

Linking Electrical and Hydraulic Conductivity through Models of Random Resistor Networks

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ABSTRACT

One of the requirements of a successful geothermal project is the ability to achieve sufficient flow rates for commercial production. In order to obtain adequate flow rates it is necessary to target areas with either high natural permeability or with potential to enhance the permeability. However, unlike temperature, the relationship between permeability and depth is complex, and difficult to predict from the surface.

We present here models of random resistor networks to relate electrical resistivity to hydraulic permeability in the upper crust. In this approach, the upper crust is modelled as a network of resistors that are either electrically and hydraulically conductive, or resistive. In the preliminary models presented here, we have used fracture diameters of 0.01 and 0.1 mm, and fracture porosities of 0.1 and 1%. The permeability is very sensitive to both of these parameters, but in particular fracture diameter, increasing to up to $4.8 \times 10^{-12} \text{ m}^2$ (from a matrix permeability of 10^{-15} m^2) in the most highly connected case. The resistivity is less sensitive to these properties.

Further modelling will be undertaken to investigate the impact of different parameters on the resistivity and permeability, in particular, fracture length. Additional modelling will also incorporate embedded random resistor networks, in order to replicate the fractal distribution of fractures in the crust.

1. INTRODUCTION

There are two geological requirements for a successful EGS geothermal prospect: elevated temperatures at accessible depths and the ability to achieve sufficient flow rates for commercial production. In order to obtain adequate flow rates it is necessary to target areas with either high natural permeability or with potential to enhance the permeability (e.g., presence of fractures that are favourably oriented for reactivation in the current stress field). However, while in most cases the temperature field varies smoothly, permeability often varies by orders of magnitude over short distances. Therefore, not only is it difficult to predict permeability from the surface, but also even when drillholes are available these may not be adequate to characterise nearby targets.

1.1 Electromagnetic methods

Electromagnetic techniques are used to map the electrical conductivity distribution of the subsurface. In sedimentary rocks, the bulk electrical resistivities can be empirically related to porosity through Archie's Law (Archie, 1942). While Archie's Law holds well in shallow basin settings, it becomes less relevant at greater depths ($>3-4\text{km}$) where the primary porosity is low permeability (and electrical current flow) is primarily in fractures. However, given the requirement for elevated temperature, EGS geothermal exploration needs to occur at these depths. Until now, there has been a lack of models to directly describe the relationship between electrical resistivity and permeability in the crust.

Magnetotelluric (MT) data collected over sedimentary basins commonly show phase splits at longer periods, corresponding to depths of about 2-3 km and deeper (Thiel et al., 2012). One interpretation of these phase splits is electrical anisotropy caused by the presence of fractures filled with an electrically conductive (e.g. saline) fluid. Time lapse MT monitoring of an enhanced geothermal system near Paralana, South Australia was performed in 2011 (Peacock et al., 2013). In this experiment, MT data were collected pre- and post-injection of an electrically conductive fluid into a fault network at 3.6 km depth. Much stronger increases in electrical conductivity were observed parallel to the strike of the fault network than perpendicular to it, implying an increase in electrical (and hydraulic) connectivity. These observations suggest that introduction of fluid filled fractures into a medium changes the 'bulk' or effective electrical conductivity of the medium that is measured by MT. Likewise; the addition of open fractures will increase the effective hydraulic conductivity of a medium. Little work has been done to quantify this relationship and link changes in fracture density and fracture characteristics to effective electrical and hydraulic conductivity.

1.2 Electrical current and fluid flow

Fluid flow through porous media is, at low flow velocities, described by Darcy's Law:

$$Q = -\frac{kA}{\mu} \nabla p \quad (1)$$

Where Q is the volumetric flow rate, k is the permeability, μ is the viscosity, p is the pressure and A is the cross sectional area of the sample. Ohm's law describes electric current flow:

$$I = -\frac{A}{\rho} \nabla v \quad (2)$$

Where I is the current, A is the cross sectional area, ρ is the resistivity and v is the voltage.

Fractures are commonly approximated by the parallel plate model, where either side of the fracture is a smooth plate with separation d and width l_y (i.e., area $A = l_y d$). The steady state solution of the Navier-Stokes equations for laminar fluid flow leads to a cubic dependence of fluid flow on aperture (Brown, 1989):

$$Q = -l_y \frac{d^3}{12\mu} \nabla p \quad (3)$$

Comparison of this equation with Darcy's Law shows that the permeability of the fracture is equal to $d^2/12$. In contrast, the electrical current flow through such a fracture has a linear dependence on d :

$$I = -l_y \frac{d}{\rho_f} \nabla v \quad (4)$$

Where ρ_f is the resistivity of the fluid. As noted by Brown (1989) equations (3) and (4) have a similar form, with the permeability/viscosity being analogous to the electrical conductivity.

2. METHOD

2.1 Random resistor networks

Bahr (1997) proposed the use of random resistor networks to evaluate the bulk electrical conductivity of a medium (Figure 1). In this type of analysis, electrical current flow is assumed to occur through a network of resistors. Resistors within this network can be defined to be "open" (i.e., high electrical conductivity) or "closed" (low conductivity). In a similar way, fluid flow through the same network can be considered in terms of a network of pipes (or in 2D, flat plates) with varying apertures, corresponding to varying hydraulic conductivity. The open resistors can be compared to faults within a host rock filled with an electrically conductive fluid, whilst the resistive parts can be compared to the background host rock and/or faults that are closed or cemented with electrically and hydraulically resistive cement. Importantly, the conductivity is controlled not only by the total number of open bonds, but also on the position of the blocked bonds within the network.

This type of analysis can be performed in a probabilistic sense by considering a suite of different networks, each with the same probability p that any particular bond within the given network is open. By repeating this process at different probabilities of connection, and by modelling the current and fluid flow in different directions, the relationship between bulk electrical conductivity (and resulting electrical anisotropy) and the probability of connection in different directions, can be explored.

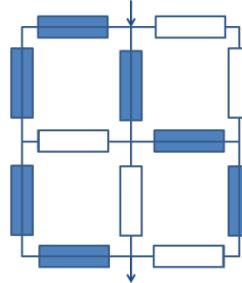


Figure 1. simple 2x2 random resistor network. Blue bonds are connected (i.e., low resistivity) bonds and white bonds are broken (or high resistivity) bonds. Modified after Bahr (1997).

In order to replicate the behaviour of faults, a third variable can be introduced, defined here as the linearity factor. This factor biases the probability of connection of any given bond depending on whether the adjacent bond (in the direction of the bond) is open or closed. The linearity factor is a multiplier and affects the relative probability of connection of a bond in any particular row depending on its position, so that the overall probability for a given network remains unchanged. For example, if a network has a linearity factor of two, the probability of connection of each bond that is adjacent to an open bond would have twice the probability of connection than one that was not adjacent to an open bond. This factor is included to make the networks more fault-like, with longer segments of high conductivity. High linearity factors are associated with longer average fault lengths, and low values are associated with short, segmented faults.

2.2 Modelling approach

We have undertaken 2D random resistor network modelling of simple (i.e., not embedded) networks. We conducted the modelling by first constructing a network of nodes. Bonds between nodes were then randomly assigned either a high or low value of permeability and electrical conductivity, according to a given probability of connection (and linearity factor) in each direction. Permeability and conductivity values were calculated based on fluid and matrix resistivity values, and an average fracture diameter. The electrical current and fluid flow was then modelled in two orthogonal directions across the network. In each model, a voltage and pressure drop of 1.0 was applied across the network. Through equation (2), the bulk resistivity is then equal to the inverse of the average current flow per unit area entering (and exiting) the network. Likewise, the bulk permeability is equal to the viscosity multiplied by the average fluid flow rate through the network.

For a given network, the fracture porosity or total void space occupied by fluid-filled fractures can be estimated using the following equation, assuming the fracture diameter is small compared to the cell size:

$$\phi = \frac{(p_x + p_z)d}{c} \quad (5)$$

Where ϕ is the porosity, p_x and p_z are the probabilities of connection in the x and z directions, d is the fracture diameter and c is the cell size. Alternatively, the porosity can be fixed, and used together with the other variables to calculate the cell size. In the models presented here, we have done this to enable the porosity to remain fixed for different fracture diameters. The modelling process was repeated 1000 times to ensure a representative sample of models. The values reported in the results are the median values.

3. PRELIMINARY RESULTS

The random resistor networks presented in this paper were designed to correspond to fluid and rock resistivities likely to be found at approximately 4 km depth in Australian sedimentary basins. The parameters chosen are based on clean fractured sandstone ($\sim 3\%$ primary porosity and a moderately low primary permeability of 10^{-15} m^2) at about 100 °C with saline fluid filling the pores and fractures. All networks have a similar fracture length distribution, but we have constructed four different models based on two different fracture porosity values, and two different mean fracture diameters. The input parameters, and bulk electrical and hydraulic properties, are presented in tables 1 and 2. The result of modelling of the network with a fracture porosity of 1 % and a mean fracture diameter of 0.1 mm is presented in Figure 2.

Table 1. Summary of fixed parameters for modelling.

Parameter	Unit	Value
Cell size	mm	5
Fluid electrical resistivity	Ωm	0.1
Matrix electrical resistivity	Ωm	100
Matrix permeability	$\times 10^{-15} \text{ m}^2$	10^{-15}
Probability of connection (X)	n/a	0.05
Probability of connection (Z)	n/a	0.45
Linearity factor	n/a	50

Table 2. Summary of variable parameters for modelling, and the results.

Parameter	Unit	Value	Value	Value	Value
Cell size	mm	5	0.5	50	5
Fracture diameter	mm	0.1	0.01	0.1	0.01
Fracture porosity	%	1	1	0.1	0.1
Vertical electrical resistivity	Ωm	11.0	11.0	53.7	53.7
Horizontal electrical resistivity	Ωm	66.3	66.4	92.6	92.6
Electrical resistivity anisotropy	n/a	6.0	6.0	1.7	1.7
Vertical permeability	$\times 10^{-15} \text{ m}^2$	4800	59.6	512	7.4
Horizontal permeability	$\times 10^{-15} \text{ m}^2$	115	3.5	15.2	1.4
Permeability anisotropy	n/a	41	17	33	5

Initial modelling shows that fluid flow tends to focus much more strongly in the longer fractures that connect all or most of the way across the network, while the electrical current is more evenly distributed amongst all fractures (Figure 2). As a result, the fluid flow appears to follow a longer path across the network than the current flow, particularly in the less connected horizontal direction (Figure 2f). Consistent with this, Table 2 shows that for the parameters considered here, the anisotropy in permeability (factor of about 5 to 41) is in general much higher than the electrical resistivity anisotropy (factor of 1.7 to 6.0).

Only small numbers of open fractures are required to increase the permeability by a substantial amount. For example, in the horizontal direction, where the probability of connection is only 5%, the permeability is increased to between $1.4 \times 10^{-15} \text{ m}^2$ and $115 \times 10^{-15} \text{ m}^2$ from the matrix value of 10^{-15} m^2 . In the vertical direction, permeability is increased to $7 \times 10^{-15} \text{ m}^2$ to $4.8 \times 10^{-12} \text{ m}^2$, a factor of up to 4800. In contrast, resistivity is only reduced to 66.4 Ωm to 92.6 Ωm in the horizontal direction and from 11.0 Ωm to 53.7 Ωm in the vertical direction, from a matrix value of 100 Ωm, a factor of up to 9.

Comparison between the different model runs shows that permeability and permeability anisotropy are much more sensitive than resistivity to these parameters. In contrast, resistivity is not sensitive at all to fracture diameter, and while it is sensitive to fracture porosity, it is to a lesser extent than permeability. These observations stem from the relationship in equations (3) and (4) in which fracture resistivity is linearly related to fracture diameter whilst fracture permeability is proportional to diameter squared. As shown by Brown (1989), equations (3) and (4) also imply that rough fractures impact more strongly on the permeability than the electrical conductivity.

CONCLUSION

We have presented here preliminary results of modelling of electrical current and fluid flow through simple 2D random resistor networks. Input parameters have been assigned based on expected values for Australian sedimentary basins. Initial modelling shows that only small numbers of open fractures are needed have a large impact on the bulk permeability. The permeability is much more sensitive than resistivity to both mean fracture diameter and porosity. Resistivity is insensitive to mean fracture diameter (for

a given fracture porosity), but it shows some sensitivity to fracture porosity. Further modelling will be undertaken to more fully explore the relationship between the different input parameters and the bulk electrical and hydraulic properties. In particular, we will look in more detail at the effect of changing the mean fracture aperture and fracture porosity, and will also investigate other parameters such as linearity factor, a proxy for mean fracture length, and the probability of connection each direction.

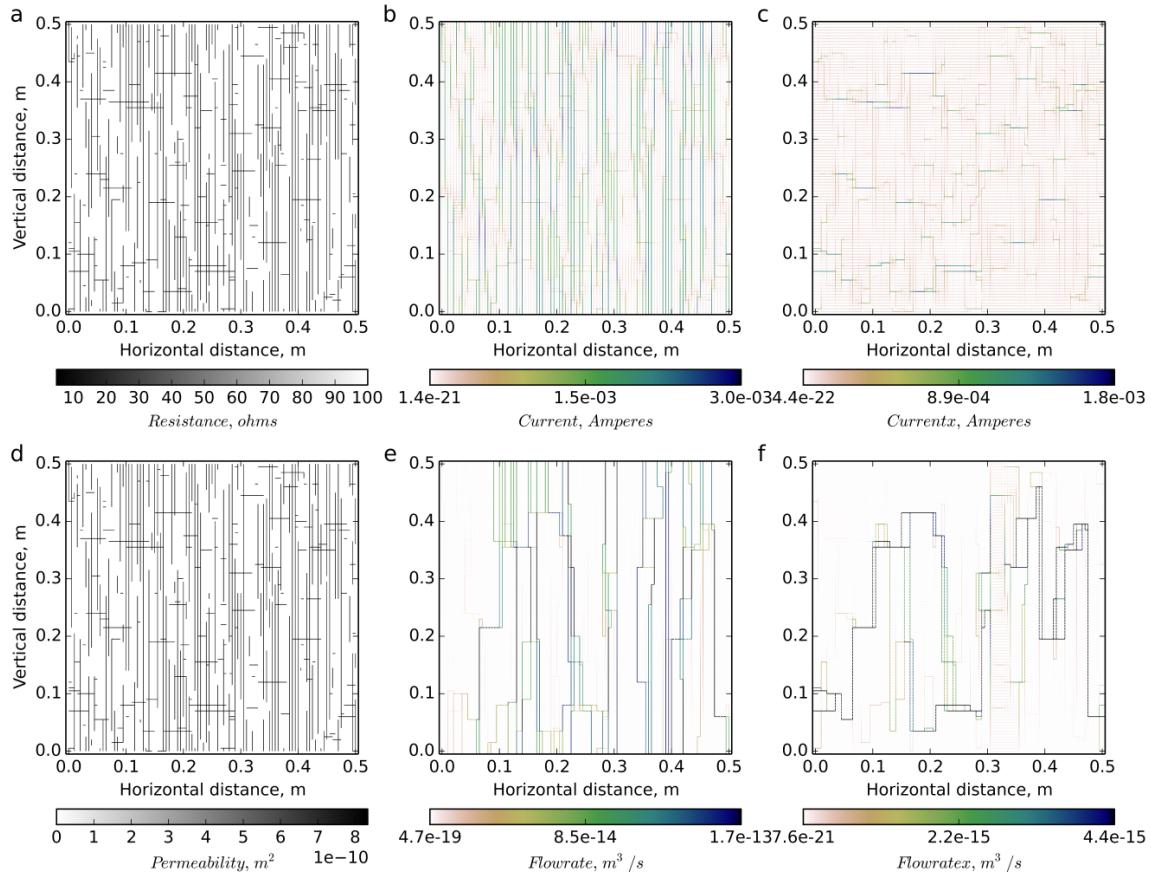


Figure 2: Results of modelling of a two dimensional electrical random resistor network representing an 0.5×0.5 m fractured rock mass with the parameters described in the first column of Table 2. (a) Resistivity, (b) Current flow (vertical direction), (c) Current flow (horizontal direction), (d) permeability, (e) Fluid flow (vertical direction), (f) Fluid flow (horizontal direction). The results (bulk electrical resistivity and permeability) are shown in Table 1.

We will also expand the modelling to 3D, and incorporate more complex networks using fractal embedded networks (Bahr, 1997). With fractal embedded networks, each bond within the network can be considered as either conductive, resistive, or alternatively, embedded. In the case that the bond is embedded, the individual bond is replaced by a network of bonds with its own probability p of connection. The network can be embedded in this way multiple times, and as a result, connectivity is evaluated on smaller and smaller scales. An embedded geometry may be more realistic for modelling fracture networks, which can exist on the scale of several kilometres down to micro-scale cracks. Using embedded networks, a range of scales can be included in a single model at relatively low computational cost. Bahr (1997) noted that it was only when embedded networks were incorporated that the high electrical anisotropy values observed in the upper crust (10-100) could be reproduced.

Modelling the effect of open fractures on resistivity and hydraulic conductivity will allow us to develop an improved understanding of the relationship between electrical resistivities obtained from electromagnetic data (in particular, anisotropy in electrical resistivity) and subsurface fracture characteristics. This will help to improve the geological interpretation of electrical resistivities interpreted from magnetotelluric data, contributing to its utility as an exploration method for EGS geothermal projects.

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