeability field κ on grids such as illustrated in Fig 2. As implied by the form of (7), K , ρ and C are here taken to be constant (representative values are given in Table 1). Our interest is how *in situ* permeability fields realized on grids as in Fig 2 can be made to create useful advective flow between EGS wellbores to promote the transfer of crustal heat from a large radius EGS heat store surface to small scale heat exchange surfaces associated with heat extraction wellbores.

We take the Fig 2 grid to represent a wellbore-centric EGS crustal section of 100m radius with central intake wellbore through which heat-depleted water can be injected and four outtake wellbores at annular radius 35m from which heat-charged water can be extracted. The outer radius of the EGS section is fixed at nominal temperature $T_o = 300^\circ\text{C}$ and ambient fluid pressure $P_o = 1\text{MPa}$. For simulations of advective flow (see Fig 5), well-to-well Darcy fluid velocity field $\mathbf{v}(x,y) = \kappa(x,y)/\mu \nabla P(x,y)$ is computed for central input wellbore pressure $P_1 = P_o + \Delta P_1$ and annular outtake wellbore pressures $P_2 = P_o - \Delta P_2$ for arbitrary permeability $\kappa(x,y)$ field; the dynamic viscosity is set at constant value $\mu = 2 \cdot 10^{-4} \text{Pa}\cdot\text{s}$ and the default permeability field is uniform at $\kappa = 10^{-14} \text{m}^2 \equiv 10\text{mDarcy}$. Changes in input and outtake pressures, and in the permeability field $\kappa(x,y)$ varies the magnitude and configuration of

the well-to-well advective flow field $\mathbf{v}(x,y)$ giving the steady-state temperature field $T(x,y)$ defined by (7). Figs 3-5 picture three EGS scenarios supported by a densified version of the Fig 2 numerical grid.

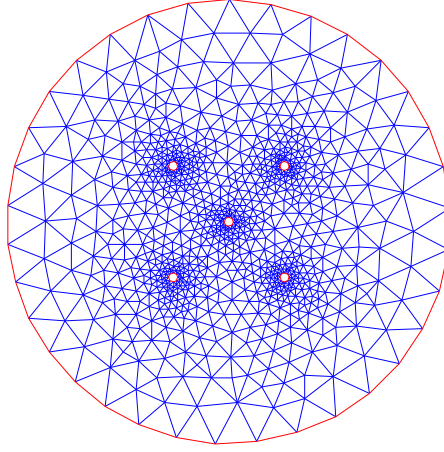


Fig 2. Matlab PDE Toolbox skeleton numerical grid for computing EGS fluid inflow from a central wellbore to an annular quartet of outflow wellbores. Permeability can vary arbitrarily throughout the grid in order to simulate the fluid flow effects of EGS permeability stimulation.

Fig 3 sets the wellbore-centric EGS scene with a temperature field for condition $\nabla^2 T(x,y) = 0$ representing conduction-only thermal gradients supplying heat to a narrow central wellbore extracting heat at a fixed rate. The temperature of the conductive gradient field is 200°C at the annular wellbore radius, but with no advective flow the annular wellbores do not influence the thermal equilibrium. The essential Fig 3 result is that at the specified central wellbore heat extraction rate the extracted fluid temperature is 50°C for conduction-only heat transport.

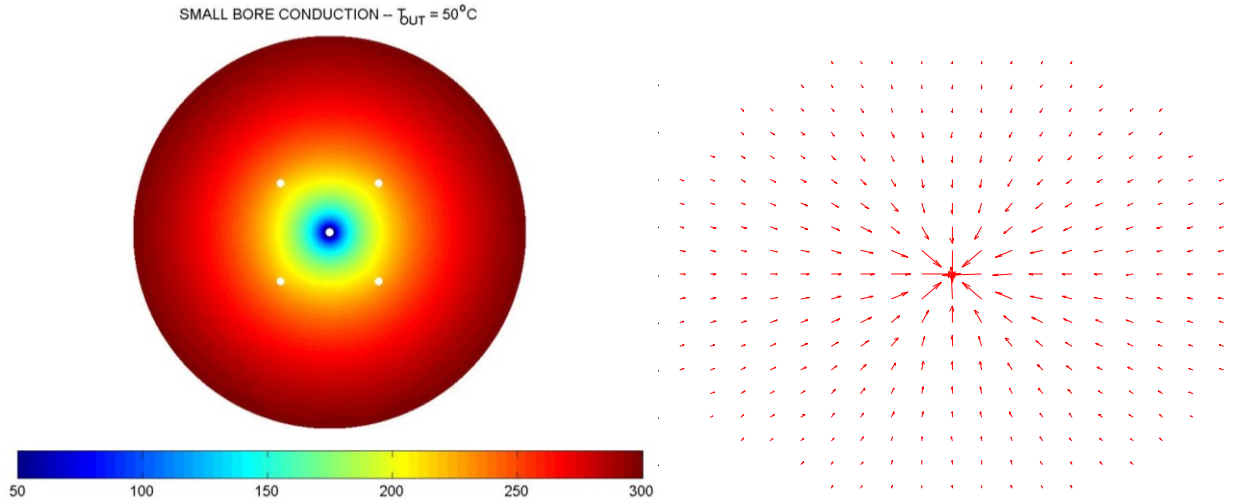


Fig 3. (Left) Thermal conduction gradients cause large heat loss at narrow wellbore heat extraction surfaces. A 100m radius crustal section with fixed boundary temperature $T_0 = 300^\circ\text{C}$ supplying a central wellbore extracting heat at fixed rate draws down the exit temperature to $T_1 = 50^\circ\text{C}$. (Right) Thermal conductivity heat flow vectors from periphery to central wellbore.

Fig 4 further sets the wellbore-centric EGS scene by noting that the same heat extracted at 50°C by the Fig 3 narrow wellbore can be extracted at 200°C if the extraction wellbore radius is 35m. It goes without saying that a 35m radius wellbore is a physical and economic impossibility, hence the need to construct advective rather than conduction heat flow realizations in order to extract heat from sufficiently large crustal volumes to meet commercial demands.

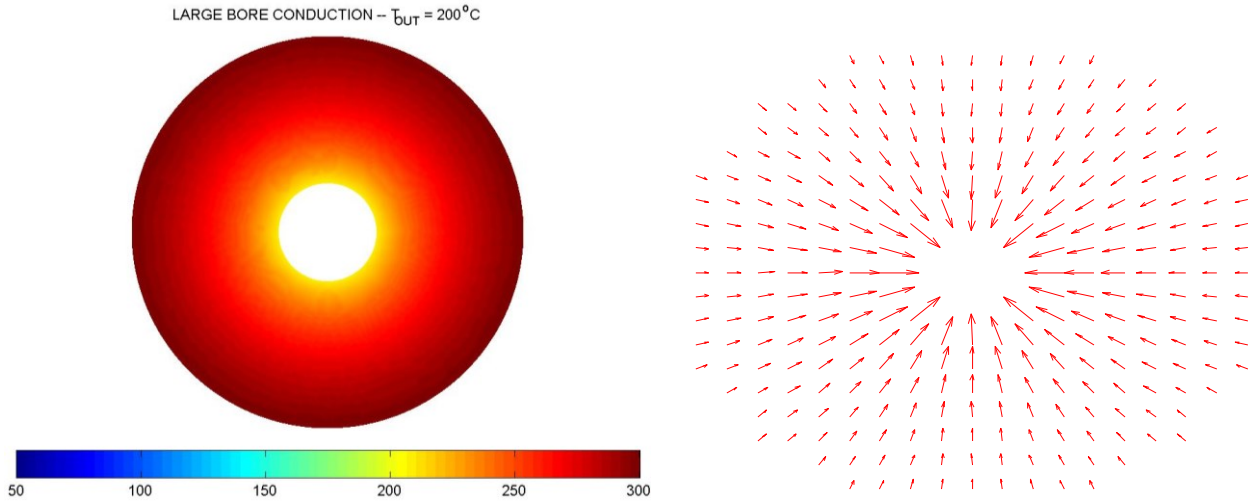


Fig 4. The same heat extraction rate as Fig 3 performed at 35m radius permits heat extraction temperature $T_2 = 200^\circ\text{C}$; the larger wellbore radius reduces the heat flow gradient hence the heat flow loss.

Fig 5 completes the wellbore-centric EGS scene by showing that economically feasible Fig 3 central wellbores can function in more productive mode through the agency of advective flow as given by $K\nabla^2 T(x,y) + \rho C \nabla \cdot (\mathbf{v}(x,y)T(x,y)) = 0$. Advective heat flow prescribed by intake pressure P_1 , outtake wellbore pressures P_2 and inverse thermal diffusion coefficient $\rho C/K \sim 10^6 \text{s/m}^2$ allows heat from the crustal ambient temperature $T_0 = 300^\circ\text{C}$ heat store at 100m radius to be extracted at temperature $T_2 \sim 200^\circ\text{C}$ by outtake wellbores at 35m effective radius in line with extraction notionally achieved by the 35m radius wellbore of Fig 4. The advective flow field $\mathbf{v}(x,y)$ giving the Fig 5 temperature field $T(x,y)$ is shown in Fig 6.

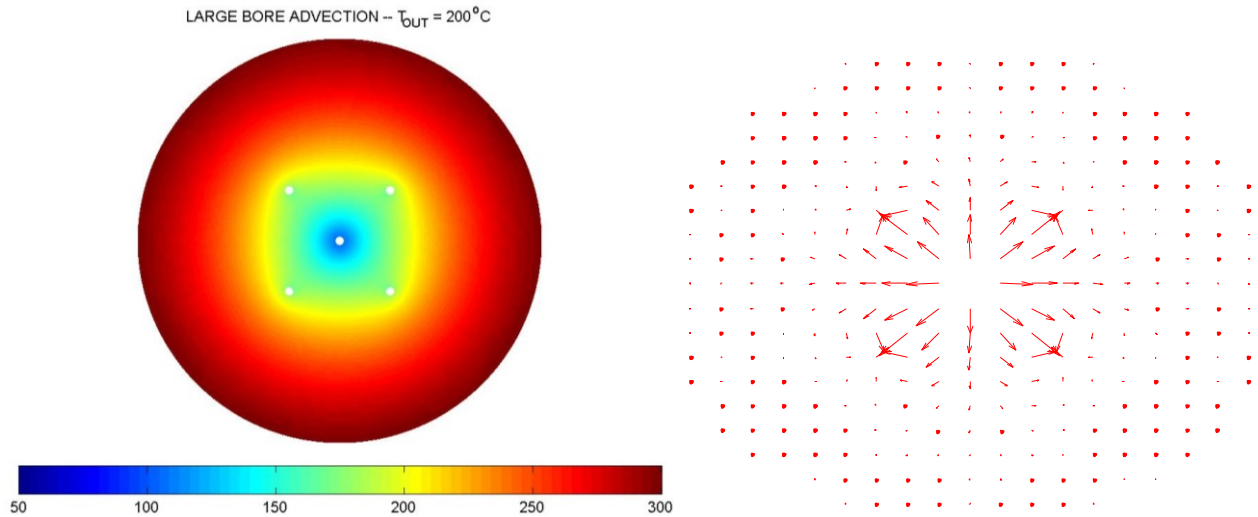


Fig 5. (Left) Advective flow from a central inflow wellbore to a quartet of outtake wellbores at 35m radius achieves the same heat extraction as Fig 4: heat extraction temperature $T_2 = 200^\circ\text{C}$. As in Fig 3 the temperature near the central wellbore is $T_1 = 50^\circ\text{C}$ but advective flow is in the opposite direction from Fig 3 outflow: low temperature inflow fluids gain heat as they flow through the advective heat exchange volume that is conductively recharged with heat from the outer radius at $T_0 = 300^\circ\text{C}$. (Right) Advective heat flow vectors from thermal conductivity recharge at the system periphery and heat-depleted fluid injection at the central wellbore to quartet of advective fluid outtake wellbores at 35m radius. Fig 6 gives details of advective fluid flow field $\mathbf{v}(x,y)$.

The Fig 3-5 EGS scenario sequence focuses on drilling wellbores for advective flow but does not include permeability stimulation (beyond the fact that for grid convenience the computations assume a 2m rather than a 20cm wellbore radius). Reasonably uniform crustal volumes of 10 mDarcy median permeability are present in some hot sedimentary aquifers (e.g., Allis et al. 2012) but are not necessarily characteristic of the very much more abundant volumes of hot crystalline rock. If the assumed EGS volume median permeability is significantly less than 10mDarcy, canonical EGS aspiration indicates wellbore-centric intervention to achieve requisite levels of permeability. The Fig 2 computational set up is readily adapted to such circumstances via prescribing the permeability field leading to the velocity field used to find the steady-state thermal equilibrium temperature field (7). Permeability stimulation is addressed in Paper III (Pogacnik et al 2015).

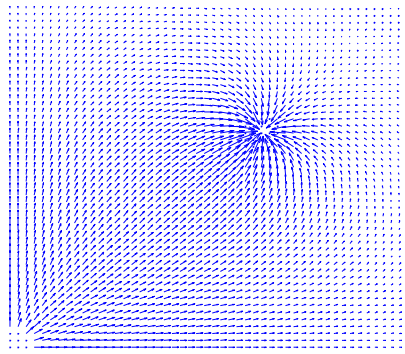


Fig 6. Darcy fluid flow field from input wellbore at lower left to outtake wellbore at upper right. For the steady-state advective flow regime shown in Fig 5, heat-depleted water at temperature $\sim 50^{\circ}\text{C}$ from the central wellbore collects heat in passing through the heat exchange volume to the outtake wells where it arrives with temperature $\sim 200^{\circ}\text{C}$. The advective heat exchange section is conductively recharged at the 35m radius with heat from the EGS section outer boundary at 300°C .

4. SUMMARY/CONCLUSION

Base-load power demands on outtake wellbore flow rate can be accommodated only by extreme interventions. Standard EGS scenarios involving planar heat-exchange interfaces require tens of km^2 of conditioned crustal rock surface. Wellbore centric EGS scenarios require tens of km of wellbore for base-load power provision. Direct-use power requires an order of magnitude smaller surface areas and/or wellbore lengths. It is likely, however, that even for direct-use standard EGS planar heat-exchange interfaces requires intervention of infeasible extent. The highly wasteful drilling experience of existing geothermal heat extraction operations to date, discussed in Paper II (Malin et al. 2015), coupled with the growing experience of shale reservoir hydrofracturing and the failure over several decades of earlier ‘proto-EGS’ projects, warn that seeking to control and maintain large-scale planar fracture structures *in situ* is highly chancy at best and essentially impossible at worst. Wellbore-centric EGS scenarios scaled to direct-use wellbore flow demands appear to provide feasible intervention opportunities. Fig 5 shows that relatively modest advective flow regimes ($v \sim 10^{-8}\text{m/s}$) performed in sedimentary aquifers with median permeabilities $\kappa \sim 10\text{mDarcy}$ can move heat across relative modest wellbore-to-wellbore (tens of meters) intervals to ‘short circuit’ thermal conductivity choke points to provide heat flows $\sim 1\text{--}3\text{W/m}^2$ in meter-thick sections of 35m radius. As discussed in Paper III (Pogacnik et al. 2015) the physical nature of *in situ* permeability can be understood in terms of grain-scale fracture-connectivity, with a logical conclusion that demands for *in situ* permeability stimulation be met by learning to increase *in situ* grain-scale fracture-connectivity in wellbore-centric volumes (as opposed to increasing heat-exchange surface areas via hydrofracture-fluid over-pressuring of rock as mechanical continuum). Any number of existing hot but low permeability geothermal field wellbores provide field settings in which to explore near-wellbore permeability enhancement designed to extend the ‘effective radius’ of drilled wellbores as represented in Fig 5. The elements of direct-use-scale power-provision seem to be available for realistic exploration for the next phase of EGS.

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