Long Term Performance Prediction of a Borehole Ground Heat Exchanger by Green's Function Method

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ABSTRACT

The ground source heat pump systems have received considerable attention in last decades as an energy efficient heating/cooling system for residential and commercial spaces. In these systems, long term performance prediction of a borehole is a critical issue since the heat exchange performance of a borehole decreases through the life time of the system. Therefore it is important to find a reliable method to determine the long term performance of a ground heat exchanger (GHE). In this study, an analytical expression for heat transfer ratio per unit length (HTR value) of a borehole GHE is derived for 1D and 2D models at constant borehole wall temperature condition by using Green's function method (GFM). A 50m-depth borehole consisting of U-shaped polyethylene pipes with 32mm outside diameter is considered. Experimental measurements are done by pumping warm water of 40 $^{\circ}$C into the pipe system for 120 hours. Variation of HTR value in time is calculated from the experimental data. Theoretical results based on GFM are validated by experimental measurements. Thermal conductivity and diffusion constant of the ground is determined by fitting the theoretical model to the experimental data and then long term HTR values are predicted. Also a comparison of the long predictions of 1D and 2D theoretical models is provided. Effectiveness of GFM is shown. Long term HTR predictions based on GFM can be used for a realistic engineering design of ground source heat pump systems.

1. INTRODUCTION

The ground source heat pump (GSHP) system has been recognized as one of the best sustainable energy technologies for heating and cooling applications in residential and commercial buildings. A GSHP system consists of a sealed loop of pipe buried in the ground and connected to a heat pump through which water or antifreeze fluid is circulated. The wide application of the GSHP technology has been hindered by its higher initial cost and substantial land areas required to install the GHE. Researchers have done a lot in terms of reducing the initial cost and broadening the applicability of the technology.

One of the important research areas of the GSHP systems is to model the heat transfer in the GHEs. Involving a time span of months or even years, the heat transfer process in the GHEs is rather complicated, and should be treated as a transient one. The fundamental task for modeling the heat transfer in the GHEs is to grasp the heat conduction process of a single borehole or pile in the GHEs. Heat transfer of multiple boreholes can then be analyzed by the superposition principle, Eskilson (1987).

The GHEs with vertical boreholes, Bose and Parker (1985), have been the mainstream for the GSHP systems, which offer better performance and require smaller land area compared with horizontal installation of pipes in trenches, but a lot of land is still needed to drill the boreholes for vertical installation of the GHEs, which remains a major obstacle to apply the GSHP technology in urban areas.

The heat transfer from the circulating fluid in the pipes to surrounding ground can be treated in a series of thermal resistances, i.e. the convective resistance inside the pipes, the resistance of the pipe walls, as well as the resistance of the borehole backfilling and that of the surrounding ground. Among them, the ground thermal resistance outside the borehole is, in general, the most significant one. It is important to assess this resistance adequately so that the GHEs can be designed properly. In a cooling mode, for instance, the warm fluid induces conductive heat flow in the surrounding cooler soil. The borehole or pile of the GHEs may be conceived as a hot rod, from which heat flows to the surrounding ground.

Analytical solutions of physical models are desirable to understand and simulate engineering problems even though numerical solutions are easily obtained with the aid of modern computers. For simulating and designing the GHEs, models and their analytical expressions with different sophistication and precision have been presented for thermal analysis of borehole GHEs.

Yavuzturk and Spitler (1999) analyzed the short time step response of the borehole by means of numerical solution with the heat capacity of the grout and pipe taken into account. Lamarche and Beauchamp (2007) have presented 1-D analytical solutions for concentric cylinders so that the heat capacity of the borehole itself can be included in the model. Their solutions are sophisticated and interesting, but much more complicated than the classical 1-D solutions mentioned above.

Classical models for the GHE's thermal analysis exist based on one-dimensional (1-D) analytical solutions. A most-widely-used1-D model for this purpose is Kelven's line source model, developed and evaluated by Carslaw and Jaeger, which has been adopted into a number of modeling systems. In this model, the borehole is replaced by a line heat source with its radial dimension neglected, so that a simple analytical solution may be obtained for the temperature response in the surrounding medium. Another best known 1-D model, referred as the cylindrical heat source model, Kavanaugh (1997) was also established by Carslaw and Jaeger (1947), but later suggested by Ingersoll and Zobel (1954) as an alternative approach to sizing ground heat exchangers.

In addition, two-or three-dimensional analytical models considering the axial heat flow have also been proposed by Eskilson, Zeng and Cui, which are developed from the line source model, and can account for the long-term effect of the limited-depth boreholes.

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This paper presents analytical solutions for heat transfer ratio per unit length (HTR value) of a borehole GHE for 1D and 2D models at constant borehole wall temperature condition by using Green's function method (GFM). This analytical model deals with the long term temperature response in the boreholes.

2. ANALYTICAL MODEL DESCRIPTION

For modeling the heat transfer of a GHE, the ground soil is usually approximated as an infinite homogeneous medium. In engineering practice the borehole diameter is normally between 0.11 m and 0.2 m, and its depth ranges from 40 m to 200 m. Compared to the depth, the borehole diameter is much smaller, as a result, the axial heat flow is often neglected, therefore 1-D models can be used for heat conduction of a single borehole.

This study is focused on long term prediction of HTR value of a borehole. Heat transfer details inside the fluid are ignored. However time dependent conductive radial heat transfer between borehole and ground is considered. To find the borehole wall temperature, following expression can be used by considering the steady state conditions inside the borehole due to its small radius:

$$T_w = \overline{T} - \frac{\dot{q}}{2\pi k_g} \ln\left(\frac{r_e}{2r_p}\right)$$

(1)

where T_w is borehole wall temperature, $\overline{T}$ is a mean temperature of fluid $\overline{T} = (T_i + T_r)/2$, r_e equivalent pipe radius defined by $r_e = 2r_p$, r_p is pipe radius, r_b is borehole radius and k_g thermal conductivity of grout. $\dot{q}$ is HTR value per borehole length (unit HTR value) and can be obtained from experimental data by following expression:

$$\dot{q} = \dot{m} c_p (T_i - T_r)$$

(2)

Where $\dot{m}$ is mass flow-rate, c_p is specific heat capacity of fluid and T_i inlet fluid temperature to ground and T_r fluid return temperature from the ground.

Borehole fluid inlet temp.: T_i

Borehole fluid return temp.: T_r

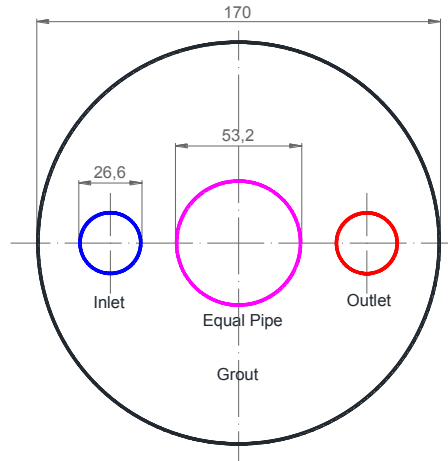Fluids mean temperature: $\overline{T} = \dfrac{T_i + T_r}{2}$

Figure 1: Cross section of borehole

2.1 Derivation of the expressions of analytical solution for the 1D solid cylindrical model at constant borehole wall temperature

In the constant borehole wall temperature model the borehole wall temperature approximated as a constant. Heat transfer equation in cylindrical coordinates for just radial and temporal variables gives

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad Assumptions : \begin{cases} \theta = \dfrac{T - T_0}{T_w - T_0} \\ \theta(\infty, t) = 0 \\ \theta(r_b, t) = 1 \\ \theta(r, 0) = 0 \end{cases}$$

(3)

where T_0 is the initial temperature value. Instead of using Green's function solution in cylindrical coordinates, it is possible to get a solution of Eq.(3) by using Green's function solution in the Cartesian coordinate given by:

$$G(x,y,z,t:x',y',z',t') = \frac{1}{8[\pi k(t-t')]^{\frac{3}{2}}}e^{[-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4k(t-t')}]}$$ (4)

As we know in cylindrical coordinates, $x = r\cos\varphi$ and $y = r\sin\varphi$. By substituting cylindrical forms, Eq.(4) becomes:

$$G(r,\varphi,z,t:r',\varphi',z',t') = \frac{1}{8[\pi k(t-t')]^{\frac{3}{2}}}e^{[-\frac{(r\cos\varphi-r'\cos\varphi')^2+(r\sin\varphi-r'\sin\varphi')^2+(z-z')^2}{4k(t-t')}]}$$ (5)

By using Green's function rules we know that temperature distribution in ground can be calculated with a following function:

$$\theta(r,\varphi,z,t) = \int_0^{2\pi} d\varphi'\int_{-\infty}^{\infty}dz'\int_0^t B\ G(r,\varphi,z,t:r',\varphi',z',t')dt'$$ (6)

For 1D problems,

$$\theta(r,t) = \int_0^{2\pi} d\varphi'\int_{-\infty}^{\infty}dz'\int_0^t \frac{B}{8[\pi k(t-t')]^{\frac{3}{2}}}e^{[-\frac{r^2-2rr_b\cos\varphi'+r_b^2+z'^2}{4k(t-t')}]}$$ (7)

where B is a constant that can be calculated from boundary conditions. As we know,

$$\int_0^{2\pi} e^{[\frac{2rr_b\cos\phi'}{4k(t-t')}]}d\varphi' = 2\pi I_0[\frac{rr_b}{2k(t-t')}]\quad \text{and}\quad \int_{-\infty}^{\infty}e^{\left[-\frac{(z-z')^2}{4k(t-t')}\right]}dz' = 2\sqrt{k\pi(t-t')}$$ (8)

where I_0 is the zeroth order modified Bessel function. After substituting Eq.(8) in Eq.(7), temperature distribution in ground is obtained as:

$$\theta(r,t) = B\int_0^t \frac{1}{t-t'}e^{\left[-\frac{r_b^2+r^2}{4\alpha(t-t')}\right]}.I_0\left[\frac{r_b r}{2\alpha(t-t')}\right]dt'$$ (9)

In order to simplify the model, we try to use the dimensionless form of the above equation, therefore we define that $R = \frac{r}{r_b}$ and

$$Fo = \frac{\alpha t}{r_b^2}$$

$$\theta(R,Fo) = \int_0^{Fo} \frac{B}{Fo-Fo'}e^{\left[-\frac{1+R^2}{4(Fo-Fo')}\right]}I_0\left[\frac{R}{2(Fo-Fo')}\right]dFo'$$ (10)

After applying the boundary conditions:

$$\theta(R,Fo) = \frac{\int_0^{Fo}\frac{1}{Fo-Fo'}e^{\left[-\frac{1+R^2}{4(Fo-Fo')}\right]}I_0\left[\frac{R}{2(Fo-Fo')}\right]dFo'}{\int_0^{Fo}\frac{1}{Fo-Fo'}e^{\left[-\frac{1}{2(Fo-Fo')}\right]}I_0\left[\frac{1}{2(Fo-Fo')}\right]dFo'}$$ (11)

2.1.1 Heat Transfer Rate per Unit Borehole Length

To find HTR value per unit borehole length, a following expression can be used:

$$\dot{q} \simeq \left.\frac{dT}{dr}\right|_{r=r_b}$$ (12)

When dimensionless terms are used in Eq.(12) we obtain:

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$$\left. (T_w - T_0) \frac{d\theta}{dR} \right|_{R=1}$$

(13)

Heat transfer rate per unit borehole in dimensionless form can be written as:

$$\frac{q}{2\pi k (T_w - T_0)} = \left. \frac{d\theta}{dR} \right|_{R=1}$$

(14)

By using Eq.(11) and (13) HTR value per unit borehole length is obtained as:

$$q = 2\pi k (T_w - T_0) \int_0^{Fo} \left(\frac{e^{-\frac{1+R^2}{4(Fo-Fo')}} I_0\left[\frac{R}{2(Fo-Fo')}\right]}{(Fo-Fo')^2} - \frac{e^{-\frac{1+R^2}{4(Fo-Fo')}}(1+R^2) I_0\left[\frac{R}{2(Fo-Fo')}\right]}{4(Fo-Fo')^3} + \frac{e^{-\frac{1+R^2}{4(Fo-Fo')}} R\, I_1\left[\frac{R}{2(Fo-Fo')}\right]}{2(Fo-Fo')^3} \right) dFo'$$

(15)

2.2 Derivation of the expressions of analytical solution for the 2D solid cylindrical model at constant borehole wall temperature

In the 2D model the cylindrical well is considered of a limited length, stretching from the boundary with constant wall temperature to a certain depth, h. The Green's function can also be used to obtain the temperature response of the medium in this case. The temperature change at any location of coordinate (r, z) and at the instants can be obtained according to the Green's function theory as

$$\theta(r,z,t) = \int_0^{2\pi} d\varphi' \int_{-\infty}^{\infty} dz' \int_0^t \frac{B}{8[\pi k(t-t')]^{\frac{3}{2}}} e^{\left[-\frac{r^2+r_0^2-2rr_b\cos\varphi'+r_b^2+(z-z')^2}{4k(t-t')}\right]} dt$$

(16)

Therefore:

$$\theta(r,z,t) = \int_0^t \int_0^h \frac{1}{t-t'} e^{\left[-\frac{r_b^2+r^2}{4\alpha(t-t')}\right]} . I_0\left[\frac{r_b r}{2\alpha(t-t')}\right] . \{e^{\left[-\frac{r^2+r_0^2+(z-z')^2}{4k(t-t')}\right]} - e^{\left[-\frac{r^2+r_0^2+(z+z')^2}{4k(t-t')}\right]}\} dz' dt'$$

(17)

In this part we follow the rule in part 2.1 and just the integration with respect to z', as we know:

$$\int \exp(-\frac{z}{a^2}) = \frac{\sqrt{\pi}}{2} a\ \mathrm{erf}\left(\frac{z}{a}\right) + A$$

(18)

By using Eq.(18) in Eq.(17), the temperature distribution is determined as:

$$\theta(r,z,t) = B \int_0^t \frac{1}{t-t'} e^{\left[-\frac{r_b^2+r^2}{4\alpha(t-t')}\right]} . I_0\left[\frac{r_b r}{2\alpha(t-t')}\right] . \{\mathrm{erf}\left[\frac{h-z}{2\sqrt{k(t-t')}}\right] + 2\mathrm{erf}\left[\frac{z}{2\sqrt{k(t-t')}}\right] - \mathrm{erf}\left[\frac{h+z}{2\sqrt{k(t-t')}}\right]\} dt$$

(19)

The dimensionless form of the Eq.(19) is:

$$\theta(r,z,t) = B \int_0^{Fo} \frac{1}{Fo-Fo'} e^{\left[-\frac{R^2+1}{4(Fo-Fo')}\right]} . I_0\left[\frac{R}{2(Fo-Fo')}\right] . \{\mathrm{erf}\left[\frac{H-Z}{2\sqrt{Fo-Fo'}}\right] + 2\mathrm{erf}\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right] - \mathrm{erf}\left[\frac{H+Z}{2\sqrt{Fo-Fo'}}\right]\} dFo'$$

(20)

where $Z = \frac{z}{h}$, $H = \frac{h}{r_b}$, $Fo = \frac{\alpha t}{r_b^2}$ and $R = \frac{r}{r_b}$.

After applying the boundary conditions, the final 2D solution for temperature distribution in ground is obtained as:

$$\theta(R,Z,Fo) = \frac{\int_0^{Fo} \frac{1}{Fo-Fo'} e^{\left[-\frac{R^2+1}{4(Fo-Fo')}\right]} . I_0\left[\frac{R}{2(Fo-Fo')}\right] . \{\mathrm{erf}\left[\frac{H-Z}{2\sqrt{Fo-Fo'}}\right] + 2\mathrm{erf}\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right] - \mathrm{erf}\left[\frac{H+Z}{2\sqrt{Fo-Fo'}}\right]\} dFo'}{\int_0^{Fo} \frac{1}{Fo-Fo'} e^{\left[-\frac{2}{4(Fo-Fo')}\right]} . I_0\left[\frac{1}{2(Fo-Fo')}\right] . \{\mathrm{erf}\left[\frac{H-Z}{2\sqrt{Fo-Fo'}}\right] + 2\mathrm{erf}\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right] - \mathrm{erf}\left[\frac{H+Z}{2\sqrt{Fo-Fo'}}\right]\} dFo'}$$

(21)

Heat transfer rate in 2D model can be derived by using the method described in the part 2.1.1. For the midpoint of the borehole, we get the following expression with Z=0.5 and H=600, for a unit HTR value:

$$\therefore \quad \cdots_w - T_0) \int_0^{Fo} \left\{ -\frac{e^{-\frac{1+R^2}{4(Fo-Fo')}} R\, I_0\left[\frac{R}{2(Fo-Fo')}\right]\left(2\text{Erf}\left[\frac{0.25}{\sqrt{Fo-Fo'}}\right] + \text{Erf}\left[\frac{29.75}{\sqrt{Fo-Fo'}}\right] - \text{Erf}\left[\frac{30.25}{\sqrt{Fo-Fo'}}\right]\right)}{2(Fo-Fo')^2} \right.$$

$$\left. + \frac{e^{-\frac{1+R^2}{4(Fo-Fo')}} I_1\left[\frac{R}{2(Fo-Fo')}\right]\left(2\text{Erf}\left[\frac{0.25}{\sqrt{Fo-Fo'}}\right] + \text{Erf}\left[\frac{29.75}{\sqrt{Fo-Fo'}}\right] - \text{Erf}\left[\frac{30.25}{\sqrt{Fo-Fo'}}\right]\right)}{2(Fo-Fo')^2} \right\} dFo$$

(22)

2.3 Experimental setup description

In the test system, flow rate, inlet and outlet temperatures are measured and recorded in real-time for each pipe by PT1000 temperature sensors and liquid turbine flow-meters. Properties of temperature sensors and flow meters are given in Table 1. Before the test system is operated, temperature sensors are calibrated in a calorimetric container to get the same results from each sensor for the temperature range of from 2 ^{0}C and 55 ^{0}C. Flow-meters are also calibrated by Siemens Mag5000 flow-meter.

Figure 2. Experimental test system

To test the borehole/s, ends of pipes were connected to the test system. After the air purged from the system, undisturbed ground temperature has to be measured before the test is stared. To determine undisturbed ground temperature as recommended by Gehlin (2002), the valves 3, 5, 6, 7 were closed (in Fig. 3) and the pump was switched on. The circulating water temperature after 15-20 minutes gives the information about the undisturbed temperature.

Later that, valves 2, 3, 7 and borehole's valves are closed, mini pump and electrical resistances with PID control are run to heat the water in the tank up to the test temperature. When the tank temperature reaches test temperature, by-pass line and valves 2 and 3 are closed, valve 7 is half opened and the others are fully opened, and then test is started. Mini pump on the tank provide homogeneous temperature in tank. Inlet temperature is measured and controlled by PID controller.

The system described above is also used for many different approaches. In this study we just use one borehole to check the accuracy of analytical method that we used. This test system can also be used to determine the performance of a GSHP system

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with more than one borehole. Testing large number of boreholes would let us to calculate the optimum distance between boreholes and reach the maximum efficiency of system.
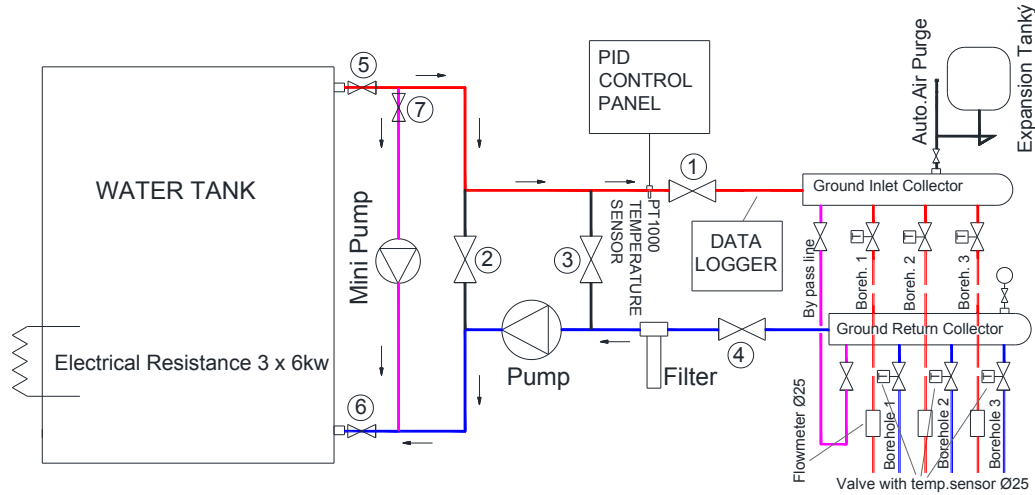

Figure 3. Constant Temperature TRT System

Table 1: Specifications of Flow-meter and Temperature Sensors

Flow meter		
Nominal Diameter	15	mm
Repeatability	±0.2	%
Accuracy - Standard	±1	%
Temperature Sensor		
Type	Pt1000	
Precision	±0.15	K

Sample test is operated between 28th of May and 7th of June. Test conditions are given in Table 2.

Table 2: Borehole specifications and test conditions

Borehole diameter	0.17	m
Borehole length	50	m
Total test duration	120	hour
Ground inlet temperature	40.0	°C
Ground avg. outlet temperature	37.5	°C
Flow-rate	25.4	lt/min
Average unit HTR value	88.0	W/m
Distance between boreholes	6	m

3. RESULTS

3.1 Validation of analytical solutions of 1D and 2D model with experimental results

Analytical solutions are often favored in engineering designs and analyses owing to their clearness and convenience of evaluation. In order to know the accuracy of the analytical solutions we always need to compare their results with experimental results or any other reliable results. The comparison between experimental results and analytical results (Eq.15 and Eq.22) is shown in the fig4.

According to the comparison shown in the fig 4, the analytical and experimental results agree with each other. Except the short initial time period, 1D and 2D results are nearly the same. But as it is seen from fig 4, 2D model is more reliable than 1D model. In 2D model the results are in good agreement with experimental ones especially in the first 50 hours. 1D model also gives acceptable results after 50 hours non-stop operation.

3.2 GHE performance prediction for 5months non-stop operation

The performance of GHE can be predicted by the aid of validated simulation model. The performance of GHE is predicted at the end of a five month non-stop operation as shown in fig5. At the end of five months, the performance of the GHE decreases up to 72 W/m and 75 W/m according to 2D and 1D models respectively. As it is seen in fig5, 1D and 2D models have a very little difference that can be neglected. However, 2D model always gives more accurate results.
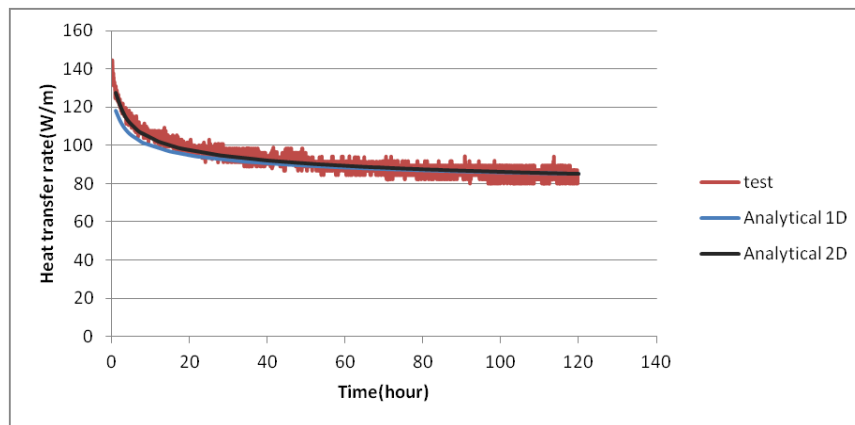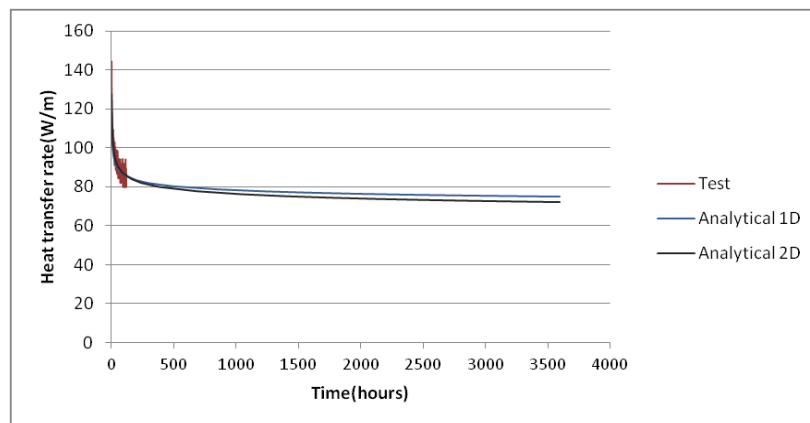
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Figure 4. Analytical model validation

Figure 5. GHE performance prediction at the end of 5months non-stop operation

4. CONCLUSIONS

The analytical method has shown success in investigating the performance prediction of a borehole. To illustrate efficiency of the method, models are validated with experimental results. Validation of the results shows that analytical model is suitable for GHE performance prediction. The analytical solutions for unit HTR value presented in this paper provide a desirable tool for simulating the GHEs. In addition to a more accurate of 2D model, 1D model also gives reasonable results for long term performance predictions of a borehole. Therefore, for simplicity, it seems to be enough to use 1D analytical model for long term predictions.

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