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Geothermal Two Phase Flow Correlation in Vertical Pipes Using Dimensional Analysis

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ABSTRACT

The flowing bottom-hole pressure of a geothermal well is the sum of the flowing wellhead pressure, the pressure exerted by the weight of the fluid column, the kinetic energy change, and the energy losses resulting from heat transfer and friction.

In this study, we propose a model which was derived using the dimensional analysis method to estimate the flowing pressure at any point along a geothermal well. The correlation includes several important parameters which have significant effect in fluid flow system in the geothermal well such as mixture density, total mass flow rate, enthalpy, pipe diameter, and pressure gradient.

For validation purpose, the measured flowing bottom-hole pressure of several wells was compared with the one calculated using the proposed correlation. The differences of flowing bottom-hole pressure are ranging from 2.016% to 12.63%. The comparison results show that the proposed model is viable to estimate the flowing bottom-hole pressure.

The model allows the application of temperature gradient change by multi-segment calculation procedure. There are little differences of flowing bottom-hole pressure calculation that are computed using the whole wellbore and two segment calculation due to round off errors. The average difference is 0.23%.

1. INTRODUCTION

Geothermal fluid undergoes pressure and temperature loss while traveling from the bottom-hole to the wellhead. The pressure loss of the fluid is due to friction with the walls of the well, acceleration and elevation of the flow. The temperature loss of the fluid is due to the change of geothermal temperature along a well which is a function of depth.

The change of the pressure and temperature of the fluid may cause the phase change of the fluid. The fluid which comes from reservoir rock is in form of either single phase or two phase flow. It depends on the reservoir pressure and temperature. In most geothermal wells, the fluid goes through phase change when the liquid begins to boil on its way up the well. The location in the well where this phase change begins is called the flashing zone or flashing horizon (Dipippo, 2008; Pora, 2013). It is due to lowered temperature and pressure during the travel of the fluid to the wellhead. Above that spot the flow is in two phases where steam and liquid travel through the well simultaneously. The flow still endures pressure drop not only due to factors mentioned above but it also looses pressure due to friction between the two phases (Kjartan, 2011).

In multi-phase flow, multiple phases exist simultaneously in flow. Two-phase flow can also be two-component flow where there are two components with different density and viscosity flowing simultaneously. Two-phase flow is the simplest type of multiphase flow. In geothermal wells there is geothermal fluid and steam flowing with a small amount of other gases (Wallis, 1969).

The objective of this paper is to propose a fluid flow correlation in vertical geothermal wells. The correlation can be used to predict flowing pressure at any point along the wells. The validation of the correlation was verified by comparison with several measurement data which shows good agreements.

2. TWO PHASE FLOW

The flow in the wells usually consists of two phase flow. Two phase flow follows all of the basic laws of fluid mechanics. However the equations are more complicated than for single phase flow since the phases have different properties. The interaction of liquid and vapor state can be described with different flow patterns. There are several different methods for evaluating the flow patterns but they depend on transport properties of the two phases, pipe roughness, the fractions of volume and mass in the pipe and velocity ratio between the two phases (Pora, 2013).

Two-phase flow in geothermal vertical wells has been studied by many investigators. The equations that describe such flow are the continuity, momentum, and energy equations. These are then used to express the total pressure drop up the wellbore in terms of its potential, acceleration, and frictional components (Jaime, 1983; Pora, 2013).

The continuity equation describes the transport of conserved quantity, in this case conservation of mass. For the case of two phase flow the continuity equation has to account for the different phase properties then the equation can be written as

$$\frac{d}{dz}\left(v_m A \rho_m\right) = \frac{d}{dz}(m_t) = 0 \tag{1}$$

Where v_m is mixture velocity of fluid that is given by

$$v_m = v_{sl} + v_{sg} \tag{2}$$

v_{sl} and v_{sg} are the superficial velocities of liquid and gas respectively and defined as

$$v_{sl} = \frac{q_l}{A} \tag{3}$$

and

$$v_{sg} = \frac{q_g}{A} \tag{4}$$

ρ_m is the density of the liquid-gas mixture which is a function of the void fraction. That is defined as:

$$\rho_m = \rho_l(1-\alpha) + \rho_g \alpha \tag{5}$$

α is also called gas holdup. It is the volume of gas or steam actually present in a given pipe section and given by (Wallis, 1969).

$$\alpha = \frac{A_g}{A} \tag{6}$$

m_t is total mass flow rate that is defined as

$$m_t = m_l + m_g = \rho_l q_l + \rho_g q_g \tag{7}$$

Gas density and liquid density can be estimated using the following equations (Wagner and Pruss, 1993)

$$\ln\left(\frac{\rho_g}{\rho_c}\right) = i_1\tau^{\frac{1}{3}} + i_2\tau^{\frac{1}{3}} + i_3\tau^{\frac{1}{3}} + i_4\tau^{\frac{1}{3}} + i_5\tau^{\frac{1}{3}} + i_6\tau^{\frac{1}{3}} \tag{8}$$

and

$$\frac{\rho_l}{\rho_c} = 1 + j_1\tau^{\frac{1}{3}} + j_2\tau^{\frac{1}{3}} + j_3\tau^{\frac{1}{3}} + j_4\tau^{\frac{1}{3}} + j_5\tau^{\frac{1}{3}} + j_6\tau^{\frac{1}{3}} \tag{9}$$

where

ρ_c = 20.102 lb/ft3	τ = 1-(T/705.103)
i_1 = -2.03150240	i_4 = -17.2991605
i_2 = -2.68302940	i_5 = -44.7586581
i_3 = -5.38626492	i_6 = -63.9201063
j_1 = 1.99274064	j_4 = -1.75493479
j_2 = 1.09965342	j_5 = -45.5170352
j_3 = -0.510839303	j_6 = -6.7469445 x 10^5

The energy equation is derived from the first law of thermodynamics. The equation is divided into four parts, namely kinetic part, change in enthalpy per unit length, gravitational potential energy, heat loss per unit length. The energy equation for two phase flow can be written as

$$\frac{d}{dz}\left[m_l\left(\frac{v_l^2}{2} + gz + h_l\right) + m_g\left(\frac{v_g^2}{2} + gz + h_g\right) + Q \right] = 0 \tag{10}$$

The momentum equation for two phase flow consists of change in inertia per unit length, Pressure changes per unit length, hydrostatic pressure gradient, hydrostatic pressure gradient and head loss. The equation is given by

$$\frac{d}{dz}\left(m_t v_m\right) + A\frac{dp}{dz} + \rho_m gA + t_f A = 0 \tag{11}$$

where t_f is the pressure drop per unit length due to friction. t_f depends on flow regime of the two phase flow. It may be expressed by

$$t_f = \frac{f}{2gD}\left[v_l^2 \rho_l (1-\alpha) + v_g^2 \rho_g \alpha\right] \tag{12}$$

The steam quality of the mixture, x, is the ratio between mass flow of gas and total mass flow through a given cross section of the pipe (Wallis, 1969). If gas rate to liquid rate ratio is based on the gas solubility in water (R_{sw}) then the steam quality may be estimated as follows

$$x = \frac{m_g}{m_l + m_g} = \frac{R_{sw}\rho_g}{\rho_l + R_{sw}\rho_g} \tag{13}$$

where R_{sw} is gas solubility in water (Ahmed, 2006)

$$R_{sw} = \frac{1}{5.615}\left(k_1 + k_2 p + k_3 p^2\right) \tag{14}$$

$$k_1 = 2.12 + 3.45(10^{-3})T - 3.59(10^{-5})T^2$$

$$k_2 = 0.0107 - 5.26(10^{-5})T + 1.48(10^{-7})T^2$$

$$k_3 = 8.75(10^{-7}) + 3.9(10^{-9})T - 1.02(10^{-11})T^2$$

A relationship between the gas hold up, steam quality, slip ratio and phase densities exists, can be expressed by (Kjartan, 2011)

$$\alpha = \frac{x\rho_l}{[(1-x)\rho_g S + x\rho_l]} \tag{15}$$

where S is slippage between the phases. This slippage is commonly termed slip ratio, S, and is defined as the ratio between the average velocity of the gas phase and the average velocity of the liquid phase in gas-liquid flow (Zhao, 2005)

$$S = \frac{v_g}{v_l} \tag{16}$$

If slip between liquid phase and gas phase is negligible, then S is equal to one.

The three governing equation can be assembled to derive a pressure drop correlation in the vertical two-phase flow. Jaime (1983) has modified Orkiszewski's method for predicting the pressure drop of the two-phase flow in geothermal wells as follows

$$\Delta p = \frac{1}{144}\left[\frac{\rho_m + t_f}{1 - \frac{(m_l + m_g)q_g}{4637A^2 p}}\right]\Delta z \tag{17}$$

3. MODELING METHOD

In this modeling, the slip between liquid phase and gas phase is negligible. This means that the velocity of the liquid phase and gas phase flow is the same. In addition, the change of flow regime of the two phase flow is not considered. The complex correlations of the two phase fluid flow in geothermal vertical well can be reduced to a simpler form using Buckingham π dimensional analysis prior to obtaining a quantitative answer. Several physical parameters which have a significant effect on the fluid flow in pipe are as follows

1. Mixture density (ρ_m)
2. Total mass flow rate (m_t)
3. Enthalpy (h)
4. Pipe diameter (d)
5. Pressure gradient ($\Delta P/L$)

The unit and dimension of the parameters are listed in the following table

Table 1: Unit and dimension of the selected parameters.

No.	Parameter	Unit	Dimension
1.	ρ_m	lbm/ft^3	ML^{-3}
2.	m_t	lbm/sec	MT^{-1}
3.	h	Btu/lb	L^2T^{-2}
4.	d	ft	L
5.	$\Delta P/L$	psi/ft	$ML^{-2}T^{-2}$

As per the Buckingham π theorem, two number of dimensionless groups can be formed. Therefore

$$f(\pi_1, \pi_2) = 0 \tag{18}$$

The first group consists of the first four variables. The variables are arranged to form the dimensionless equation as follows

$$\pi_1 = \frac{\rho_m^2 d^4 h}{m_t^2} \tag{19}$$

Using the same way, the second dimensionless group is arranged by replacing the fourth variable with the fifth one. The equation is as follows

$$\pi_2 = \frac{\rho_m d^5 \left(\frac{\Delta P}{L}\right)}{m_t^2} \tag{20}$$

where:

$$\Delta P = P_{wf} - P_{wh} \tag{21}$$

Subtituting Eq. (21) into Eq. (20) and rearranging, the flowing bottom-hole pressure is obtained:

$$P_{wf} = P_{wh} + \frac{L m_t^2 \pi_2}{\rho_m d^5} \tag{22}$$

In order to obtain relation between the two dimensionless groups, several variables given in Table 2 were randomly varied to result in 350 sets of data. The interval of the variables fixed in Table 2 were based on some references (Kjartan,2011; Iqbal, K.Z, etc. 1977; Chi, 1992). The values of the data were input into a simulator developed by Jaime (1983) to generate 350 values of the two dimensionless groups. Plot of the values as depicted in Figure 1, shows the relationship between the two groups as follows:

$$\pi_2 = 0.0000092033 \quad \pi_1^{0.966377} \tag{23}$$

Table 2: Parameter used to correlate the two dimensionless groups.

Parameter	minimum	Maximum
d(ft)	0.72	1.0335
L(ft)	2280	8299
e(in)	0.00006	0.0001
P_{wh}(psi)	195	995
P_{bh}(psi)	658	3073
T_{wh}(^oF)	50	534
T_{bh}(^oF)	182	757
m_t(lb/sec)	25	154
h(btu/lb)	21	528
U (btu/hr/sqft/F)	289	1386
γ_w	0.75	1.13

Procedure of the model application for bottom-hole pressure calculation (P_{wf}) at a given depth below the surface is handled by trial-and error method. This is caused the P_{wf} is used to obtain an average pressure. While the parameters which are used to calculate the P_{wf}, are evaluated at the average pressure. The procedure to estimate the flowing bottom is given as follows:

1. Assume a value of flowing bottom-hole pressure (P_{wfa}).
2. Calculate average pressure and average temperature using the following equations

$$\bar{P} = \frac{P_{wh} + P_{wfa}}{2} \tag{24}$$

$$\bar{T} = \frac{T_{wh} + T_{wf}}{2} \tag{25}$$

3. Calculate or measure mixture density, (ρ_m), total mass flow rate (m_t), enthalpy, and diameter pipe at $\bar{P}$ and $\bar{T}$.
4. Calculate the first and second dimensionless equations using Eqs. (19) and (23), respectively.
5. Calculate flowing bottom-hole pressure using Eq. (22).
6. If the different between the calculated P_{wf} (in step 5) and the assumed P_{wfa} (in step 1) is less than 1 psi, the procedure ends. If not, assume P_{wfa} = P_{wf} and go to step 2.

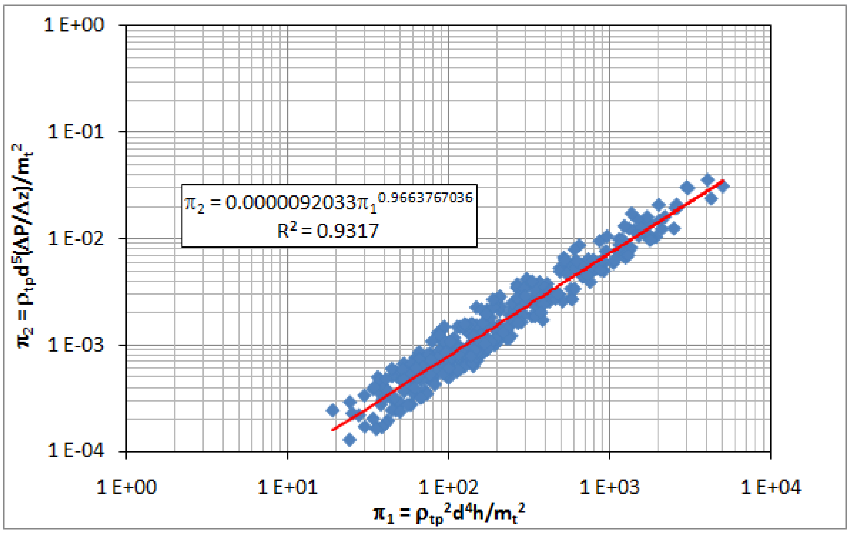

Figure 1: Correlation of the two dimensionless groups.

4. RESULTS AND DISCUSSION

Tables 3 and 4 provide the measurement results of several geothermal wells in Iceland. The data referred from the work of Kjartan (2011) and Póra (2013) come from three wells at Hellisheiði field, three wells at Reykjanes field and a well at Svartsengi field. The data are used to validate the proposed model by comparing the measured and calculated flowing bottom-hole pressure.

Table 3: Data of four geothermal wells at Hellisheiði and Reykjanes fields (Kjartan, 2011).

Parameter	HE-05	HE-48	HE-54	RN-15
Depth, ft	2595.1	2719.8	2467.2	2296.6
ID pd.Casing, ft	0.8021	1.0335	1.0335	1.0335
P_{wh}, psi	275.57	210.30	942.75	580.15
T_{wh}, ^oF	402.80	386.60	536.00	473.00
T_{wf}, ^oF	500.00	467.60	557.60	512.60
h(btu/lb)	513.33	460.88	782.89	674.98
m_t(lb/sec)	108.03	86.421	86.421	78.925
P_{wf}, psi	688.93	529.10	1000.8	739.69

Table 4: Data of three geothermal wells at Reykjanes and Svartsengi Fields (Póra, 2013).

Parameter	RN-11	RN-12	SV-21
Depth, ft	2260.5	2253.9	2752.6
ID pd.Casing, ft	0.9843	0.9843	1.0335
P_{wh}, psi	594.65	587.40	210.30
T_{wh}, ^oF	473.00	473.00	383.00
T_{wf}, ^oF	511.98	511.87	462.20
h(btu/lb)	558.90	558.90	442.82
m_t(lb/sec)	110.23	108.03	154.32
P_{wf}, psi	868.78	841.22	565.65

Table 5 shows the comparisons of flowing bottom-hole between the two methods. The table shows that the results obtained from the proposed correlation agree with the data. The differences of the flowing bottom-hole pressure values are ranging from 2.016% to 12.63%, with an average difference of 6.29%. The deviation of the computed flowing bottom-hole pressure tends to increase if the data of the wells given in Tables 3 and 4 are beyond the interval of parameter set up in Table 2. The tables indicate that enthalpy contributes a significant deviation. Wells HE-54, RN-15, RN-11, and RN-12 which have a greater value of enthalpy than the maximum value of the parameter established in Table 2 shows greater differences than Wells HE-05, HE-48, and SV-21.

Table 5: Comparisons of data and calculation of flowing bottom-hole pressure.

Well	Flowing Bottom-hole Pressure, P_{wf}, psi		
	Data (Kjartan,2001)	Proposed Model	% difference
HE-05	688.93	675.04	2.016
HE-48	529.10	545.40	3.082
HE-54	1000.8	1127.1	12.63
RN-11	868.78	798.97	8.036
RN-12	841.22	792.52	5.789
RN-15	739.69	810.90	9.627
SV-21	565.65	549.69	2.821

The proposed correlation assumes that the temperature gradient is constant. Therefore, the all parameter were evaluated at an average temperature. In reality, the temperature gradient is not constant at all along the well. In order to apply the change of temperature gradient, one can divide the well depth into several segments. The parameter of a segment is evaluated at an average temperature of the segment. Table 6 shows the application of two segment calculations to estimate the flowing bottom-hole pressure of the wells. There are little differences of flowing bottom-hole pressure that are computed using the whole wellbore calculation (second column of Table 6) and two segment calculation (third column of Table 6). The differences were most likely due to round off errors. The differences of the flowing bottom-hole pressure values are ranging from 0.003% to 0.556%, with average difference of 0.23%.

Table 6: Comparisons of flowing bottom-hole pressure estimation using whole well and two segment calculations.

Well	Flowing Bottom-hole Pressure, P_{wf}, psi		
	Whole well	Two Segments	% difference
HE-05	675.04	671.59	0.511
HE-48	545.40	542.37	0.556
HE-54	1127.1	1127.3	0.018
RN-11	798.97	798.94	0.004
RN-12	792.52	792.50	0.003
RN-15	810.90	810.93	0.004
SV-21	549.69	546.69	0.546

5. CONCLUSIONS AND RECOMMENDATION

Based on the analysis and discussion shown above, several statements are made as follows:

1. Comparisons of the flowing bottom-hole pressure show that the proposed correlation gave good agreement with the measured data, where their average different was 6.29%.

2. The model can be applied for non linear temperature gradient case by using multi segment calculation. In case of constant temperature gradient, the average difference of flowing bottom-hole pressure calculation between the whole wellbore calculation and two segment calculation is 0.23%.

3. The application of the proposed correlation is restricted by the limitation of the range of parameters involved. The deviation of the computed flowing bottom-hole pressure to measurement tends to increase if the data of the wells are beyond the range.

4. The correlation may be improved by including viscosity parameter and considering the change of flow regime of fluid flow.

NOMENCLATURE

A = cross-sectional area of pipe, ft^2

d = inner pipe diameter, ft

e = absolute roughness of pipe, in.

f = friction factor, dimensionless

g = gravity acceleration, ft/sec

h = enthalpy, Btu/lbm

L = length of well, ft

m = total mass flow rate, lbm/hr

P = pressure, psi

P_{wh} = wellhead pressure, psi

P_{wf} = flowing bottom-hole pressure, psi

Q = heat transferred to surroundings, Btu/lbm

R_{sw} = gas solubility in water, dimensionless

q = volumetric flow rate, ft/sec

T = temperature, ^{o}F

T_{wh} = wellhead temperature, ^{o}F

T_{wf} = flowing bottom-hole temperature, ^{o}F

t = time, sec

t_f = friction-loss gradient, lb/ft

U = overall heat transfer coeff., Btu/hr-ft^2-F

v = velocity, ft/sec

v_{sg} = superficial gas velocity, ft/sec

v_{sl} = superficial liquid velocity, ft/sec

z = depth, ft

α = void fraction, dimensionless

γ = specific gravity, dimensionless

ρ = density, lbm/ft

Subscripts:

g = gas

l = liquid

m = mixture

t = total

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