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Hydraulic Fracture Simulation Based on Coupled Discrete Element Method and Lattice Boltzmann Method

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ABSTRACT

In this paper, we present a fully coupled solid-fluid code based on discrete element method (DEM) and lattice Boltzmann method (LBM). The DEM is used to model the deformation and fracture in solid, while the LBM is used to model the fluid flow. The two methods are two-way coupled, i.e., the solid part provides a moving boundary condition and transfers momentum to fluid, while the fluid exerts a dragging force to the solid. The preliminary 2-D simulations of hydraulic fracturing induced by injection of fluid into a borehole, are carried out to validate the integrated code.

1. INTRODUCTION

The geothermal energy extraction process typically involves drilling deep boreholes into hot rocks and injecting high pressure fluid, causing rocks to fracture or slip along the existing joints. The fracture and slip of these joints create high permeability pathways for fluid flow. The thermal energy trapped in the rocks within this region can then be extracted by circulating water between the injection and production wells through this permeable fracture system. One of the challenges in geothermal energy extraction is how to generate an efficient hydraulic subsurface heat exchange (fracture) system and how to stimulate and sustain the flow of fluid through the geothermal field. Understanding the characteristics of hydraulic fracture, porous flow and heat transfer in fractured rock is an important step to achieve this goal, and numerical simulations based on the fully developed thermo-hydro-mechanical coupling models can provide a powerful approach for systematically and thoroughly investigating these problems. The aim of this paper is to present a fully coulped solid-fluid code and the preliminary simulation of hydraulic fracturing process. The thermal effects and coupling is under developing and is beyond the scope of this paper.

A good hydraulic fracturing model should include mechanical deformation and fracturing propagation of the solid, flow of fluid within the fracture and fluid pressure applied to the solid – a typical two-way solid-fluid coupled problem. There are already some numerical models proposed in this field. For the solid fracturing simulations, most approaches are based on continuum methodology (Zhang et al., 2009; Secchi and Schrefler, 2012; Hunsweck et al., 2013). Although the continuum based models have been shown to be valuable in modelling deformation and plasticity, they are not efficient to capture the full physics of the modeled fractured rocks and evolution of complicated fracture systems which is a consequence of microcropic process. The discrete element method (DEM) naturally overcomes such difficulties since displacements and detachment of solid fragments can be simulated.

When modeling fluid flow, the classical continuum approach is based on the numerical solution of Navier-Stokes (NS) equations. There are also other microscopic and mesoscopic approaches, such as the Molecular Dynamics method (MD), the Lattice Boltzmann Method (LBM), and the Smoothed Particle Hydrodynamics method (SPH). The LBM method is based on kinetic gas theory and simulates fluid flows by tracking the evolution of the single fluid particle distributions. Advantages of LBM over the classical NS approach include its ease in implementation and parallelization, and its ability to handle boundary conditions of complicated geometries (Chen et al, 2003).

In this study, an integrated code coupling DEM and LBM is developed by combining two well developed open source codes: the ESyS_Particle and the OpenLB. The coupled approach is tested by preliminary 2-D simulations of hydraulic fracturing induced by injection of fluid into a borehole.

2. DISCRETE ELEMENT METHOD

The Discrete Element Method (Cundall and Strack, 1979) is based on the concept that the modeled material can be represented as a collection of discrete solid particles interacting with one another at their contacts. At each time step, the calculations performed in DEM alternate between integrating equations of motion with respect to time for each particle and applying the force-displacement law at each contact, through which the contact forces are updated based on the relative motions between two particles and their relevant contact stiffness. One kind of DEM allows particles to be bonded so that tensile forces can be transmitted. Fracturing is represented explicitly as broken bonds, which form and coalesce into macroscopic fractures.

The ESyS_Particle, an open source DEM software, is used in this study. The ESyS_Particle has been utilised in the study of rock fracture and earthquake dynamics (Mora and Place, 1993; 1994). The details of the ESyS_Particle code can be found in open literature (Wang, 2009; Wang and Mora, 2009; Wang and Alonso-Marroquin, 2009; Wang et al., 2012a).

Here an example is given to show the capability of the code in modeling rock fracturing. Fig. 1 presents the results for a simulation of typical progressive borehole breakout development. Two groups of conjugate cracks, dominated by shear fractures, start and intersect at both sides of the borehole surface, and then release crushed grains (Fig. 1, left). With the removal of some particles and continuous loading, stresses concentrate at the corner of the previous fault intersection, inducing several groups of shear fracture bands, and producing another major layer of fractures. Finally at both sides of the wall, a group of shear conjugate fractures develops progressively, forming a "dog eared" shape (Fig. 1, middle and right). Such "dog eared" fracture patterns have been widely observed in laboratory tests and in-situ surveys (Wang et al., 2012b).

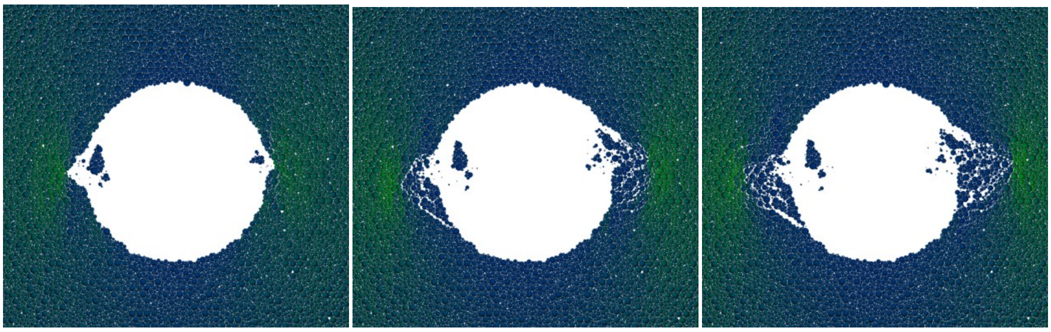

Figure 1: Progressive fracture development under bi-axial compression. Dog-eared shaped breakouts are formed.

3. LATTICE BOLTZMANN METHOD

LBM solves the fluid particle distribution function *f*. The completely discretized equation, with the time step Δt and space step Δx, is given by BGK model (Chen and Doolen, 1998):

$$f_\alpha(\boldsymbol{x}_i + \boldsymbol{e}_\alpha \Delta t, t + \Delta t) - f_\alpha(\boldsymbol{x}_i, t) = -\frac{1}{\tau}\left[f_\alpha(\boldsymbol{x}_i, t) - f_\alpha^{eq}(\boldsymbol{x}_i, t)\right] \quad (1)$$

where τ denotes the lattice relaxation time, $\boldsymbol{e}_\alpha$ is the discrete lattice velocity in direction α, $\boldsymbol{x}_i$ is a point in the discretized physical space, and f_α^{eq} is the equilibrium distribution function . Eq. 1 is usually solved in the following two steps: collision step and streaming step

$$\widetilde{f}_\alpha(\boldsymbol{x}_i, t + \Delta t) - f_\alpha(\boldsymbol{x}_i, t) = -\frac{1}{\tau}\left[f_\alpha(\boldsymbol{x}_i, t) - f_\alpha^{eq}(\boldsymbol{x}_i, t)\right] \quad (2)$$

$$f_\alpha(\boldsymbol{x}_i + \boldsymbol{e}_\alpha \Delta t, t + \Delta t) = \widetilde{f}_\alpha(\boldsymbol{x}_i, t + \Delta t) \quad (3)$$

where $\widetilde{f}_\alpha$ represents the post-collision state. LBM uses the Eulerian approach to describe the motion of fluid particles, therefore the fixed Euler grids are adopted. A square and nine-velocity lattice model (D2Q9) is used in this study.

4. TWO-WAY COUPLING OF DEM AND LBM

In order to couple LBM and DEM, the following issues need to be considered: moving boundary conditions of a curved shape for LBM which handles the momentum transfer from solid particle to fluid and reflection of fluid at the solid boundaries, and force (and momentum) transfer from fluid to solid particles. The curved boundary condition proposed by Yu et al. (Yu et al., 2003) is used here. Due to the arbitrary position and/or curvature of the solid particles, the particle surface can intersect the link between two nodes of LBM at an arbitrary distance with a ratio of

$$\delta = \frac{|\boldsymbol{x}_f - \boldsymbol{x}_w|}{|\boldsymbol{x}_f - \boldsymbol{x}_b|} \in (0,1] \quad (4)$$

where $\boldsymbol{x}_f$ and $\boldsymbol{x}_b$ denote the lattice nodes on the fluid side of the boundary and that on the solid side respectively. $\boldsymbol{x}_w$ denotes the intersection of the wall with lattice link. The reflected distribution function at node $\boldsymbol{x}_f$ can be calculated using an interpolation scheme (Yu et al., 2003)

$$f_{\tilde{\alpha}}(\boldsymbol{x}_f, t + \Delta t) = \frac{1}{1+\delta}\left[(1-\delta)\cdot f_\alpha(\boldsymbol{x}_f, t + \Delta t) + \delta \cdot f_\alpha(\boldsymbol{x}_b, t + \Delta t) + \delta \cdot f_{\tilde{\alpha}}(\boldsymbol{x}_{f2}, t + \Delta t) - 6 w_\alpha \rho_w \boldsymbol{e}_\alpha \cdot \boldsymbol{u}_w / c^2\right] \quad (5)$$

where w_α is the weight factor, ρ_w is the fluid density at node $\boldsymbol{x}_f$, $\boldsymbol{u}_w$ is the velocity of the solid particle, $c = \Delta x / \Delta t$ is the lattice speed and $\boldsymbol{x}_{f2} = \boldsymbol{x}_f - \boldsymbol{e}_\alpha \Delta t$ (Fig. 2). The last term in Eq. 5 represents the momentum transferred from solid particle to fluid. The fluid force acting on the solid particle surface can be obtained using the formula

$$\boldsymbol{f}_F = \sum_{x_b} \sum_{\alpha=1}^{9} \boldsymbol{e}_\alpha \left[f_\alpha(\boldsymbol{x}_b, t) + f_{\tilde{\alpha}}(\boldsymbol{x}_f, t + \Delta t)\right](\Delta x)^2 / \Delta t \quad (6)$$

where the first summation is taken over all solid boundary nodes ($\boldsymbol{x}_b$s) adjacent to the fluid nodes and the second summation is taken over lattice directions pointing from $\boldsymbol{x}_b$ towards all possible neighboring fluid nodes around each solid node $\boldsymbol{x}_b$.

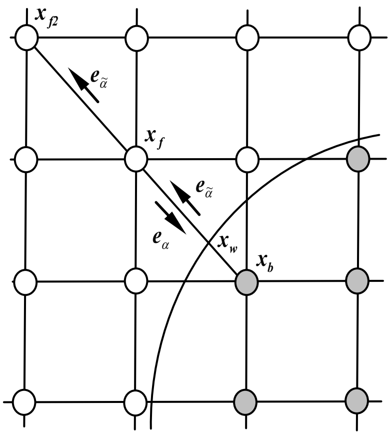

Figure 2: The moving curved wall boundary condition. The curved line represents the boundary wall between a solid particle and the fluid.

5. PRELIMINARY RESULTS

To assess the performance of the coupled DEM-LBM approach, the preliminary hydraulic fracturing simulations are presented here. Figures 3 to 5 show a 2-D simulation of a hydraulic fracturing process. In this test, rock is modeled as 7317 DEM particles with a hole in the middle. Circular particles with sizes ranging from 0.1 to 1 unit are bonded by brittle elastic bonds. The fluid flow is simulated using 1210×1210 LBM grids. The size of the fluid grid is half of the minimum solid particle radius. Four motionless walls provide constant displacement boundary conditions, and the friction between walls and particles is set to zero to minimize influence of the boundary. Constant pressure boundary conditions are set for the LBM fluid. Fluid pressure in the centre of the hole is increased slowly, modeling the injection of water. A pressure increase is realized by adding a source term on the right-hand side of Eq. 1, resulting in

$$f_\alpha(\boldsymbol{x}_i + \boldsymbol{e}_\alpha \Delta t, t + \Delta t) - f_\alpha(\boldsymbol{x}_i, t) = -\frac{1}{\tau}\left[f_\alpha(\boldsymbol{x}_i, t) - f_\alpha^{eq}(\boldsymbol{x}_i, t)\right] + \Delta f_\alpha(\boldsymbol{x}_i, t) \tag{7}$$

where $\Delta f_\alpha(\boldsymbol{x}_i, t)$ in each direction is proportional to $f_\alpha(\boldsymbol{x}_i, t)$ and the increase of fluid pressure in one time step is determined by $\Delta p = c_s^2 \sum_{\alpha=0}^{8} \Delta f_\alpha$, where $c_s = c/\sqrt{3}$ is the speed of sound in this model.

Fig. 3 shows several snapshots of fracture initiation and propagation due to hydraulic loading. Different colors in this figure represent the different magnitudes of solid particle displacement in horizontal direction. The cracks are observed to start from the surface of the borehole, caused by the increasing fluid pressure on the surface of the borehole, and propagate inside the rock with the continuous breakage of bonds and infiltration of pressured fluid. Fig. 4 plots evolution of the micro-fracturing of bonds. The sizes of the event represent fracturing energy released. It is clearly seen that tensile fractures are dominant in the beginning of the crack initiation. The fluid pressure is visualized in Fig. 5. Since there are many LBM grids (1210×1210), these plots can rather be interpreted as the presence and extension of the fluid. In the initial state, fluid only occupies the borehole area. With the occurrence of fractures, fluid is driven by pressure into the cracks, and then exerts further pressure on the crack surface, which results in more fractures generated.

6. DISCUSSIONS AND CONCLUSIONS

This paper presents a two-way coupled DEM-LBM scheme based on two open source codes, the ESyS_Particle and the OpenLB. The ESyS_Particle models solid particle motion, interaction between particles, and fracturing of bonds, while the OpenLB is responsible for fluid motion on the fixed Euler grids. The two-way coupling is achieved through drag forces applied to DEM particles by fluid and momentum transfer from moving solid particles to fluid grids. The basic features of hydraulic fracturing are reproduced in the preliminary example.

The current simulations are still rudimental and un-calibrated against laboratory and in-situ testing, and the influence of input parameters on the fracturing behavior is not investigated. However, these small scaled simulations can be used as a qualitative display of the capability and potential of the coupled approach. The coupled code includes the major mechanisms of hydraulic fracturing process: deformation and fracturing of solids induced by the fluid pressure, flow of fluid within the fracture and progressive fracture propagation. Furthermore, there are no specific assumptions about how fluid flows inside cracks and how a solid breaks, i.e., they are treated in a straightforward way in the new coupled model. This study is the first step towards developing a fully coupled T (thermal) – H (hydraulic) - M (mechanical) code. While the results in this paper demonstrate the promising applications of the new modeling approach, the coupled code requires significant improvements to realize its full potentials. This may includes extension of the coupled code to 3-D, implementation of thermal coupling, large scale simulations and realistic parameter calibration.

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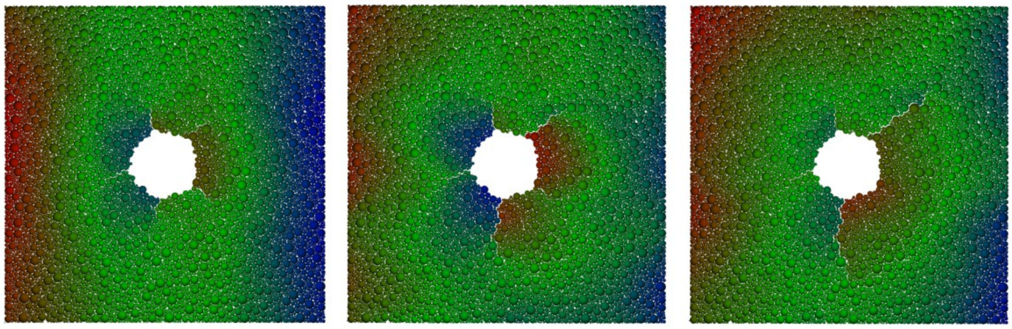

Figure 3 Snapshots of hydraulic fracture. Colors represent solid particle displacement in horizontal direction. Fracture initiation and propagation due to hydraulic loading is clearly seen.

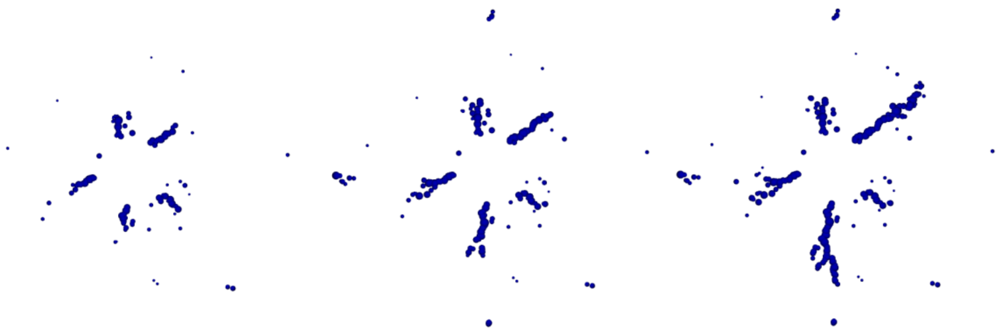

Figure 4 Accumulated fracturing event distribution. Tensile fractures are dominant in the process of the crack initiation and extension.

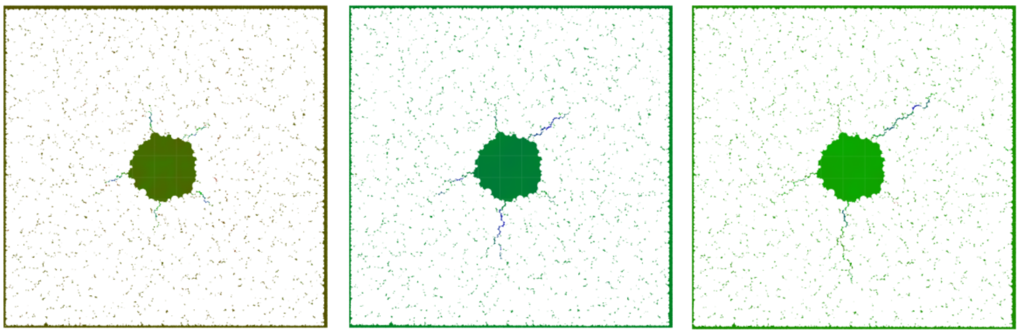

Figure 5 Fluid pressure during hydraulic fracturing. Only those LBM grids covered by fluid are visualized, representing the process of cracking and fluid flowing in the hydraulically generated fracturing tunnels.

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