

The Analytical Solution of the Water-Rock Heat Transfer Coefficient and Sensitivity Analyses of Parameters

Guowei Zhang, Jialing Zhu, Jun Li and Qiulin Wang

Key Laboratory of Efficient Utilization of Low and Medium Grade Energy of Ministry of Education, Tianjin University, Tianjin 300072, China

Lijun79@tju.edu.cn

Keywords: EGS, single fracture, heat transfer coefficient, analytical solution, sensitivity analysis, correlation equation

ABSTRACT

Most previous works with respect to Enhanced Geothermal Systems (EGS) applied local thermal equilibrium method to study water-rock heat transfer. In order to use the local thermal non-equilibrium method that is close to actual condition, for the single fracture model, water-rock heat transfer coefficient analytical solutions are deduced using the energy balance. For the parameters related to the calculation, except the inner surface temperature, all the parameters can be obtained easily, but the inner surface temperature is necessary to the heat transfer coefficient. In one semi-circle section, by 2D heat conduction the heat fluxes from the inner surface and the inner surface temperature can be calculated, so the analytical solution of heat transfer coefficient is given. Then the measurable parameters from the reference are substituted into the solution and calculate heat transfer coefficient. Comparing the results with the reference, almost all the relative errors are less than 30%, and it shows that this method is receivable. At the conditions of four different outer surface temperatures, sensitivity analyses about two main parameters (flow velocity and aperture) are completed, and it indicates that the heat transfer coefficient is more sensitive to the flow velocity, but less to the aperture. Finally, for water-rock laminar heat convection model, the correlation equation is given, which indicates the importance of Prandtl number to heat transfer.

1. INTRODUCTION

EGS involves many kinds of subjects, such as heat transfer, flow mechanics and rock mechanics. At present, theoretical studies and engineering studies are two major ways. Especially, for theoretical studies, numerical simulations are dominated, which can obtain the results easily for different temperature gradient, heat reservoir dimension, geological conditions.

The fluid-rock (fracture) heat transfer is significant to study EGS. To deal with heat transfer, there are two scenarios: local thermal equilibrium and local thermal non-equilibrium^[1]. The former considers that fluid-rock heat transfer is finished instantaneously and fluid temperature is same as rock temperature, which is widely applied to simplify works^[2, 3]. The later viewpoint thinks that temperature difference between fluid and rock exists and two energy equations should be used to describe heat transfer in the fluid and in the rock^[1, 4, 5]. Generally, the local thermal equilibrium method leads to inaccurate estimation for heat extraction^[6]. If the fracture spacing is more than 2-3m then local thermal equilibrium isn't a valid assumption^[7].

The local thermal non-equilibrium is an actual theory, but it is difficult to analyze than local thermal equilibrium. Using the local thermal non-equilibrium, some researches have been performed^[8-10]. Furthermore, Dong^[11] introduced sensitivity analysis to fluid-rock heat transfer and discussed the affection of several parameters on temperature field of the rock.

For the local thermal non-equilibrium method, heat transfer coefficient is an important parameter, which is useful to determine temperature field or total heat exchange amount. The last work about heat transfer coefficient can be traced to 1993^[12]. Hence, at this study, the calculation of the heat transfer coefficient is focused again. By the energy balance, the analytical solution of the heat transfer coefficient is deduced and sensitivity analyses of two parameters (flow velocity and aperture) are done. In addition, for water-rock laminar heat convection model, the correlation equation is given.

2. MODEL DESCRIPTION

2.1 Physical model

The geometry of the water-rock heat transfer model is a cylinder with a single fracture and the fracture runs along the Z-direction (Fig.1, the fracture is exaggerated). Fluid flows from one side to another in the fracture and heat transfer with the hot matrix. Because of removing the heat from the rock to the fluid, the inner surface temperature (the fracture surface temperature) decreases. The outer surface temperature of the model is constant, and the temperature difference between inner and outer surfaces leads to heat conduction. Overall, the heat conducted from the matrix is equal to the heat removed by fluid from the fracture surfaces; it is also equal to the heat convected.

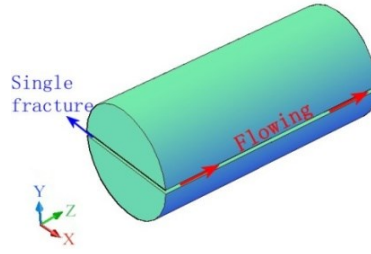


Fig.1 Physical model of water-rock heat transfer with a single fracture

2.2 Mathematic model

For the above physical process, some assumptions are made as follows.

- (1) Porosity of the rock is neglected, and the rock is treated as the isotropic material.
- (2) The fluid is treated as incompressible liquid and constant physical property. No gasification due to the high pressure.
- (3) No consideration the radiation heat transfer.
- (4) No consideration the end effect.
- (5) No consideration the heat loss.

The length of the cylinder is L , radius r_0 . The outer surface temperature is maintained constant T_0 . The fluid inlet temperature and outlet temperature are t_1, t_2 respectively, and the fluid flow velocity is u .

Heat removed by fluid through the fracture can be expressed as

$$Q = c_p q_m \Delta t = c_p q_m (t_2 - t_1) \quad (1)$$

where, c_p —the isobaric heat capacity, J/°C;

q_m —the fluid mass flow-rate, kg/s;

Δt —the temperature difference of fluid, °C.

Due to flowing, the heat convection exists between the fluid and the rock, and the heat convected is expressed as

$$Q = hA\Delta t = hA(T - t) \quad (2)$$

$$t_f = \frac{1}{2}(t_1 + t_2) \quad (3)$$

where, h —the heat convection coefficient, W/(m² · K);

A —the heat convection exchange area, m²;

$\Delta t'$ —the temperature difference between fluid and inner surface, °C;

T_i —the arithmetic meat temperature of the inner surface, °C;

t_f —the fluid temperature in the fracture, is taken as the arithmetic mean of the inlet and outlet temperatures, °C.

Because the inner surface temperature isn't uniform, so it is difficult to calculate the heat conducted. Assume that any point temperature is the surface average temperature T_i . Only half of the cylinder is studied due to symmetry. (Fig.2)

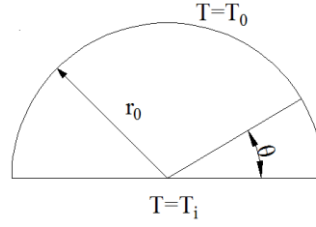


Fig.2 Semi-circle section of the model

For the real situations, the boundary conditions are complicated and dynamic. But for the theoretical study, in order to analyze conduction and convection conveniently, the uniform heat flux and the uniform wall temperature are applied generally [13]. So at present work, the constant outer surface temperature boundary and adiabatic two end faces boundary are put into use.

Using cylindrical coordinates, the time-independent differential equation of heat conduction and boundary conditions are written as,

$$\left\{ \begin{array}{l} \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \\ (4) \\ r = r_0, \quad T = T_0 \\ \theta = 0 \text{ or } \theta = \pi, \quad T = T_i \end{array} \right.$$

3. SOLUTION OF HEAT TRANSFER COEFFICIENT AND VALIDATION

3.1 Calculation of heat transfer coefficient

By the eq. (1) (2) (3), the heat transfer coefficient can be expressed as

$$h = \frac{cq_m(t_2 - t_1)}{A \left(T_i - \frac{t_1 + t_2}{2} \right)} \quad (5)$$

The mass flow rate of the fluid q_m is calculated as follow

$$q_m = u \times \delta \times 2r_0 \times \rho \quad (6)$$

where, u —the fluid flow velocity, m/s ; δ —the aperture, m ; r_0 —the radius of the section, m ; ρ —the density of the fluid, kg/m³ .

Fluid flows in the fracture touching the two rectangular fracture surfaces, so the heat convection exchange area can be given as

$$A = 2 \times 2r_0 \times L \quad (7)$$

where, L —the length of the cylinder, m .

Eqs. (6) and (7) are substituted into eq. (5), and the heat transfer coefficient is rewritten as

$$h = \frac{c_p u \delta \rho (t_2 - t_1)}{2L \left(T_i - \frac{t_1 + t_2}{2} \right)} \quad (8)$$

From eq. (8), the heat transfer coefficient is a function of many parameters. The isobaric heat capacity and the density are dependent on the thermal physical property; the length of cylinder lies on the geometry; both the fluid flow velocity, the aperture, the inlet temperature and the outlet temperature are measureable parameters. They are set or measured easily, but it is difficult to obtain the inner surface temperature.

Following the analytical method is given to calculate the inner surface temperature.

For eq. (4), the analytical solution of the temperature filed is [14]

$$T = T_i + \frac{4}{\pi} (T_0 - T_i) \sum_{v=0}^{\infty} \frac{(r/r_0)^{2v+1}}{2v+1} \sin[(2v+1)\theta] \quad (9)$$

Through the transformation of coordinates $r = \sqrt{x^2 + y^2}$, $\theta = \arctan\left(\frac{y}{x}\right)$, the cylindrical coordinate's temperature filed is transformed as rectangular coordinate's

$$T = T_i + \frac{4}{\pi}(T_0 - T_i) \sum_{v=0}^{\infty} \frac{\left(\sqrt{x^2 + y^2}/r_0\right)^{2v+1}}{2v+1} \sin\left[(2v+1)\arctan\left(\frac{y}{x}\right)\right] \quad (10)$$

The heat conducted can be calculated by the integral for temperature filed eq. (10). But there is a singularity at the point $x = r_0, y = 0$, at which physical significance is unreasonable (two kinds of boundary conditions). So the integral for temperature filed is done from 0 to $0.99r_0$. The heat conducted of the rock from the outer surface to the inner surface is calculated.

$$Q = \left| 4 \int_0^{0.99r_0} \left(-\lambda \frac{\partial T}{\partial y} \right)_{y=0} L dx \right| \quad (11)$$

where, λ —the thermal conductivity, $W/(m \cdot K)$.

Eq. (10) is substituted into eq. (11)

$$Q = \left| \frac{16}{\pi} \lambda r_0 L (T_0 - T_i) \int_0^{0.99r_0} \frac{1}{x^2 - r_0^2} dx \right| = \left| \frac{8}{\pi} \lambda L (T_0 - T_i) (-\ln 199) \right| = \frac{42.32}{\pi} \lambda L (T_0 - T_i) \quad (12)$$

By eq. (1) and eq. (12), the inner surface temperature can be calculated

$$T_i = T_0 - \frac{\pi c_p q_m (t_2 - t_1)}{42.32 \lambda L} \quad (13)$$

Eq. (6) is substituted into eq. (13)

$$T_i = T_0 - \frac{\pi c_p r_0 \rho u \delta (t_2 - t_1)}{21.16 \lambda L} \quad (14)$$

The inner surface temperature is given, all the parameters are measureable, eq. (14) is substituted into eq. (8)

$$h = \frac{21.16 c_p \lambda \rho u \delta (t_2 - t_1)}{42.32 \lambda L T_0 - 2 \pi c_p r_0 \rho u \delta (t_2 - t_1) - 21.16 \lambda L (t_1 + t_2)} \quad (15)$$

3.2 Validation

For the experimental tiny scale model, taking into consideration of the experimental rock sample from the reference and rock triaxial texting system, the granite and water are chosen and the geometric and physical parameters are listed as follows.

$$L = 102 \text{ mm}, r_0 = 25.5 \text{ mm}, c_p = 4200 \text{ J/(kg} \cdot ^\circ\text{C)},$$

$$\rho = 1000 \text{ kg/m}^3, \lambda = 3.5 \text{ W/(m} \cdot \text{K)}.$$

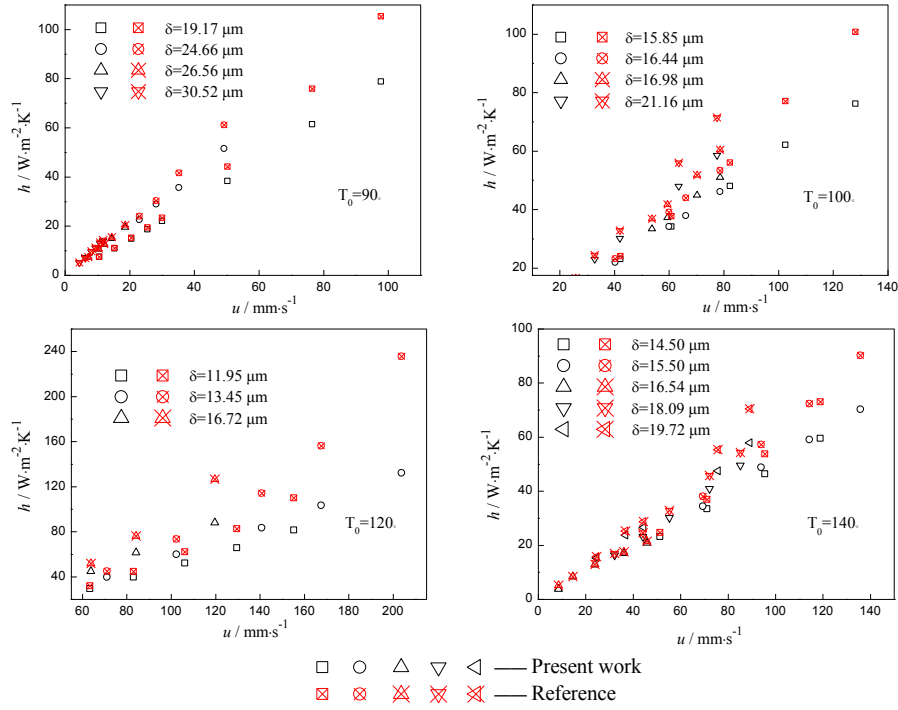
Above parameters are substituted into the equation (15). The heat transfer coefficient is a function of the flow velocity, the aperture, the inlet temperature and the outlet temperature.

$$h = \frac{0.31105 u \delta (t_2 - t_1)}{15.10824 T_0 - 0.00067 u \delta (t_2 - t_1) - 7.55412 (t_1 + t_2)} \quad (16)$$

where, δ —the aperture, μm ;

u —the fluid flow velocity, mm/s .

Using the experimental parameters from the reference, 78 data are calculated by eq. (16), and the comparison of results is presented in Fig. 4



**Fig.3 Comparing the present's heat transfer coefficients
with the results from the reference**

Through the error analysis for 78 data, the minimum relative error is only 0.02%, and almost all the relative errors are less than 30%. Especially, for low flow velocity (less than 100mm/s), the relative errors aren't greater than 20%. Just for one data, when the flow velocity is 203.72mm/s, which seriously exceed the regular velocity for the HDR, the relative error is 43.9%. From above figure, it indicates that the errors increase with the flow velocity and two groups of results are in good agreement at low flow velocity.

4. SENSITIVITY ANALYSIS

4.1 Theory

For a system, the system characteristics P is influenced by n parameters, $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \dots\}$, $P = f\{\alpha_1, \alpha_2, \alpha_3, \dots\}$. When $\alpha = \alpha^* = \{\alpha_1^*, \alpha_2^*, \alpha_3^*, \dots\}$, system characteristics is P^* , $P^* = f\{\alpha_1^*, \alpha_2^*, \alpha_3^*, \dots\}$. The sensitivity analysis is defined in order to investigate the trend and level of system characteristics P deviating standard state P^* because of parameters ranging, when every parameter ranges within suitable scope^[15].

Dimensionless sensitivity function is defined as

$$S(\alpha_k) = \left| \frac{dP(\alpha_k)}{d\alpha_k} \right| \frac{\alpha_k}{P} \quad (17)$$

when $\alpha_k = \alpha_k^*$, the sensitivity factor is given as

$$S^*(\alpha_k) = \left| \frac{dP(\alpha_k)}{d\alpha_k} \right|_{\alpha_k = \alpha_k^*} \frac{\alpha_k^*}{P^*} \quad (18)$$

Comparing the sensitivity factors, the effect extent of the parameters on the system can be determined. The greater factors, it shows that slight fluctuation of parameters can influence systems seriously.

4.2 Results

At the different outer surface temperatures, sensitivity analyses are done for two major parameters, flow velocity u and aperture δ . The heat transfer coefficient is regarded as system characteristics P , $P = h = f\{u, \delta\}$. When $u = u^*$ or $\delta = \delta^*$, $h = h^*$.

For example, $T_0 = 90^\circ\text{C}$, inlet temperature $t_1 = 40^\circ\text{C}$, outlet temperature $t_2 = 88^\circ\text{C}$, flow velocity ranges from 8.2mm/s to 97.75mm/s, aperture ranges from 19.17 μm to 30.52 μm , $u^* = 49\text{mm/s}$ and $\delta^* = 24.5\mu\text{m}$.

When $u = u^* = 49 \text{ mm/s}$ and $\delta = \delta^* = 24.5 \mu\text{m}$, the functions fitting $h = h(u)$ and $h = h(\delta)$ are shown in Fig.5 respectively.

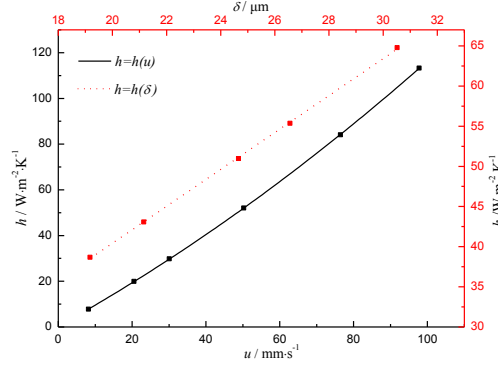


Fig.4 The functions fitting for $h = h(u)$ and $h = h(\delta)$

Functions fitting $h = h(u)$ and $h = h(\delta)$ are

$$h(u) = 0.003u^2 + 0.896u + 0.357$$

$$h(\delta) = 2.298\delta - 5.524$$

By eq. (17), sensitivity functions are given as

$$S_u = \frac{0.006u^2 + 0.896u}{0.003u^2 + 0.896u + 0.357} \quad (19)$$

$$S_\delta = \frac{2.298}{2.298\delta - 5.524} \quad (20)$$

By eq. (18), sensitivity factors are given as

$$S_u^* = 1.133, \quad S_\delta^* = 1.109$$

Applying above methods, for the different outer surface temperatures $T_0 = 100^\circ\text{C}$, $T_0 = 120^\circ\text{C}$ and $T_0 = 140^\circ\text{C}$, the sensitivity factors are listed in Table 1.

Table 1 Heat transfer coefficient sensitivity factors for the flow velocity and aperture at different outer surface temperatures

$T_0 / ^\circ\text{C}$	$u^* / (\text{mm} \cdot \text{s}^{-1})$	$\delta^* / \mu\text{m}$	$t_1 / ^\circ\text{C}$	$t_2 / ^\circ\text{C}$	S_u^*	S_δ^*
100	70	18.5	57	95	1.108	1.098
120	102	14	62	117	1.140	1.134
140	76	17	73	129	1.094	1.090

For all the outer surface temperatures, comparing sensitivity factors, the results ($S_u^* > S_\delta^*$) show that heat transfer coefficient is affected more by flow velocity than the aperture.

5. CORRELATION EQUATION OF HEAT CONVECTION

By the experimental results, the aperture chose as characteristic length, and Reynolds number and Nusselt number are calculated. Reynolds number range from 0.31-8.04, so flowing state is laminar. Using software, correlation equation $Nu = f(Re, Pr)$ can be given.

The correlation equation $Nu = CRe^a Pr^b$ is the basic equation, and taking logarithms for both sides of equation the correlation equation is changed as $\lg Nu = \lg C + a \lg Re + b \lg Pr$. By the multiple linear regression, the result is $\lg Nu = -4.06 + 0.92 \lg Re + 1.89 \lg Pr$, where $\lg C = -4.06, a = 0.92, b = 1.89$. So for the laminar flow water-rock heat transfer the correlation equation is $Nu = 0.000087 Re^{0.92} Pr^{1.89}$ (as shown in fig.5)

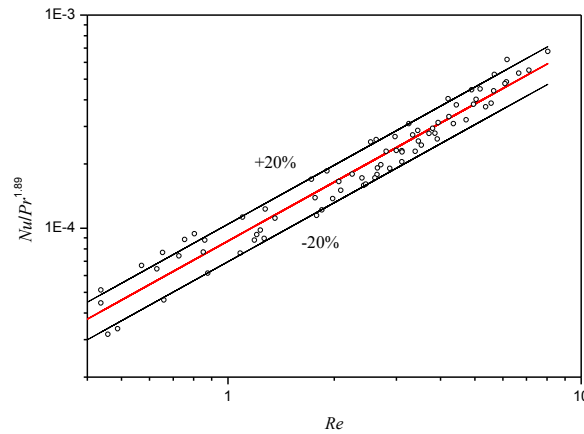


Fig.5 The correlation equation and errors

From fig.5, the relative errors are seen, and about 80% data are in the reasonable range of $\pm 20\%$. Additionally, the power exponent of Prandtl number is higher than Reynolds number, and it indicates that the influence of fluid viscosity on heat transfer isn't insignificant. So the correlation equation $Nu = f(Re)$ can't be used, which is another usual equation for heat convection.

5. CONCLUSIONS

In this paper, the calculation method of the heat transfer coefficient is mainly emphasized. The inner surface temperature, which is only one non-measurable parameter, can be calculated and then the heat transfer coefficient be obtained easily. In terms of experimental parameters from the reference and analytical solution of heat transfer coefficient, 78 data are presented and compared with the results from the reference. Due to use the different solutions, the results are smaller than reference, and almost all the relative errors are less than 30%, which indicates that the analytical solution in this work is credible. Moreover, the errors increase gradually with flow velocity, and there is a good fit for low flow velocity. When flow velocity is below 100mm/s, the error isn't greater than 20%. Only for one condition, flow velocity 203.72mm/s, the relative error is 43.9%, but this velocity goes far beyond the general velocity in EGS.

By sensitivity analysis, for four different outer surface temperatures $T_0 = 90^\circ\text{C}$ $T_0 = 100^\circ\text{C}$ $T_0 = 120^\circ\text{C}$ $T_0 = 140^\circ\text{C}$, the sensitivity factors of flow velocity and aperture are 1.133, 1.108, 1.140, 1.094 and 1.109, 1.098, 1.134, 1.090 respectively. This results ($S > S$) demonstrate that flow velocity plays a more important role in determining heat transfer coefficient than aperture.

Finally, the correlation equation $Nu = 0.000087Re^{0.92}Pr^{1.89}$ is presented. The Power exponent of Prandtl number is higher than others correlation equations for heat convection in the duct. It shows that the fluid viscosity is an important parameter for heat convection in this model.

REFERENCES <HEADING 1 STYLE>

- [1] Jiang P. X., Ren Z. P.: Numerical investigation of forced convection heat transfer in porous media using a thermal non-equilibrium model, *International Journal of Heat and Fluid Flow*, **22**, (2001), 102-110.
- [2] McTigue D. F.: Thermoelastic response of fluid-saturated porous rock, *Journal of Geophysical Research*, **91**, (1986), 9533-9542.
- [3] Kurashige M.: A thermoelastic theory of fluid-filled porous materials, *International Journal of Solids Structures*, **25**, (1989), 1039-1052.
- [4] Rees D. A. S., Bassom A. P., Siddheshwar P. G.: Local thermal non-equilibrium effects arising from the injection of a hot fluid into a porous medium, *Journal of Fluid Mechanics*, **594**, (2008), 379-398.
- [5] He L. W., Jin Z. H.: A local thermal non-equilibrium poroelastic theory for fluid saturated porous media, *Journal of Thermal Stresses*, **33**, (2010), 799-813.
- [6] Guo J., Chen J. L., Cao W. J., Jiang F. M.: Research Review on Enhanced Geothermal System, *Electric Power Construction*, **35**, (2014), 10-24.
- [7] Sanyal S. K., Granados E. E., Butler S. J., Horne R. N.: An alternative and Modular Approach to Enhanced Geothermal Systems, World Geothermal Congress 2005, Antalya, Turkey, 2005.
- [8] Shaik A. R., Rahmam S. S., Tran N. H., Tran T.: Numerical simulation of Fluid-Rock coupling heat transfer in naturally fractured geothermal system, *Applied Thermal Engineering*, **31**, (2011), 1600-1606.
- [9] Zhao J.: Analytical and experimental studies of heat convection by water flow in rock fractures, The 33th U.S. Symposium on Rock Mechanics (USRMS) [C], Santa Fe, New Mexico, 1992.
- [10] Jiang F. M., Luo L., Chen J. L.: A novel three-dimensional transient model for subsurface heat exchange in enhanced geothermal systems, *International Communication in Heat and Mass Transfer*, **41**, (2013), 57-62.
- [11] Dong H. Z., Luo R. H., Zhang L.: Seepage and heat transfer model of a single fracture of rock and relevant parameter sensitivity analysis, *Journal of Hohai University (Natural Sciences)*, **41**, (2013), 42-47.

Zhang, Zhu, Li and Wang.

- [12] Zhao J., Tso C. P.: Heat transfer by water flow in rock fractures and the application to Hot Dry Rock geothermal systems, *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, **30**, (1993), 633-641.
- [13] Yang S. M., Tao W. Q.: Heat Transfer, Beijing: Higher Education Press, (2010), 47.
- [14] Zhang H. J.: Heat conduction, Beijing: Higher Education Press, (1992), 147.
- [15] Zhang G., Zhu W. S.: Parameter sensitivity analysis and optimizing for text programs, *Rock and Soli Mechanics*, **14**, (1993), 51-58.