

Optimization of Reservoir Simulation using Hybrid Kalman Filter

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ABSTRACT

An important method for reservoir assesment is to build a numerical reservoir simulation model. Part of good reservoir management is that the simulation model should be able to capture both thermodynamic processes and uncertainty based on geoscience or other studies. Sometimes, the simulation model cannot represent the reservoir condition due to errors from data uncertainty, numerical computation or the conceptual model itself. This research tries to optimize the reservoir simulation using Kalman filter which has been known as an algorithm to estimate variables by assimilating simulation results and measurement data.

A test or synthetic model of steam dominated geothermal system was built as an object of this study to be validated using a known Kalman Filter of Hybrid type (HKF). Its ability to estimate rock permeability using pressure data was studied by comparing the output, of both the test model and reservoir simulation coupled with HKF. Three performance parameters were analyzed; the effect of the number of ensemble, initial guess and range of ensemble initialization.

In this study, the HKF algorithm has been successfully implemented in geothermal reservoir simulation. This algorithm is able to eliminate uncertainty either in the data or numerical approach to get rock permeability distribution. From the sensitivity study, using 40, 100 and 200 number of ensembles, it concluded that the ensemble number does not significantly affect the HKF performance. Using initial guess for the RMSE (Root Mean Square Error) of 124 md, the HKF resulted 93 md, 91.77 md and 91.79 md for number of ensembles of 40, 100 and 200 ensembles, respectively. There are two factors that leads to this result, first that the model is not sensitive with the variation of permeability within propagation process. The second factor is that there was a problem in the software for parallel simulation which disabled the pressure output for some blocks if the initial condition was specified by saturation and temperature.

The initial guess played an important role to the accuracy of the algorithm. Changing the initial guess would result in a RMSE reduction of up to 5 md. The range of ensemble initialization also had a significant effect to the HKF's performance. In the first experiment, using a wide range of values from 1 to 400 md, for a good permeability layer of the reservoir, it would converged only from 124 md to 92 md. While in the second experiment, using a narrower range of 150 to 250 md, it would converge to 16 md. Furthermore, there was no singularity effect found using HKF, which strengthens the capability of the algorithm.

In order to improve the computation, it is suggested to couple the HKF algorithm with TOUGH2 by having an interface to bridge simulator and the HKF algorithm.

1. INTRODUCTION

According to Grant and Bixley (2011), reservoir assessment in geothermal is approached by two ways. First, by collecting all the possible information and use it to define physical properties from the subsurface object that is being explored. Second, by investigating the processes that might happen in the subsurface to see what role it will play in the reservoir. With the both of them used together in practice. There are many ways to do it, from a simple method to a complicated method. One of the method is with reservoir simulation.

Reservoir simulation involves creating a computer model that can simulate the reservoir performance in the *natural state* or before production and after or in production state (*history matching*). Reservoir simulation begins with creating a conceptual model that can represent or describe the geothermal system, accommodating the heat source, the reservoir zone, injection and discharge zone. In special systems, it includes a cap rock zone where the geothermal fluid is held temporarily, causing intensive heating and thus creating steam. Next, we have to create a computer model that can represent the conceptual model. A detailed model is desired, but it's limited to the computing power available. The next step involves calibration that should be done to ensure that the model represents the conceptual model. There are two calibration steps, *natural state* which is the state before exploitation and *history matching*, the state after the exploitation begin.

For *natural state* calibration, the model responses should match with the field data, temperature and pressure in varying depth of the exploration wells. For *history matching* stage, there are many variables assess the simulation's success. Basically, pressure decreasing pattern of our production wells is the parameter to be matched by adjusting the permeability or the porosity of the rock. According to Sullivan, et. al. (2001), for some cases other variables may be used as the parameter. For the wells that produce two phases of fluid, enthalpy could be used as a parameter. For a hot water system that has produced for short time or a steam dominated system, enthalpy cannot be used as a parameter. Well flow rate can be used as a parameter when the deliverability is assumed to be fix because of the stability of the production scenario.

Both of calibration processes need time and is difficult to be done. The mathematical model involves a strongly coupled equation and is also non-linear. There are many inputs and it is solved by a numerical method, not an analysis method, so there are numerical

errors involved. Often this means, that there is more than one solution or also known as *non-uniqueness*. Permeability, porosity or other variables adjustment can be done in two ways, manually or automatically. In a manual method, adjustments are made by the engineer to meet the sufficient condition. In an automatic method, not fully-automatic, it uses an inversion method to do the adjustment by minimizing the gap between simulation output and the field data.

To cope with the *non-uniqueness* of the solution, to make the simulation process simpler, and to get an alternative automatic method, this research tries to implement the *Hybrid Kalman Filter*(HKF) algorithm coupled with reservoir simulation. This HKF algorithm is a combination of the *Ensemble Kalman Filter* (EnKF) and *Reduced-Rank Square Root* (RRSQRRT) *Kalman Filter*.

A test or synthetic model of steam dominated geothermal system was built as the object of this study to be validated using a known Kalman Filter of Hybrid type (HKF). Its ability to estimate rock permeability using pressure data was being studied by comparing the output, both of the test model and reservoir simulation coupled with HKF. There are 3 (three) performance parameters that are being analyzed; the number of ensembles, initial guess and the range of ensemble initialization.

2. METHODOLOGY

Hybrid Kalman filter is one variant of the Kalman filter that is known to be a reliable algorithm to estimate a variable. Kalman filter was found in 1958 by Kalman. From Grewal and Andrews (2008), there are two functions of the Kalman filter :

1. **Estimating a state of a dynamical system.** Almost everything is a dynamic system. The problem is that we cannot always know about the dynamics accurately. Instead of saying “we don’t know”, there is a meaningful and informative parameter i.e. probability. This Kalman filter allows us to estimate the dynamic state by creating a random sample and using a statistical parameter such as probability.
2. **Performance analysis of the estimator system.** The performance parameter is usually estimation accuracy and cost of the system (memory, processor capability, etc).

Kalman filter uses the parametric character from the probability distribution of estimation error to calculate the optimal filtering gain. This probability distribution can be used to test the performance as a function of the design parameter of an estimation system.

Kalman filter continuation begins with a solution for the linear problem called linear filtering. Then it is developed into a solution for the non-linear problem such as *Extended Kalman Filter* (EKF) and *Unscented Kalman Filter* (UKF). And then the development continues by *Ensemble Kalman Filter* (EnKF) and *Reduced Rank Square Root* (RRSQRRT). Recent development is the *Hybrid Kalman Filter* which is the combination between EnKF and RRSQRRT, with some differences. This is a summary from Lewis, et.al. (2006):

Dynamic system and observation modelling

$$\mathbf{x}_{k+1} = \mathbf{M}(\mathbf{x}_k)\mathbf{x}_k + \mathbf{w}_{k+1} \quad (1)$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad (2)$$

Generating Ensemble

$$\widehat{\xi}_k(i) = \widehat{\mathbf{x}}_k + \widehat{\mathbf{S}}_k(1:p)\mathbf{y}(i) \quad (3)$$

with

$$\{\widehat{\xi}_k(i) | 1 \leq i \leq N1\} = E\{\widehat{\mathbf{x}}_k, \widehat{\mathbf{S}}_k(1:p)\}$$

Propagate the ensemble

$$\xi_{k+1}^f = \mathbf{M}(\widehat{\xi}_k(i)) \text{ for } 1 \leq i \leq N1 \quad (4)$$

$$\xi_{k+1}^f = \mathbf{M}(\widehat{\mathbf{x}}_k) + \mathbf{w}_{k+1}(i) \text{ for } 1 \leq i \leq N2$$

Prediction calculation

$$(\mathbf{x}_{k+1}^f, \mathbf{S}_{k+1}^f(1:p)) = E^{-1}\{\xi_{k+1}^f(i) | 1 \leq i \leq N1 + N2\} \quad (5)$$

Data Assimilation

Calculate $\widehat{\mathbf{x}}_{k+1}$ and $\widehat{\mathbf{S}}_{k+1}(1:p)$

$$\mathbf{K} = \mathbf{S}_{k+1}^f(N) \mathbf{H}_{k+1}^T [\mathbf{H}_{k+1} \mathbf{S}_{k+1}^f(N) \mathbf{H}_{k+1}^T + \mathbf{R}_{k+1}]^{-1} \quad (6)$$

$$\begin{aligned} \widehat{\mathbf{x}}_{k+1}(N) &= \frac{1}{N} \sum_{i=1}^N \widehat{\xi}_{k+1}(i) \\ &= \mathbf{x}_{k+1}^f(N) + \mathbf{K} [\bar{\mathbf{z}}_{k+1}(N) - \mathbf{H}_{k+1} \mathbf{x}_{k+1}^f(N)] \end{aligned} \quad (7)$$

$$\begin{aligned}
\widehat{\mathbf{S}}_{k+1}(N) &= \frac{1}{N-1} \sum_{i=1}^N \widehat{\mathbf{e}}_{k+1}(i) [\mathbf{e}_{k+1}^f(i)]^T \\
&= (\mathbf{I} - \mathbf{K}\mathbf{H}_{k+1}) \mathbf{S}_{k+1}^f(N) (\mathbf{I} - \mathbf{K}\mathbf{H}_{k+1})^T + \\
&\quad \mathbf{K}\mathbf{R}_{k+1}(N)\mathbf{K}^T
\end{aligned} \tag{8}$$

3. SYNTHETIC MODEL

In this research, a steam-dominated geothermal system is used as the synthetic model. General model of steam dominated system is characterized by a low immobile water fraction so that the production wells discharges dry steam (Kaya & O'Sullivan, 2010). In the reservoir zone, there is a high temperature together with a low pressure state. It produces from a high permeability region in the reservoir zone (around 10 to 200 mD) and a low permeability around it (10^{-1} to 10^{-2} mD). Steam dominated is a special case where Neumann boundary condition is not appropriate; we have to use Dirichlet boundary condition.

The synthetic model used is a model with sizes of (7500 x 2000 x 3500) with zonation based on its permeability like in the Table 1 below. And the general rock properties are:

- Rock density (kg/m³) : 2500
- Porosity (fraction) : 0.10
- Heat conductivity (W/mC) : 2.5
- Specific heat (J/kg C) : 1000

Table 1. Layer zonation and true material rocks

Layer	Depth range (m)	True Permeability (mD)	
		Horizontal	Vertical
Atmosfer	1000 s/d 0	0,00001	0
Layer 2	0 s/d -150	0,09	0,045
Layer 3	-150 s/d -500	0,06	0,03
Layer 4	-500 s/d -1100	200	100
Layer 5	-1100 s/d -1900	40	20
Layer 6	-1900 s/d -2300	8	4
Layer 7	0 s/d -1100	0,2	0,1
Layer 8	-1100 s/d -1900	0,1	0,05
Layer 9	-1900 s/d -2500	0,01	0,005

To test HKF's ability to improve the estimation while we update the observation data, we set a drilling scenario as below:

1. **First phase.** There are three wells and we run it for five years production
2. **Second phase.** Addition of four wells and simulate it for 7 years
3. **Third phase.** Addition of 4 wells and run it for 7 years.

All phases have rock types and x-axis permeability distribution like presented below:

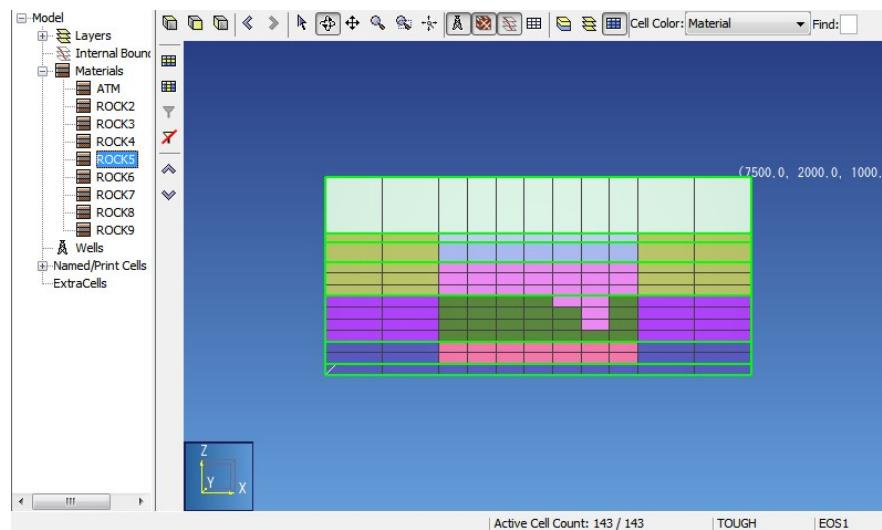


Figure 1.rock types distribution with true data

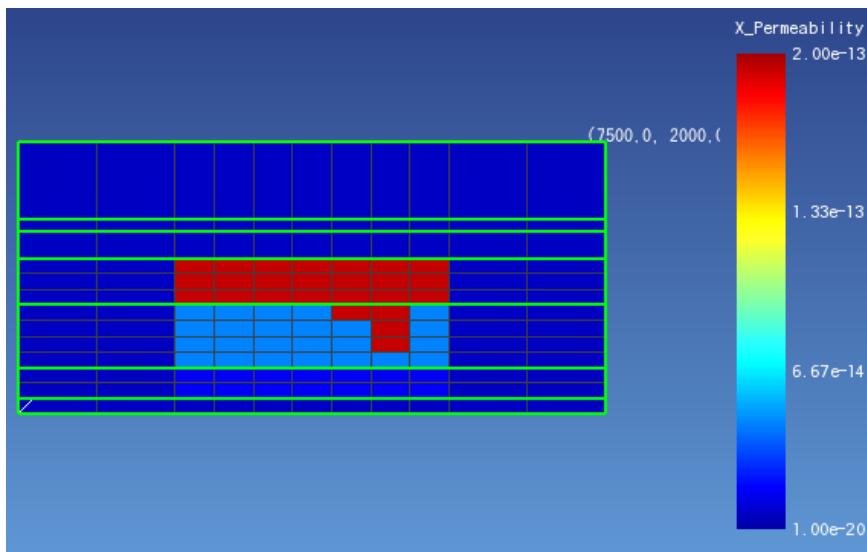


Figure 2.x-axis true permeability distribution

The wells trajectory and properties are:

Table 2. Wells properties and trajectory

	Trajectory	Completion depth (m)	Productivity index (m ³)	Well Flowing Pressure (PWF) (bara)
Sumur 1	(5250, 1000, 0)	-400 s/d -600	1.3E-12	25
	(5250, 1000, -600)			
Sumur 2	(2100, 900, 0)	-300 s/d -500	2.0E-11	25
	(2100, 900, -500)			
Sumur 3	(4250, 1700, 0)	-300 s/d -500	4.0E-13	25
	(4250, 1700, -500)			
Sumur 4	(3200, 500, 0)	-500 s/d -700	1.7E-11	25
	(3200, 500, -700)			

Sumur 5	(3205, 500, 0) (3205, 600, -500) (3205, 700, -700)	-500 s/d -700	1.4E-13	25
Sumur 6	(4200, 1700, 0) (4250, 1500, -500) (4400, 1500, -700)	-500 s/d -700	3.0E-12	24
Sumur 7	(5200, 1000, 0) (5100, 1000, -600) (5000, 1000, -700)	-500 s/d -700	1.9E-11	26
Sumur 8	(2150, 900, 0) (2300, 1100, -500) (2500, 1100, -700)	-500 s/d -700	3.0E-12	25
Sumur 9	(3195, 500, 0) (3000, 600, -500) (2900, 600, -800)	-500 s/d -800	1.0E-11	25
Sumur 10	(4150, 1700, 0) (4150, 1700, -200) (4000, 1500, -500) (4000, 1300, -800)	-600 s/d -800	3.3E-12	25
Sumur 11	(2750, 1600, 0) (2750, 1600, -600)	-400 s/d -600	3.6E-11	25

4. RESULTS AND DISCUSSION

There are two scenarios to test the performance of the HKF, i.e. with some different initial guesses and with some different ensemble numbers, described by Fig. 3.

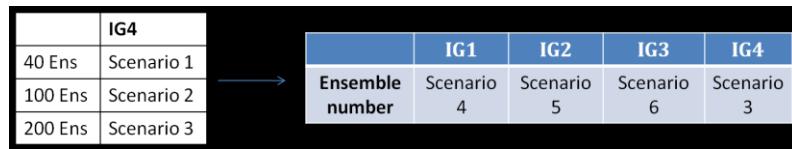


Figure 3. Scenario names and the conditions

And we calculated the RMSE to assess the performance:

$$\sqrt{\frac{1}{a} \sum_{i=1}^a (x_i^t - x_i)^2} \quad (9)$$

The effect of the ensemble number is shown in Fig. 4. By increasing the ensemble number, there was a small improvement, so we conclude that the ensemble number does not affect the performance, but still we try to use the biggest number in scenario with different initial guess. For different initial guesses, the results can be seen in Fig. 5. As can be seen, HKF is able to decrease the RMSE on IG400% and IG_rand, but not on the other two. It can be seen that all scenarios tend to one direction that is to 85 mD RMSE. There are two possible conclusions i.e. HKF was not able to converge to 0 (zero) mD RMSE or there are some characteristics in the IG125% and IG_rand2 scenarios that cannot be convergent. For that, we look at one of the IG_rand's

ensemble to be simulated but with a lower RMSE than before and call it IG_rand3. The results of this can be observed in Fig.6. As can be seen, it still produces a convergent result. So the appropriate conclusion is that there are some current characteristics which made IG125% and IG_rand2 non convergent.

The last step is to test the responses of the model for permeability estimation, i.e. the pressure compared with model respond when the true data applied. When we apply permeability estimation as shown in Fig. 7, this produces the responses in Fig.8. There is still an over-estimation in the permeability data but we obtained sufficient responses in Fig.8. This still shows a non-unique solution. There are some possibilities to cause this non-uniqueness. First, too large spatial dimension, resulting in difficulty for the HKF to do the estimation. And the second is too large range of a generated ensemble. So we tested the narrower range in generating the ensemble with larger wells flow rate. With a narrower range in Table.3 and flow rate in Table.4, we got result of Fig.9 and Fig.10. We got significant changes in the pressure versus time and obtained closer responses.

5. CONCLUSION

In this study, the HKF algorithm was successfully implemented for geothermal reservoir simulation. This algorithm is able to eliminate uncertainty either in the data or numerical approach to get the rock permeability distribution. The initial guess played an important role to the accuracy of the algorithm. Changing the initial guess could result in a RMSE reduction of up to 5 md. The range of ensemble initialization also had a significant effect to HKF's performance. In the first experiment, using a wide value of range of 1 to 400 md for the good permeability layer of the reservoir, it converged between 124 md to 92 md. While in the second experiment, using a narrower range from 150 to 250 mD, it would converge to 16 md.

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NOMENCLATURE

k = time; \mathbf{x} = dynamic system variable; \mathbf{w} = noise system; \mathbf{z} = observation variable; \mathbf{H} = observability matrix; \mathbf{v} = measurement noise; ξ = ensemble; \mathbf{S} = reduced-rank square root from p-rank covariant \mathbf{P} ; E = expectation value of a variable; \mathbf{x}^f = forecasted variable; \mathbf{S}^f = forecasted square root; \mathbf{K} = Kalman Gain; \mathbf{R} = covariant of observation model; N = number of the ensemble

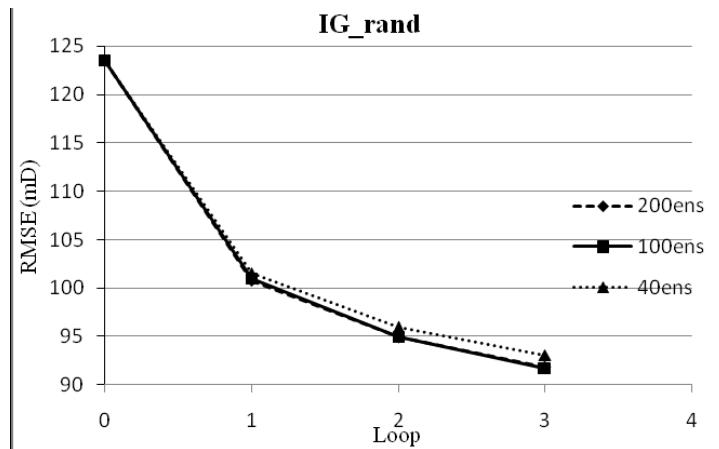


Figure 4. Comparison of RMSE changes with data updating for different ensemble numbers

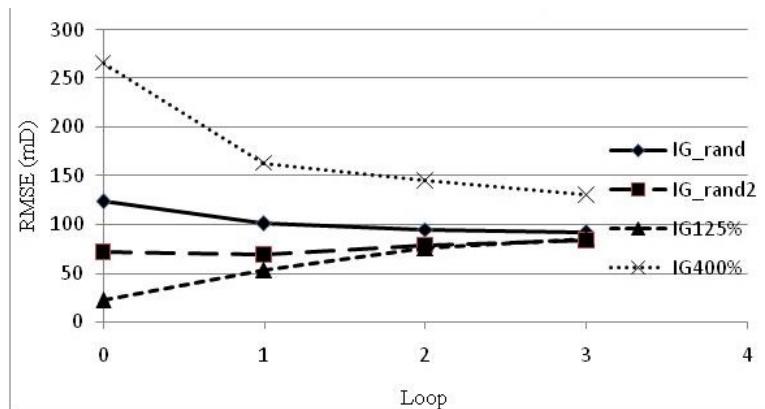


Figure 5. HKF estimation performance for different initial guesses

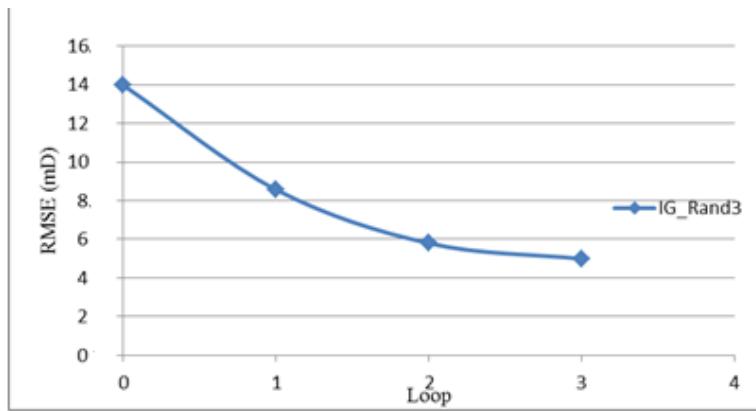


Figure 6. RMSE changes for IG_rand3 with 200 ensemble

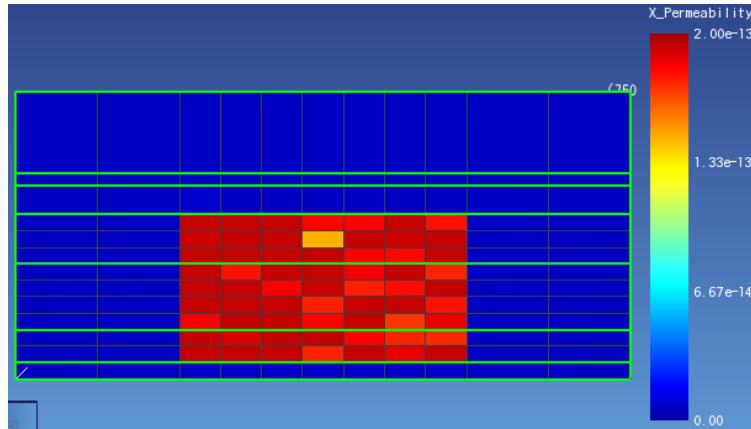


Figure 7. Permeability estimate for IG_Rand

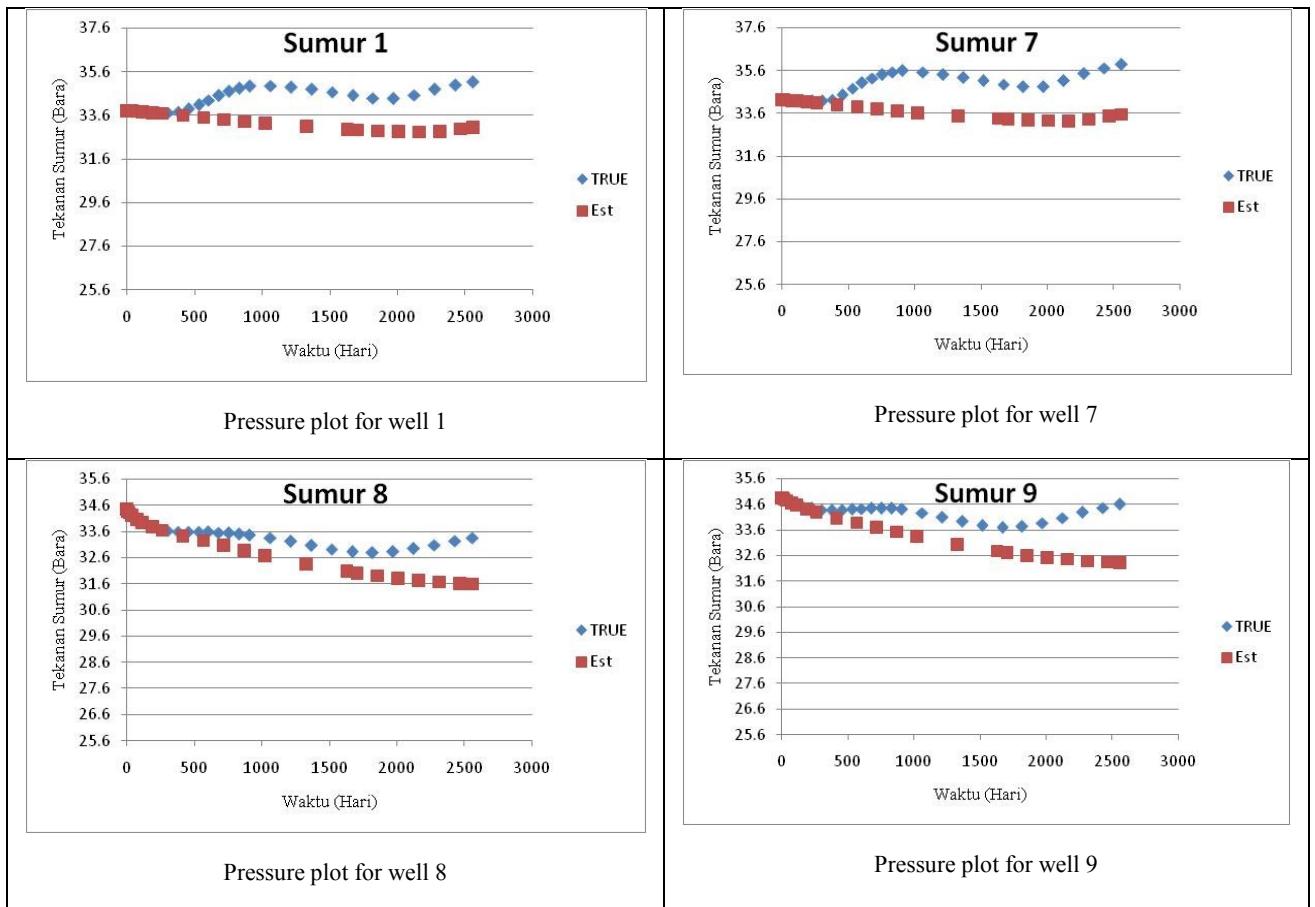
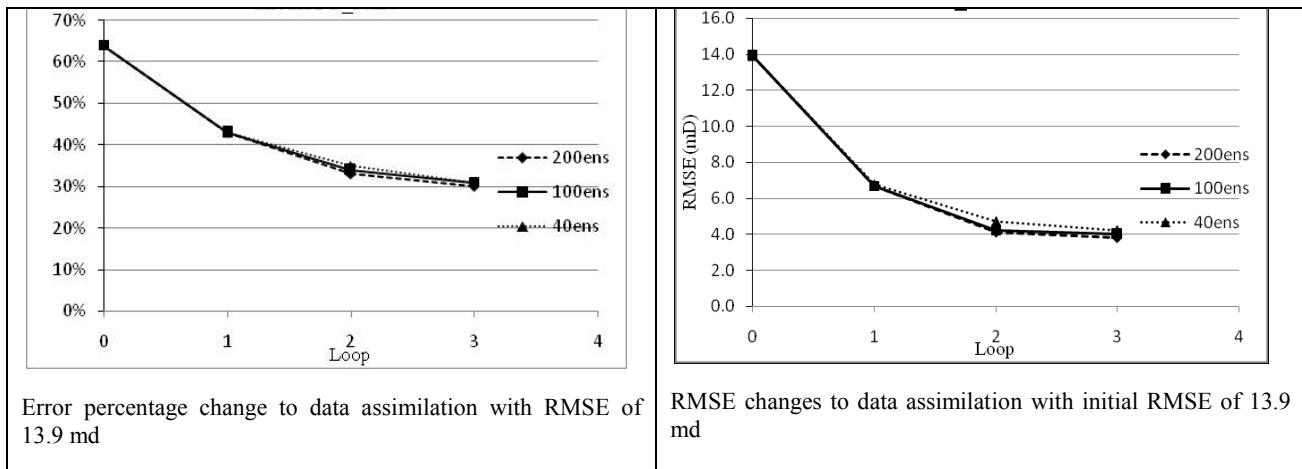
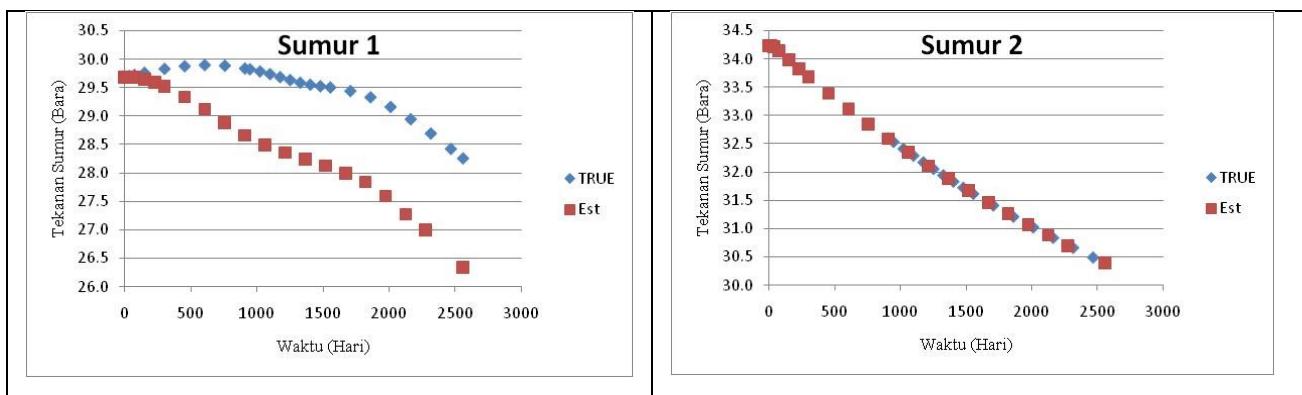


Figure 8. Pressure plot for IG_rand**Table 3. Narrower range generated ensemble**

Layer	Depth range (m)	Random (md)	
		Min	Max
Layer 2	0 s/d -150	0.01	0.1
Layer 3	-150 s/d -500	0.01	0.1
Layer 4	-500 s/d -1100	150	250
Layer 5	-1100 s/d -1900	15	45
Layer 6	-1900 s/d -2300	1	15
Layer 7	0 s/d -1100	0.1	1
Layer 8	-1100 s/d -1900	0.01	0.1
Layer 9	-1900 s/d -2500	0.001	0.01

Table 4. List of the well and mass flow rate changes

Well name	Mass flow rate (kg/s)
Well 1	90
Well 7	60
Well 8	90

**Figure 9. Error percentage and RMSE changes with narrower range of ensemble and flow rate changes**

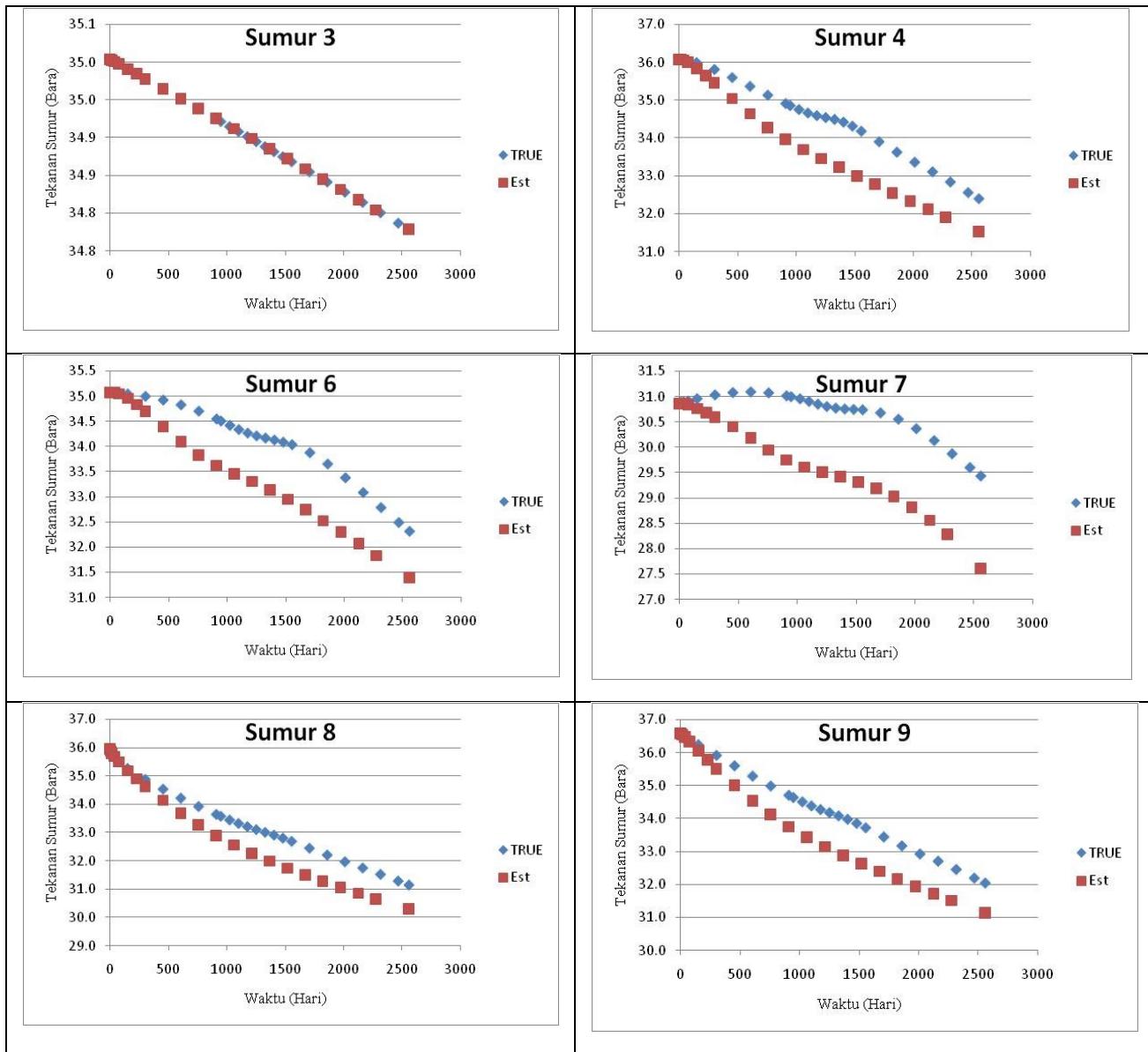


Figure 10. Plot pressure responses with narrower range of ensemble and flow rate changes

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