Characterization of Mass/Heat Transfer in Fractured Geothermal Reservoirs by Means of Mathematical Model for Complex Systems

Anna Suzuki[1], Yuichi NIIBORI[2], Sergei A. FOMIN[3], Vladimir A. CHUGUNOV[4], and Toshiyuki HASHIDA[5]

[1] Tohoku University, 6-6-11-707, Aramaki-Aza-Aoba,Aoba, Sendai 980-8579, Japan

[2] Tohoku University, 6-6-01-2, Aramaki-Aza-Aoba,Aoba, Sendai 980-8579, Japan

[3]California State University, Chico, CA 95929, USA

[4]Kazan Federal University,Kazan 420008, Russia

[5]Tohoku University, 6-6-11-709, Aramaki-Aza-Aoba,Aoba, Sendai 980-8579, Japan

e-mail: anna3@stanford.edu

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ABSTRACT

A fractional heat transfer equation (fHTE) has been developed in order to characterize thermal responses in a geothermal reservoir on a basis of fractional advection-dispersion equation (fADE). The fHTE takes into account thermal diffusion from main conduit toward the outside and the inside of a reservoir. Numerical simulations of fluid and heat flow in two models were conducted by using TOUGH2: one is a fault model implemented permeability distribution due to a structure of fault zones to reproduce thermal diffusivity into the outside of the reservoir, and the other is MINC treating interactions between matrix and fractures to generate thermal diffusivity into the inside. The results for the fault model showed that the difference of permeability distribution in the surrounding rocks affected the rate at which temperature dropped. The fHTE was in good agreement with simulated temperature profiles, compared with conventional models. The MINC yielded the intermediate responses in between a porous and a fracture medium. Fitted results of fHTE on the MINC results suggested that the fHTE is efficient to characterize thermal response from a fractured medium to a porous medium and would provide a time-saving and more precise approach for characterizing thermal response in a fractured reservoir.

1. INTRODUCTION

Reservoir models have been developed from qualitative and quantitative information gathered during the exploration phase of a project. Comprehensive computer simulation identifies thermal and hydrologic behavior of the reservoir with assembled data through field observation, exploratory drilling, and well tests by relying on continuum approaches or discrete fracture network approaches. The main problem with applying the numerical simulation is to require accurate site-specific geological measurement in order to determine numerous constitutive parameters. Unfortunately, most measurement data are obtained from limited points and are much less accurate to describe a fractured medium. Inverse analysis of constitutive parameters has been still try-and-error and can be time-consuming.

In a simple way, specific mathematical models can be constructed to evaluate and optimize geothermal utilization schemes. Migration of cold-water front for a single-phase liquid flow that conducts heat from a semi-infinite matrix is fairly well understood through the work of Lauwerier (1955) and Bodvarsson (1972). Consequently, the temperature changes over time at the production wells were estimated (Gringatten and Sauty, 1975; Bodvarsson and Tsang, 1982; Kocabas, 2004). Those models have been widely used to estimate cooling effects of reinjected fluid in several geothermal fields but still lead to overestimation of temperature decline (i.e., Aksoy et al, 2008).

Complexity in a fractured geothermal reservoir can arise from the large number of natural fractures. In order to describe the complex behavior in a fractured reservoir, an alternative modeling methodology based on mathematical models has been discussed in recent years, which takes into account the fracture nature of self-organized natural system. Fractional derivatives is used to develop a mass transport model, resulting in the fractional advection–dispersion equation (fADE) (Benson et al. 2000). Using fractional derivatives in time and space, Fomin et al. (2005) derived an equation that accounts for diffusion of solute into matrix and surrounding rocks. In the last decade, many authors have made notable contribution to both theory and application of the fADE as reviewed by Zhang et al. (2009). The advantage of using the fADE is its ability to characterize mass transport in heterogeneous media by using few parameters. Further research on the application of fADE should be conducted to evaluate and optimize geothermal development.

Analogous behaviors of heat and mass transfer have been long recognized (Welty et al., 2007). The purpose of this study is to develop a complex heat transfer equation based on the fADE, which is called fractional heat transfer equation (fHTE). Little has been known yet about the relationship between the geological structures and the fHTE constitutional parameters. Numerical simulation results generated by a general reservoir simulator, TOUGH2, were used as synthetic field performance data to validate the applicability of fHTE to a geothermal field.

2. MATHEMATICAL MODEL

2.1 fractional Heat Transfer Equation (fHTE)

Fomin et al. (2011) proposed the one-dimensional fractional advection-dispersion equation (fADE) to model solute transport in a fracture system. Detailed derivations of the fADE can be found in the literature (Fomin et al. 2005, 2011). In their model, a

fractured reservoir consists of a complex distribution of natural fractures, which is characterized through fractal geometry. The governing equation of the fADE is derived as follows:

$$\frac{\partial C}{\partial \tau} + b_3 \frac{\partial^{\gamma} C}{\partial \tau^{\gamma}} + b_1 \frac{\partial^{\beta} C}{\partial \tau^{\beta}} = \frac{1}{Pe} \frac{\partial}{\partial X}\left(p \frac{\partial^{\alpha} C}{\partial X^{\alpha}} + (1-p) \frac{\partial^{\alpha} C}{\partial (-X)^{\alpha}} \right) - \frac{\partial C}{\partial X} \tag{1}$$

where C, T, and X are concentration, time and distance, which are normalized with respect to each representative value. b_1 and b_3 are the retardation coefficient and Pe is Péclet number. p is the skewness parameters. α ($0 < \alpha \leq 1$), β ($0 < \beta \leq 1$) and γ ($0 \leq \gamma \leq 1$) are the orders of the fractional temporal derivatives. Here, the first term on the left side of Eq. (1) is the accumulation term, and the second term models the retardation process associated with dispersion into secondary branched fractures. The term can describe the thermal diffusivity into matrix, namely the inside of the reservoir. The third term describes the process of vertical diffusion into the surrounding rock masses, which means the diffusion into the outside from the reservoir. The first term on the right side expresses dispersion within the fractured reservoir, and the second term is the advection term.

Analogous behaviors of heat and mass transfer have been long recognized (Welty et al., 2007). Based on this analogy, the above fADE is suggested to lead to an equation to characterize heat transfer in a fractured reservoir. The governing equation of fHTE is presented as follows:

$$\frac{\partial T}{\partial \tau} + e_3 \frac{\partial^{\gamma'} T}{\partial \tau^{\gamma'}} + e_1 \frac{\partial^{\beta'} T}{\partial \tau^{\beta'}} = -\frac{\varphi_2 \rho_2 C_{pw}}{\overline{\rho C_{p2}}} \frac{\partial T}{\partial X} \tag{2}$$

where $\overline{\rho C_{P2}} = m_2 \rho_w C_{Pw} + (1-m_2)\rho_r C_{P2}$ and ρ_w and ρ_r are the density of water and rock, respectively; C_{pw2} and C_{pr2} are the heat capacity of water and rock in the reservoir, respectively. T is temperature. The coefficients e_3 and e_1 represent retardation processes caused by the heat diffusivities into the matrix and into the surrounding rocks, respectively. The orders β' ($0 < \beta \leq 1$) and γ' ($0 \leq \gamma \leq 1$) are the orders of the fractional derivatives. In Eq. (2), thermal conduction inside the reservoir is negligible as indicated by Woods and Fitzgerald (1993).

The schematic of a fractured reservoir for the fADE is represented in Fig. 1. All the considered fault zones are characterized by three architectural scales. The numbers of subscripts, i, for some variables represent the objective scales, respectively. At a larger scale ($i = 1$), a fault zone system consists of a fault core and damage zones, in which a main conduit with high permeability is formed along the fault core. At this scale, the major flow is assumed to be one dimensional, but diffusion into the surrounding rocks perpendicular to the fault core occurs. At an intermediate scale ($i = 2$), the main conduit is characterized by complex fracture networks inside the damage zone. At a smaller scale ($i = 3$), the rock matrix includes secondary branched fractures and minor faults.
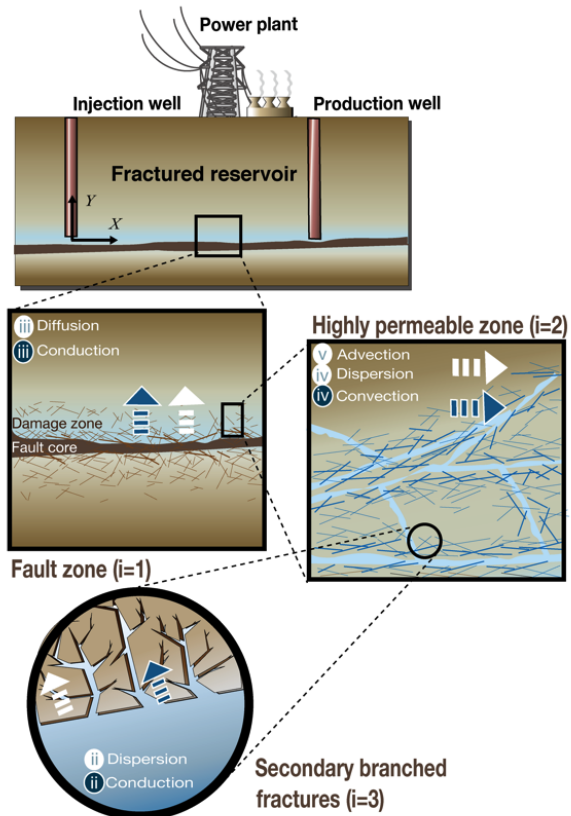

Figure 1: Schematic of a fractured reservoir.

3. NUMERICAL SIMULATION

As mentioned above, the fADE and the fHTE take into account diffusivities toward the outside and the inside of a reservoir. For the sake of simplification, this study discussed the effects of above diffusivities separately. Fluid, mass, and heat flow simulations for both reservoirs were carried out with the general-purpose reservoir simulator TOUGH2 (Pruess et al., 1999).

Considering the diffusivity from a main conduit into the outside, the feature of fault structure was implemented in TOUGH2. This model is called fault model in this paper. The numerical properties are summarized in Table 1. The thickness H and length L of the calculation domain were set to $H = 20$ m and $L = 100$ m, respectively.

A main conduit with high permeability was set up at the bottom of domain, of which the thickness H_0 was 0.2 m. The inlet and the outlet were located at the sides of the main conduit. The cell sizes were set to 1 m × 1 m × 1 m, except for the refined cells around the main conduit of which the cell sizes were set to 1 m × 0.1 m × 1 m. Heterogeneous distribution of fracture was neglected, and the simulated reservoirs were assumed to be homogeneous. In order to express decrease in fracture density in a damage zone, the permeability distribution in the outside of the main conduit was defined in the following form

$$K_s(Y) = K_0 (Y - H_0)^{-\theta},$$
(3)

where K_0 and H_0 are the constant permeability and the thickness of the main conduit, respectively; Y is the distance from the main conduit and the direction is perpendicular to the main conduit; and θ is the index of anomalous diffusion in the surrounding rocks.

Subsequently, in order to take into account the effect of thermal diffusion toward the inside of the reservoir, we employed the method of "multiple interacting continua" (MINC) (Pruess and Narasimhan, 1982, 1985). The numerical properties are summarized in Table 2. The MINC assumes two interactive continua, i.e., matrix and fractures, and contributes to a representation of diffusive processes in fractured media where matrix rock is partitioned into several interacting continua. In this study, ten sub-continua for each grid were implemented in the MINC partitioning process.

The fracture continuum had a volume fraction of 0.045 and a ''intrinsic'' porosity of 0.5. Volume fractions of 0.051, 0.058, 0.067, 0.079, 0.097, 0.125, 0.176, and 0.3 were used for the other nine matrix continua. The thickness H and length L of the calculation domain were set to $H = 11$ m and $L = 80$ m, respectively. The MINC grid consisted of 880 (=80 × 11) blocks. The inlet and the outlet were located at the sides of the calculation domain.

Table 1: Numerical properties of the fault model used in TOUGH2

Property	
Fault zone model	
Calculation domain	
Thickness T	20 m
Length L	100 m
Thickness of reservoir T_r	0.2 m
Reservoir permeability	1.0×10^{-13} m^2
Porosity	0.1
Rock density	2600 kg/m^3
Rock heat capacity	1 kJ/kg °C
Thermal conductivity	0 W/m °C
Initial pressure	10 MPa
Initial temperature	175 °C
Injection rate	0.2 kg/s
Injection temperature	35 °C
Productivity Index	1×10^{-8} m^3
Production pressure	9 MPa

Table 2: Numerical properties of the MINC used in TOUGH2.

Property	
Calculation domain	
Thickness T	68.57 m
Length L	500 m
Permeability	
Fracture	1.0×10^{-13} m^2
Matrix	1.0×10^{-18} m^2
Porosity	
Fracture	0.5
Matrix	0.1
Rock density	2600 kg/m^3
Rock heat capacity	1 kJ/kg °C
Thermal conductivity	0 W/m °C
Initial pressure	10 MPa
Initial temperature	175 °C
Injection rate	0.2 kg/s
Injection temperature	35 °C
Productivity Index	1×10^{-12} m^3
Production pressure	9.65 MPa

For both model, the initial temperature was set to 175 °C. Water was injected from the inlet. The temperature of injected water was constant at 35 °C. The temperature profiles were observed at the outlet. Provided that the rock grains are sufficiently small and the fluid velocities sufficiently low, local thermodynamic equilibrium between the rock and fluid can be assumed. It was also assumed that the rock is incompressible, that both rock and fluid have constant thermal properties, and that thermal conduction is neglected. Note that tracers were used to determine the average travel time of water migration.

The simulated temperature profiles were analyzed by using numerical solutions of fHTE (Eq. (2)). The finite difference solution has been discussed in Suzuki et al. (2014). The thermal responses simulated by TOUGH2 were normalized via representative physical variables: the representative distance was the well spacing. The representative time was the average travel time of water migration. The normalized temperature is expressed as:

$$T(t)[-] = \frac{T_{pro}(t) - T_{in}}{T_r - T_{in}}$$ (4)

where T_{pro} is the temperature of fluid at the outlet, T_r is the initial temperature of the reservoir, and T_{in} is the temperature of injected water.

Each of the porosity, the density, and the heat capacity in Eq. (2) were set to the same values given in TOUGH2. The other parameters, such as e_3, e_1, β', and γ', were determined by an optimization approach based on the root mean squared error (RMSE) to fit temperature profiles, which is described as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (T_i^{fHTE} - T_i^{TOUGH2})^2}$$ (5)

where N is the number of data points, T_i^{fHTE} is the i th temperature calculated by the fHTE, and T_i^{TOUGH2} is the i th temperature obtained using TOUGH2. For the fault model, the second term on the left-side hand in Eq. (2) was negligible, namely the fitting parameters were e_1 and β'. On the other hand, for MINC model, the fitting parameters were e_3 and γ'.

4. NUMERICAL RESULTS

First, the effect of thermal diffusion into surrounding rocks was investigated by using the fault model. Temperature profiles of produced water for different θ are plotted in Fig. 2. The temperature for larger θ showed a rapid decline. In the case of $\theta \rightarrow \infty$, the surrounding rocks are impermeable, and the flow can be govened solely by advection. On the other hand, the injected water for smaller θ penetrated into the sorrounding rocks widly and leaded to imcomplete thermal sweep.
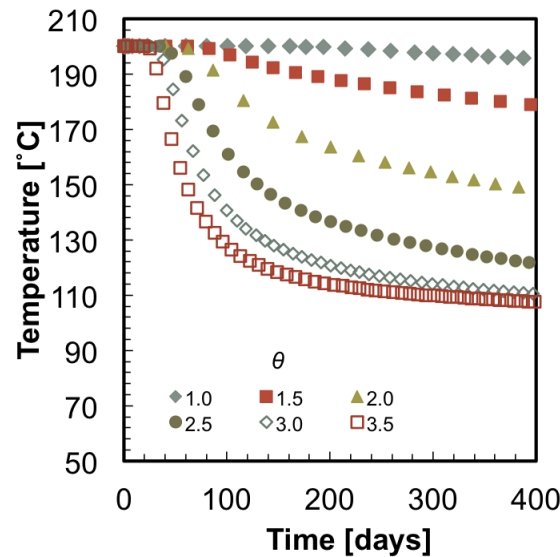
Figure 2: Temperature profile by TOUGH2 using the fault zone model for different decay rate of permeability θ.

Subsequently, we investigated the effect of thermal diffusion into the inside of the reservoir by using MINC model. The simulated temperature profiles for different fracture spacing are shown in Fig. 3. It is shown that the temperature for fracture spacing of 0.5 m declined rapidly. This excellent thermal sweep was a similar finding to the former result for larger θ as shown in Fig. 2. The result for smaller fracture spacing indicates that the effect of fractures was neglected and that the model coincided with a porous medium virtually. In contrast, the results for fracture spacing of 20 m yielded the early start of thermal breakthrough and the gradual decline of temperature profile. This result suggests that the fracture formed substantial bypassing inside the reservoir.
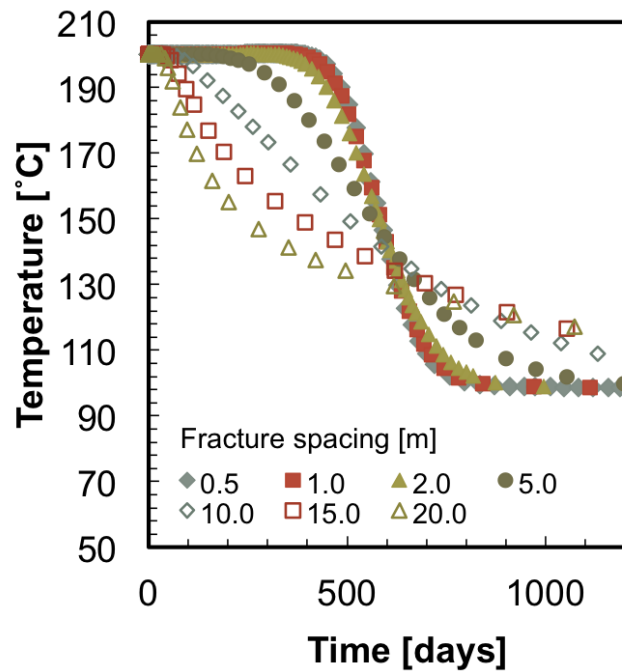
Figure 3: Temperature profile by TOUGH2 using the MINC for different fracture spacing.

The temperature profiles obtained by TOUGH2 (Figs. 2 and 3) were used to verify the applicability of the fHTE. For comparison, the conventional mathematical models of characterizing thermal response were also applied. The basic equation for subsurface temperature field was derived for rocks with intergranular flow by Bodvarsson (1972). This model expresses convective heat transfer in a porous medium, which is called porous model in this paper. Lauwerier (1955) developed an analytical solution for the heat transfer during one-dimensional flow with heat loss to confining beds according to the Fick's law. Gringarten and Sauty(1975) extended the ideas to reinjection problems in geothermal fields. This model corresponds to description of heat flow in a single fracture, which is called fracture model in this paper. This model can also express thermal diffusion into matrix virtually.

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Figure 4 shows the best-fit curves of the fHTE onto the simulated temperature profile for θ = 1.0 -2.5 for the fault model. The error values of fitting results by the fHTE, the porous model, and the fracture model are plotted in Fig. 5. As shown in Fig. 4, the fitted results by the porous model significantly departed from the simulated result of TOUGH2, especially after thermal breakthrough occurred. As heat loss from reservoir was not incorporated into the porous model, the temperature of produced water inevitably declined to the injected temperature for the porous model. Thus, the porous model yielded poor performance. The fitted results by the fracture model were more satisfactory than those by the porous model, as shown in Fig. 4. However, some simulated temperatures for the fracture model showed somewhat difference of decline rate during thermal breakthrough. The fracture model following the Fick's law is incapable of reproducing thermal diffusion process into the surrounding rocks. In other words, the thermal diffusion in to the surrounding rocks can be non-Fickian. Compared with the results of conventional models, the fHTE was in good agreement with the temperature profile for the fault model. The better performance of fHTE was also supported by its larger R^2 and smaller RMSE values than those of the porous model and the fracture model, as shown in Fig. 5.

Similar comparison tests by using temperature profiles of MINC are shown in Fig. 6. It can be found that the fHTE is capable of describing the temperature decline calculated by MINC in TOUGH2. This conclusion is also supported by checking their R^2 and RMSE values as evident from Fig. 7. Interestingly, the porous model characterized the result of MINC for smaller fracture spacing, while the fracture model agreed with the results of MINC for larger fracture spacing. Hence, the comparison results revealed that MINC can reproduce thermal response in intermediate between porous fracture media by changing the fracture spacing.
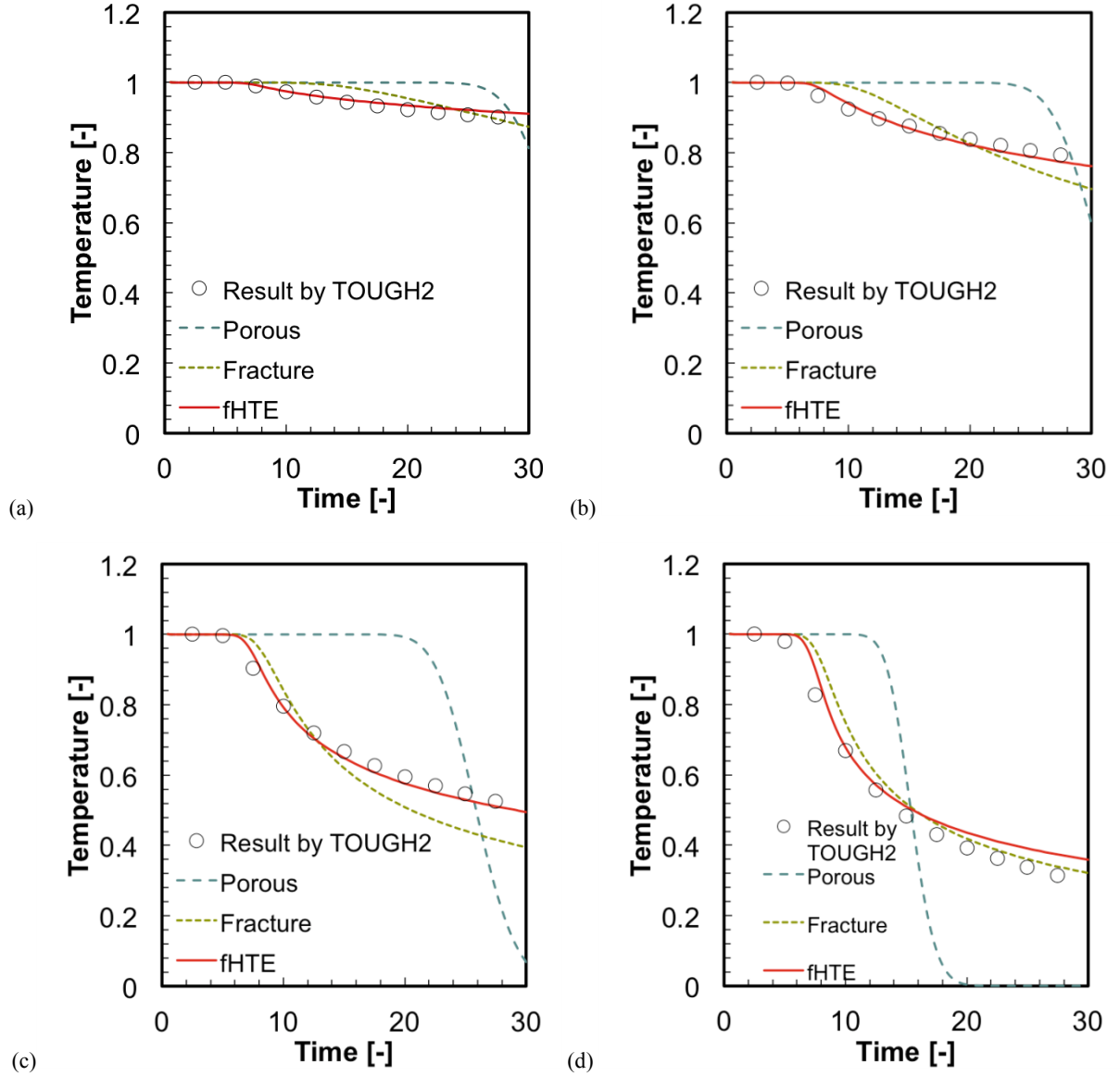

(a)

(b)

(c)

(d)

Figure 4: Simulated temperature profiles by TOUGH2 using the fault model for θ = (a)1.0, (b)1.5, (c)2.0, and (d)2.5, and the best fit with the fHTE, the porous model and the fracture model.
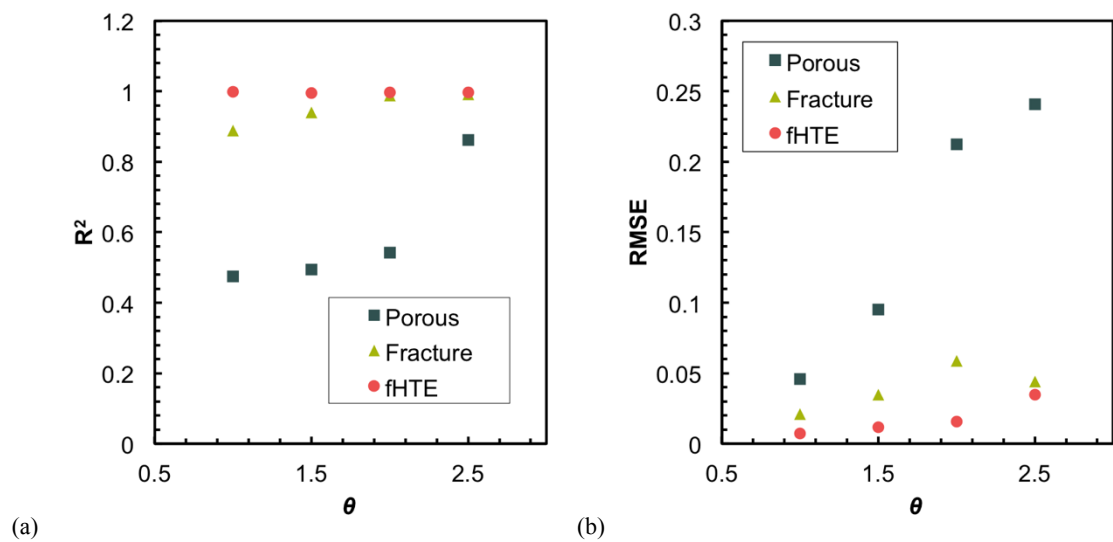
(a)

(b)

Figure 5: Goodness of fit for temperature profiles simulated by TOUGH2 using the fault model for different permeability of the surrounding rocks.
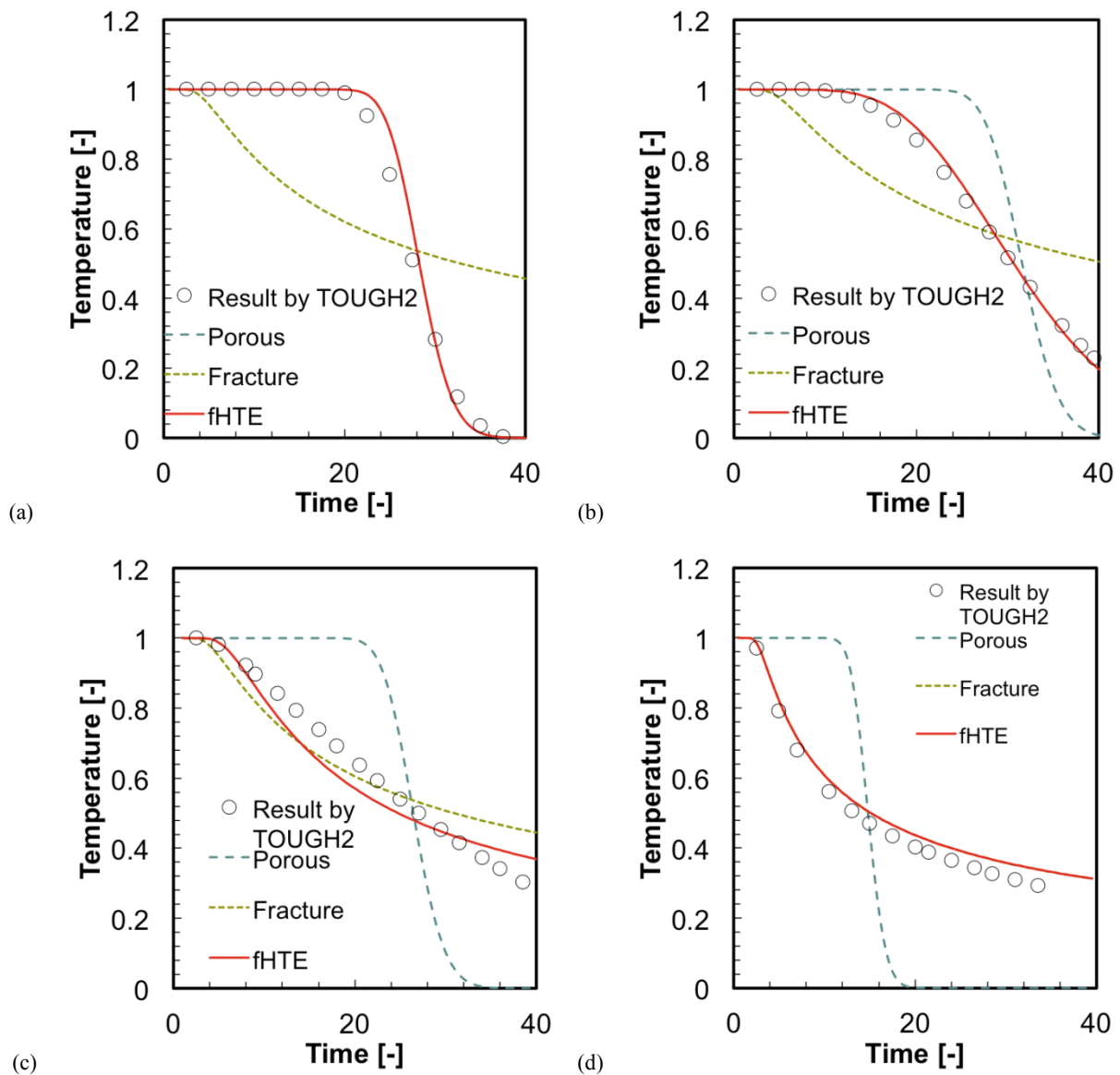

(a)

(b)

(c)

(d)

Figure 6: Simulated temperature profiles by TOUGH2 using the MINC model for fracture spacing of (a)0.5 m, (b)5.0 m, (c)10 m, and (d)20 m and the best fit with the fHTE, the porous model, and the fracture model.
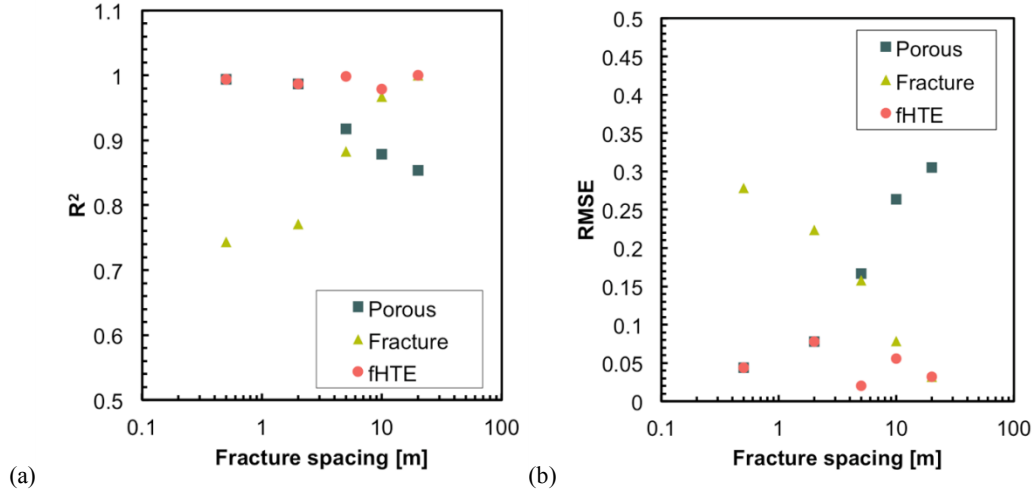
Figure 7: Goodness of fit for temperature profiles simulated by TOUGH2 using the MINC for different fracture spacings.

The correlations between θ and the best-fit constitutional parameters in the fHTE are plotted in Fig. 8. The constitutional parameters, e_1, and β', were determined by minimization of the root mean squared error in Eq. (5). Here, smaller θ caused higher penetration into the surrounding rocks. The smaller θ leaded to increase in the retardation parameters e_1 and decrease in the orders of the fractional derivative β', respectively. The correlations between fracture spacing and the best-fit constitutional parameters in the fHTE are plotted in Fig. 9. The smaller fracture spacing leaded to increase in the retardation parameters e_3 and decrease in the orders of the fractional derivative γ', respectively.
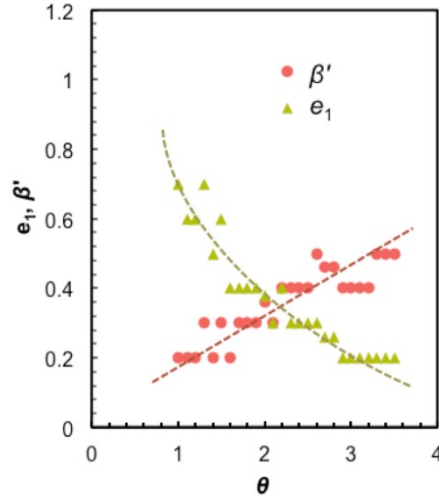

Figure 8: Relationship between θ and the fHTE constitutional parameters.
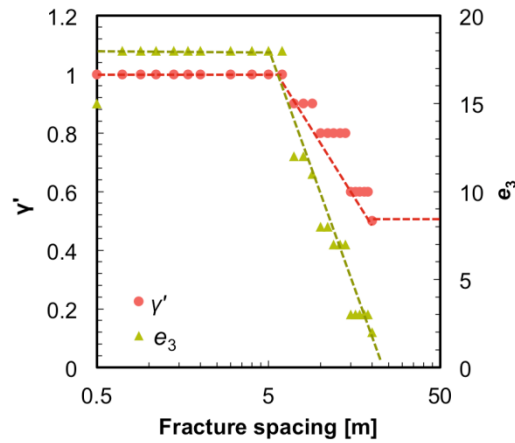

Figure 9: Relationship between fracture spacing and the fHTE constitutional parameters.

5. DISCUSSION AND CONCLUSIONS

In this study, TOUGH2 was regarded as a representative of numerical simulation model. By implementing the MINC in the simulation, it can characterize a fractured reservoir with consideration of interaction between fracture and matrix. As shown in Fig. 6, comparison with MINC results showed that the porous model is suitable for the case of smaller fracture spacing, while the fracture model is appropriate for the case of larger fracture spacing. Both models are incapable of characterizing intermediate behaviors between fracture and porous media, which can be expressed by MINC. In addition, this study demonstrated that fHTE can satisfactorily describe non-Fickian thermal behaviors in fractured media. The fHTE can describe complex phenomena by using parameter, such as fractional derivatives. Fomin et al. (2011) derived the analytical solution of fADE, which characterize mass transport in a fractured reservoir and became the basis of the fHTE. They mentioned that in the particular case of $\beta = 1/2$, the solution reduces to the well- documented formulation of Fickian diffusion. The order of fractional derivatives may characterize the interplay between sub- (weak) and superdiffusion (strong diffusion) with the boundary of 0.5. As shown in Fig. 9, the smaller fracture spacing leaded to $\gamma = 1$. This indicates that the fHTE exhibits a similar pattern with the porous model. On the other hand, the value of γ for the larger fracture spacing approached 0.5, indicating that the fHTE shows the same formation as the fracture model. Therefore, employing the order of fractional derivative as variable parameters, the fHTE can become an effective model to characterize thermal response in an intermediate reservoir between a fracture and a porous medium.

The advantages of using the fHTE in place of the MINC is time-saving to determine constitutional parameters and higher calculation accuracy because of less limitation of the number of calculation elements. Therefore, the fHTE will provide a powerful tool to characterize thermal response at early stage of geothermal development or with insufficient information obtained from field. In this regard, the development of fHTE in this study was only tested by numerically simulated results. Further study is needed to apply the method to laboratory experiments or field measurements.

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