

Mathematical Modelling of Geothermal Fields in the Black Sea

Simeon Kostyanov, Georgi Trapov, Velislav Stoyanov

University of Mining and Geology "St. Ivan Rilski" - Sofia; Bulgaria

simeon44@yahoo.co.uk, trapov@abv.bg, velislavs@yahoo.com

Keywords: Mathematical Modeling, Black Sea, Temperature, Heat Flow, Heat Production

ABSTRACT

A mathematical modelling to the thermal conductivity equation was carried out along three profiles; the Varna-Sukhumi profile and two transverse profiles. The purpose of this paper is a more detailed study of the distribution in depth of the thermal field in the light of the latest geological and geophysical data concerning the age and structure of the sedimentary rocks and the Black Sea basement. Specified seismic and tomographic data about the sedimentary formation and the region basement were obtained and employed in order to precise the results obtained from the previous studies. Calculations were carried out along a geological profile using real properties of sedimentary rocks and basement and they have shown that the regional variation of temperature along the Moho plane varies from 420 to 754°C. The heat flow along the same plane varies from 15-20 to 29-41 mW/m². The part of the heat flow that is caused by radiogenic sources amounts to 17-30 mW/m². The modelling results are presented as sections that illustrate the distribution of temperature and heat flow in depth.

1. INTRODUCTION

Mathematical modelling of geothermal fields is of great significance for the creation of structural models of the Black Sea lithosphere. The main task while modelling the geothermal field is to determine the parameters of the heat source, the thermophysical properties of the rocks, the heat flows and temperatures that best conform to the examined field and to the geological and geophysical data. Solving this problem is a process characterised by a number of peculiarities that are due to the nature of the geothermal field and its anomalies.

The formation of the heat flows at the bottom of the Black Sea is significantly influenced by the following: the radiogenic heat of the crust and the upper mantle; the conditions of heat transfer which depend on the temperature and vary in space and in time; the heat arising from depths that is connected with the global cooling of certain horizons in the tectonosphere. The heat flow measured at the bottom of the Black Sea is a major input parameter in calculating the temperature in depth. Possible flaws in defining this parameter may result in considerable errors in temperature calculations [16, 17].

The information about the heat flow distribution at the bottom of the Black Sea is of primary importance for the studies on the structure of the Earth's crust and the lithosphere evolution. This information, combined with data on the depth distribution of thermal conductivity and heat generation, makes it possible to extrapolate the surface values of the heat flow at greater depths. This is achieved through a numerical solution to the thermal conductivity equation taking into account the corresponding boundary conditions.

A considerable number of geothermal measurements have been carried out in the Black Sea. The heat flow has been studied by many scientists and research workers [1 ÷ 14]. So far, more than 750 measurements of heat flow values have been performed in the Black Sea. This allows a better-grounded approach to the analysis of the thermal field and to the interpretation of its heterogeneity. These data help understand the general principles of heat flow distribution, yet they are not sufficient for the detailed characterisation of the separate structures and zones as they were obtained in stations that are located extremely unevenly and relatively scarcely. The majority of stations are concentrated at the western-most sector of the Black Sea and in the central part of the West basin. The East basin has not been explored in detail (Fig.1).

The numerous experimental data on the heat flow density [4, 5, 6, 9, 10] show that low heat flows are predominant in most of the aquatory of the Black Sea. This fact has been subject to discussion on a number of occasions, yet no satisfactory uniform explanation has been given so far. Thus, in the central parts of the basin, the heat flow does not exceed 20-40 mW/m². Along the periphery of the basin, within the boundaries of the shelf, though, these values increase. The range of variation widens – from 15-20 to 100-150 mW/m². Some scientists associate these high values with processes in the mantle. Others are prone to attribute them to local reasons that are connected with peculiarities in the structure of the crust, or with specific conditions of the heat transfer and heat exchange within the upper parts of the crust. Studies of recent years have confirmed the general principles of heat flow distribution. At the same time, they point out to considerable variations both of heat flows and of other geothermal parameters that reflect the variety of heat transfer conditions as well as the presence of active physicochemical processes in the upper part of the Earth's crust.

Heat flow is influenced by a large number of factors, mostly by the tectonic activity, by the deposition of sediments, by the conditions of heat exchange with the water medium, by the relief at the bottom of the sea, etc. All of these require a special approach to the processing, analysis and interpretation of the heat flow values, as well as maximum attention and consideration of the geological-geophysical information.

This article presents numerical models of the thermal conductivity equation taking into account the respective boundary conditions for Profiles I, II, III (Fig.1, 2, 3). The modelling is used for a more detailed study of the thermal field distribution in depth in the light of the latest geological and geophysical data on the age and structure of the Black Sea sedimentary rocks and basement.

Specified data are employed in the model: seismic, tomographic, and at the boundaries of the sedimentary formations in the region. This has made it possible to precise the results obtained by previous research workers.

2. A BRIEF GEOLOGICAL AND TECTONIC OUTLINE OF THE BLACK SEA

The Black Sea depression is one of the deepest depressions in the Alpine-Mediterranean zone. This intercontinental depression is located between two mountain ridges: the Crimea and the Caucasus Mountains to the north and the Pontides to the south. The origin of the Black Sea depression, as well as of the other basins in the Mediterranean zone, is related to the evolution of the Tethys and the Neotethys Oceans, and above all to the final stage of the evolution of the latter. The Black Sea depression is a flat hollow whose sea bottom stretches at 2 km in depth without any relief variations and which covers two large sedimentary basins: a west and an east one (the thickness of the sedimentary rocks in those two is 16-18 km and 10-12 km respectively – see Fig.1). These two depressions are separated by the linear structure of the Central Black Sea Uplift connecting the Upper Crimean to the Pontides structures. A number of uplifts are observed along the periphery of the depression (Fig.1).

The deep geological structure of the Black Sea depression is characterised by the lack of a granite layer in its central part. In the western part of the basin, sediments have precipitated directly onto the basement. The thickness of the Earth's crust in the central part of the depression is 22-28 km. Below the Central Black Sea Uplift, the thickness of the granite layer increases to 5-7 km and the thickness of the Earth's crust – to 30 km.

The formation of the Black Sea region has been significantly influenced by the fault disturbances among which we could outline the deep faults of mantle origin that protrude far beyond the boundaries of the depression and separate large tectonic blocks of the Earth's crust. Along the periphery of the Black Sea depression, a system of faults is traced that is inextricably bound up with the formation and evolution of the depression. Currently, these faults' activity is still preserved which is confirmed by their seismicity, by the structure of the Earth's crust and the sedimentary depositions, by the shape of the sea bed relief, etc.

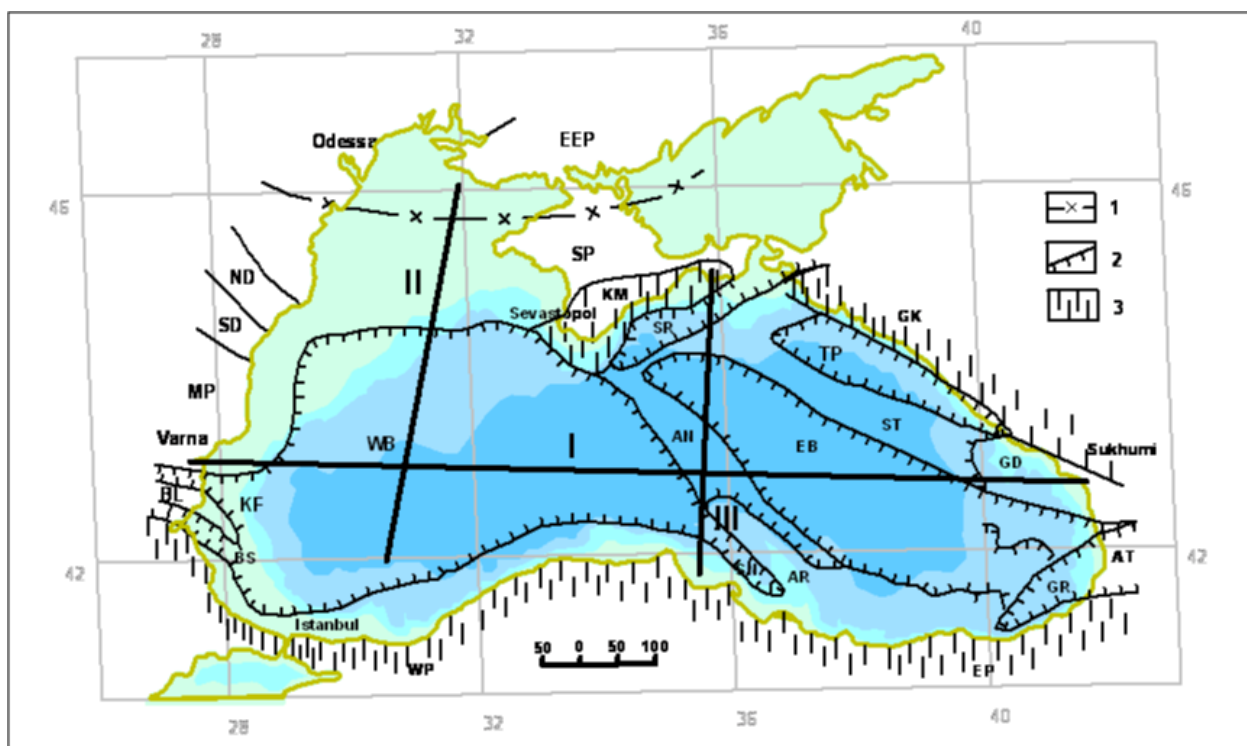


Figure 1: Scheme of the main geological structure of the Black Sea area: 1 = boundary of the East European Platform; 2 = boundary of the West and East Black Sea basins, uplift and trough; 3 = Alpine folded area; WB, EB = West and East basins; AN, AR, ST = Andrusov (Mid-Black Sea), Arkhangelsky and Shatsky ridges, respectively; GD = Gudauta uplift; SR, TR, GR, SN = Sorokin, Tuapse, Curian, Sinoptrouth, respectively; KF = Kemchian fore deep; BS = Burgas depression; EEP = East European Platform; SP, MP = Scythian and Moesian plates; ND, SD = North and South Dobrogea; BL = Balkanides; WP, EP = West and East Pontides; AT = Adzaro-Trialet system; GK = Great Caucasus; KM = Crimea Mountains; A-A' = profile Varna-Sukhumi; I, II, III = Geothermal profiles

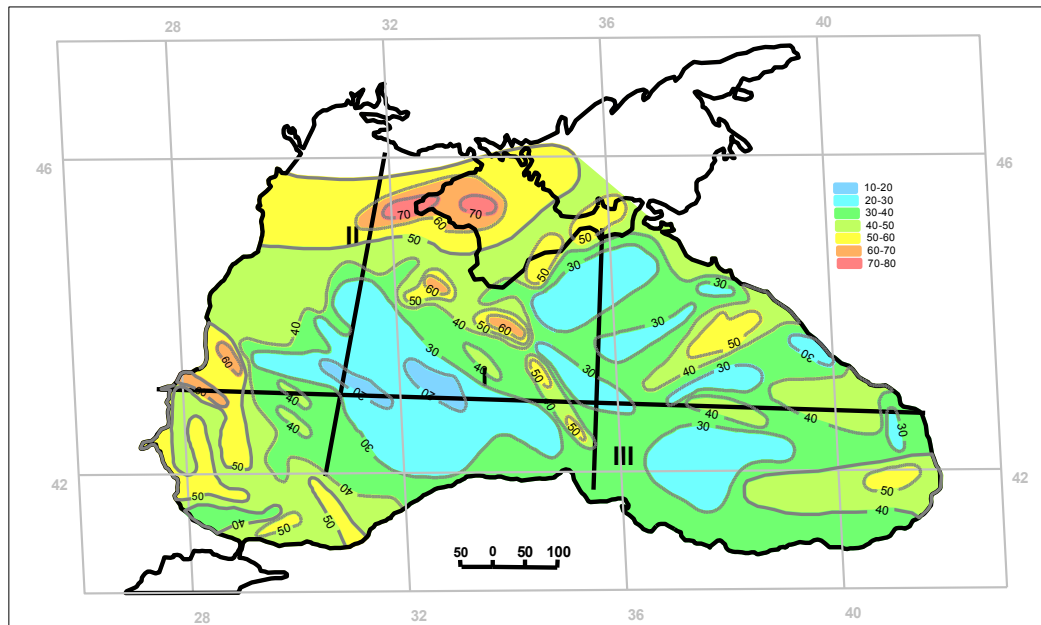


Figure 2: Map of the measured heat flow in seafloor sediments of the Black Sea (in mW/m^2); I, II, III = Geothermal profiles

It should be noted that despite the active studies of the Black Sea depression through geological and geophysical methods, so far there is no uniform picture of the deep structure and the geodynamics of the Black Sea region.

3. CHARACTERISTICS OF THE BLACK SEA GEOTHERMAL FIELD

We are going to present brief characteristics of the geothermal field of the Black Sea basin after some published sources [1-14].

A large number of heat flow measurements have been carried out in the Black Sea – 750 stations. One characteristic feature is that low values of the heat flow predominate ($20\text{--}40 \text{ mW/m}^2$). The lowest values of the heat flow (less than 25 mW/m^2) have been registered in the central parts of the West and East trough where young sediments are with the maximum thickness (Fig.2). Between these troughs there is a zone of increased heat flow values ($40\text{--}60 \text{ mW/m}^2$). This is the zone of the so called Andrusov ridge (Figs.1, 2).

The geothermal field in the West Black Sea trough is slightly differentiated. The low heat flow values cover practically the whole area of the trough. A gradual increase of the heat flow is observed in the central part of the trough ($20\text{--}30 \text{ mW/m}^2$), towards the periphery where it reaches relatively high values in the northern and western shelf ($50\text{--}60 \text{ mW/m}^2$). In the western part of the Black Sea trough, the heat flow varies from 35 to 60 mW/m^2 . This structure correlates to the tectonic background which is a proof not just of the spatial but also of the genetic uniformity of the heat anomalies along the coastal zones of the sea and the land.

The area of increased heat flows along the Andrusov ridge consists of several anomalies of different sizes and intensity. The maximum heat flow values ($50\text{--}60 \text{ mW/m}^2$) are registered in the Upper Crimean faults.

In the eastern part of the Black Sea trough, the geothermal field is better differentiated compared to that in the western part. In this region, as well as along the slopes of the Andrusov and Shatsky ridges, a number of high value anomalies are observed that vary within a wide range: from 50 to 100 mW/m^2 .

The factors influencing the heat flow values are numerous. They can mainly be divided into depth factors and surface factors. Of the latter, the most influential is the deposition of seafloor sediments that, as a rule, manifest low levels of thermal conductivity. Particularly influential are the young anthropogenic sediments that are characterized by a fast sedimentation rate. A lot of attempts, based on analytical solutions, have been made to provide a quantitative assessment of the distorting action of sediments on the Black Sea thermal field [12]. The obtained results are essentially non-uniform since it is not possible to theoretically describe the whole complexity of processes during sedimentation. Therefore Kutas et al. [12] attempt to assess the influence of sedimentation in the Black Sea depression taking into account the experimental data on the heat flow and the actual values of the thickness of sediments, their age, and the rate of deposition. The undisturbed heat flow values range from 40 to 50 mW/m^2 . Kutas et al. [18] present a variant of a distribution pattern for the heat flows in the Black Sea depression that takes into consideration the influence of the anthropogenic sediments. The introduced corrections reflect the influence of “sedimentation”. They alter the structure and level of the heat flow but its characteristic features remain unchanged.

It should be noted that the distortion of the thermal field at the sea bottom may be caused by temperature variations in the seafloor water layer that are connected with climatic changes and/or disturbances in the hydrodynamic conditions. According to Galushkin et al. [3], in order to explain the present-day low heat flow values in the deep-water basins of the Black Sea, the effect of sedimentation should be added to the low heat flow and to the temperature influence of warm and salty waters that penetrated from the Mediterranean about 7000 years ago. Despite the considerable number of studies, however, the issue of the nature of the low heat flow in the two Black Sea basins remains open.

In [14] analyses the influence of various processes on the heat flow value in the Black Sea. A conclusion is drawn that the corrections to the flow caused by the irregular relief of the sea bottom do not exceed 8% and the heat flow added by the vertical motion of water that is due to the compaction of the Pliocene-Quaternary depositions is not more than 1 mW/m^2 , since velocity does not go beyond $3 \times 10^{-12} \text{ m/s}$. The movement of water along faults and weakened zones is of greater importance for the Black Sea basin. This is obvious from the large number of shallow volcanoes, gas vent flows that are generally found along the continental slope. The water and gas outlets form dome-shaped structures on the sea bottom that can be located by means of acoustic methods. The heat flows measured in such structures vary from 0 to more than 100 mW/m^2 .

Heat flow disturbances are generated by a number of depth processes altering the heat flow distribution and the conditions of heat exchange. The physical and geological nature of these processes is not always clear. Only through utilizing maximum geological and geophysical information it will be possible to build realistic models of the energy processes and the thermal state of the Earth's crust and the upper mantle.

4. NUMERICAL MODELS

Kutas, Kobolev, and Tzyvashenko [18] modelled the Black Sea thermal field along the Varna-Sukhumi profile. Expedition temperature data were used over which corrections for "sedimentation" were entered. To produce a geological section, seismic data obtained by 1996 were used. Gobarenko and Egorova [2] employ local seismic tomography to specify the geological section along the profile, as well as along two more profiles that are transverse to the above (Figs.1, 2).

This paper aims at a more detailed study of the thermal field distribution in depth which is based on the latest geological and geophysical data about the age and the structure of the Earth's crust and the upper mantle [2].

a) A general case of conductive heat transfer.

To determine the thermal field in every point of the heterogeneous space, it is necessary to solve an equation of the type

$$\text{div} \left[K(x, y, z, T) \text{grad} T(x, y, z, t) \right] + A(x, y, z) = C(x, y, z) \cdot \rho(x, y, z) \frac{\partial T(x, y, z, t)}{\partial t} \quad (1)$$

with respect to the initial and boundary conditions that describe the known geological and geophysical information [19-25], where T is the temperature, K is the thermal conductivity factor, C is the specific heat capacity factor, ρ is density, A is the distribution of heat sources.

In the course of the modelling, the problem is divided into two sub-problems: stationary and non-stationary. In cases of heterogeneous thermal conductivity and heat generation, as is the case in the Black Sea depression, these two sub-problems are solved through numerical methods.

b) Stationary numerical model.

To obtain the temperature distribution in the Earth's crust and the upper mantle along the Profiles I, II and III (Figs.1, 2), the boundary problem of the stationary equation of thermal conductivity in a heterogeneous medium is numerically solved

$$\text{div} [K(x, y, z, T) \text{grad} T(x, y, z)] + A(x, y, z) = 0 \quad (2)$$

$$T(x, y, z=0) = T_0(x, y) \quad (3)$$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = 0, \text{ at } x=0, x=L, y=0, y=N \quad (4)$$

$$T(x, y, z=M) - \text{unknown}; \quad k \frac{\partial T}{\partial z} \Big|_{z=M} - \text{unknown} \quad (5)$$

where $K(x, y, z, T)$ is thermal conductivity, $A(x, y, z)$ is heat generation, $T(x, y, z)$ is temperature, x, y, z are Cartesian coordinates, $T_0(x, y)$ is a known value – the temperature measured at the sea bottom, L и N – are the length and width of the studied area, M stands for the mantle.

Boundary condition (3) requires that the model temperature at the sea floor correspond to the measured temperature; (4) describes the lack of heat exchange through the lateral surfaces; (5) – i.e. the temperature distribution $T_M(x, y)$ or the heat flow $Q_M(x, y)$ at the lower boundary of the area are unknown.

This reverse problem is not correctly set. It does not allow for a correct setting. Nevertheless, a stable quasi-solution of the problem can be obtained by using the observed heat flow distribution at the sea bottom in the capacity of additional information $Q(x, y, z=0) = Q_0(x, y)$. Besides, it is assumed that the heat flow Q_0 is a sum of two components, Q_R and Q_M , i.e.

$$Q_0 = Q_R + Q_M, \quad (6)$$

where Q_R is the flow generated by the radiogenic heat release within the studied region, and Q_M is the flow from the mantle.

The boundary problem (1-5) for the outlined Profiles is solved with an arbitrary distribution of Q_M . The temperature distribution is obtained and hence we can calculate the heat flow Q_C at $z=0$ (Q_C). We consecutively correct Q_M in conformity with the difference $(Q_O - Q_C)$ and solving the problem again (1-5) we could obtain the function $Q_M(x, y)$ with which the difference between the actual and the calculated heat flow will be within the boundaries of the specified accuracy. In principle, Q_M can be obtained to which corresponds the heat flow at $z=0$ that slightly varies from the actual heat flow Q_O . To perform this procedure, 3 or 4 iterations are required but their number can be reduced should we take the initial distribution of Q_M from the solution of a one-dimensional problem and following the algorithm described by Moysenko and Smislov [15]. The temperature of the Earth's crust is calculated by solving the boundary problem about a stationary one-dimensional thermal conductivity equation:

$$\frac{d}{dz} \left[K(z) \frac{dT}{dz} \right] = -A(z), \quad (7)$$

$$T(0) = 0; \quad K \frac{dT}{dz} = Q_O, \text{ at } z = 0, \quad (8)$$

where T is the temperature at depth z , K is thermal conductivity, A is function of the distribution of radiogenic heat sources, and Q_O is the density of the heat flow measured at the sea bottom.

The formula that we have based our temperature calculations on by means of software of our own reads as follows:

$$T_i = T_{i-1} + \left(\frac{1}{K_{i-1}} \right) \left[Q_{i-1} - A_{i-1} \left(\frac{h}{2} \right) \right] h, \quad (9)$$

where T_i and T_{i-1} are temperatures at the bottom of layers i and $(i-1)$ respectively, K_{i-1} is the thermal conductivity of layer $(i-1)$ at temperature T_{i-1} , Q_{i-1} is the heat flow density of layer $(i-1)$, A_{i-1} is the heat generation in layer $(i-1)$, and h is the layer thickness.

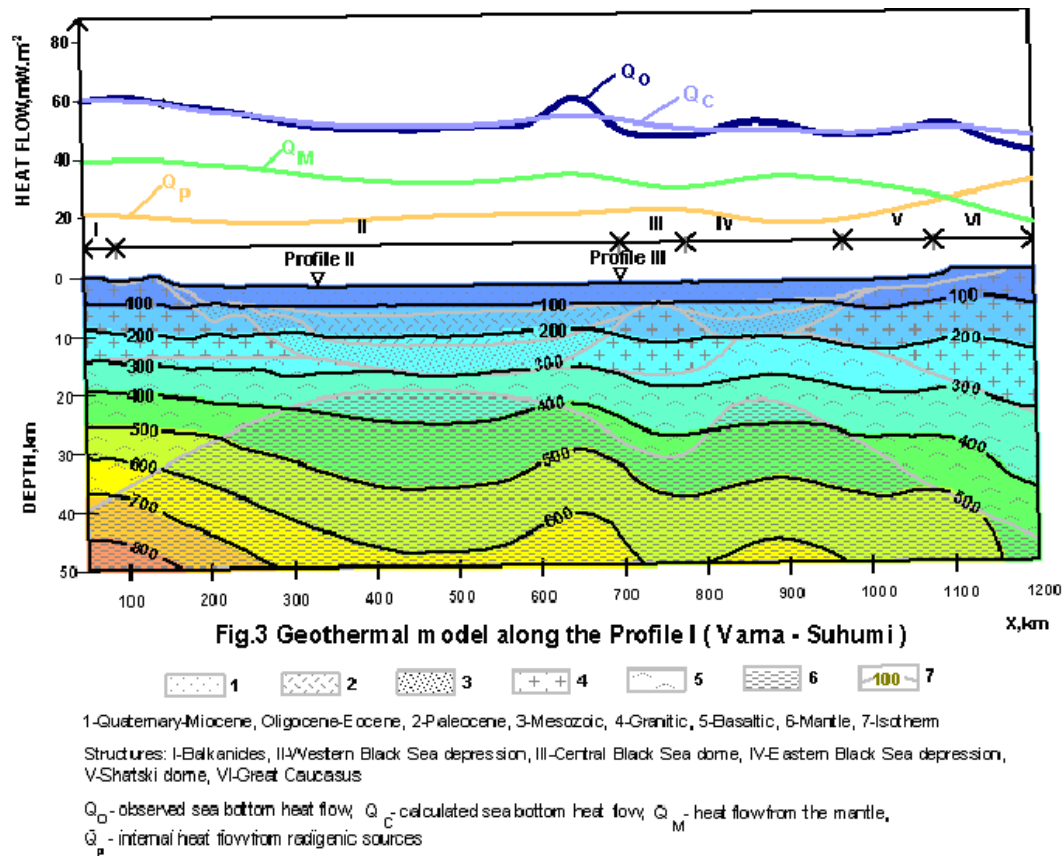
c) Heat conductivity and heat generation.

Among the parameters determining temperature distribution in depth, is the thermal conductivity K . This factor is either set in conformity with the experimental rock data as temperature dependent, or is taken as constant within the layer. Thermal conductivity of non-compacted layers at the sea bottom of the Black Sea ranges from 0,7 to 1,1 W/m.K [18]. Thermal conductivity of the deeper sedimentary layers increases.

Another important parameter is the radioactive heat release factor which is a major source of heat in the Earth's crust. Its value can be measured through the amount of radioactive elements per unit of volume. However, this can be carried out over specific rock samples only. The distribution of radioactive elements in the deeper layers of the crust can only be evaluated indirectly considering the velocities of seismic waves [20]. The numerical values of rock thermal conductivity and heat generation that are employed in the model are given in Table 1.

Table 1: Thermal conductivity K (W/m.K), temperature conductivity a (m²/s) and heat generation A (μW/m³) of the crust in the Black Sea [18].

Layer of the Earth's crust	K (W/m.K)	a (m ² /s)	A (μW/m ³)
Quaternary-Miocene	1,0	6,3	1,3
Oligocene-Eocene	1,7	6,1	1,3
Palaeocene	2,2	7,0	1,2
Mesozoic	2,5	8,0	1,0
Granite layer	2,7	10	1,0
Basalt layer	2,4	7,5	0,3
Upper mantle	3,3	11	0,04



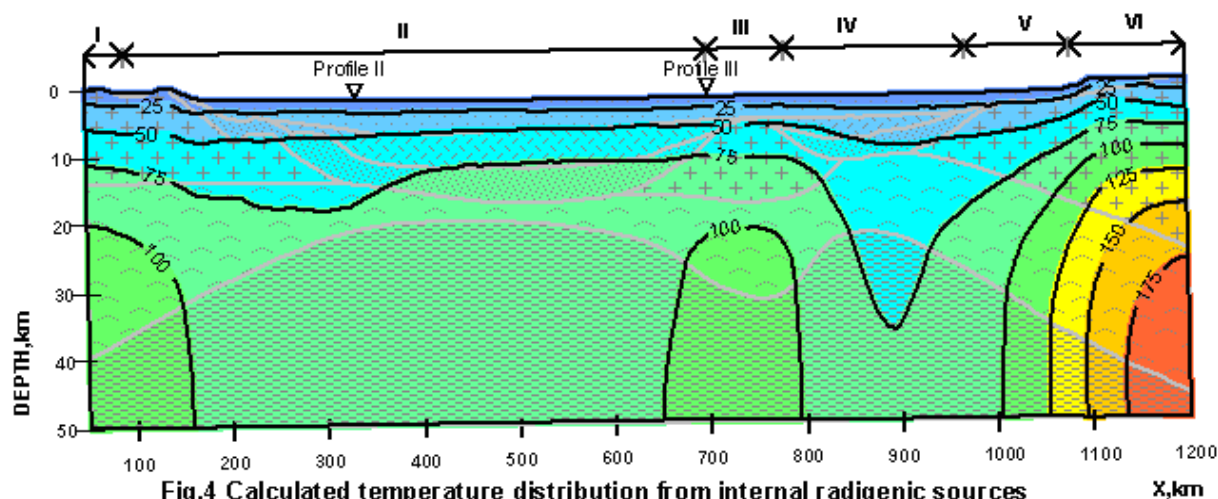
d) Distribution of temperature in depth.

As was pointed out previously, to study temperatures in depth, the inverse problem needs to be solved, i.e. to obtain the heat flow at the lower boundary of the area out of the heat flow at the sea bottom. As heat generation in the mantle is comparatively small, this flow is approximately equal to the heat flow on the Moho surface (Q_m). Usually, we begin at $Q_m = 0$. Further, the values of Q_m are consecutively corrected in conformity with the difference of flows Q_o and Q_c calculated at the surface of the sea bottom. The objective is to get a concurrence between those flows that ranges from 2 to 5 mW/m², for which purpose several extra iterations are necessary. In order to reduce their number, calculations begin at Q_m , pre-calculated, with the one-dimensional equation of thermal conductivity [15]. The criterion is for the horizontal gradients of Q_m и Q_o to be comparable. It has proved to be more appropriate to model the heat flow and temperatures that were produced only by the radiogenic heat sources in the Earth's crust with the absence of a heat flow at the lower boundary. Thus, we determine the heat flow value and the temperature distribution from heat sources that are arbitrarily distributed throughout the Earth's crust which is heterogeneous in terms of thermal conductivity. Then we employ the difference ($Q_o - Q_c$) in place of Q_m .

To solve the boundary problem of the thermal conductivity equation, a calculation scheme was developed based on a regular rectangular grid with a 50-km step along the horizontal axis and a 1-km step along the vertical axis. Calculations were performed employing the software packages Solid Works 2011. The geological cross-sections along the profiles, obtained through seismotomographic research work, are borrowed from Gobarenko and Egorova [2].

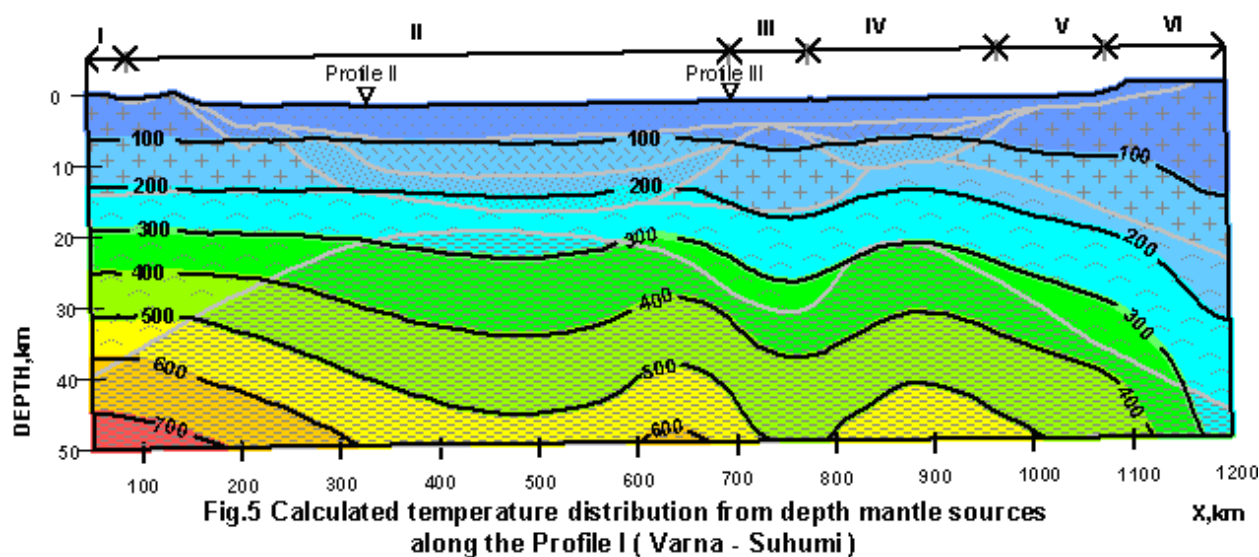
Table 2: Calculated temperatures and heat flows in depth along the Varna-Sukhumi Profile

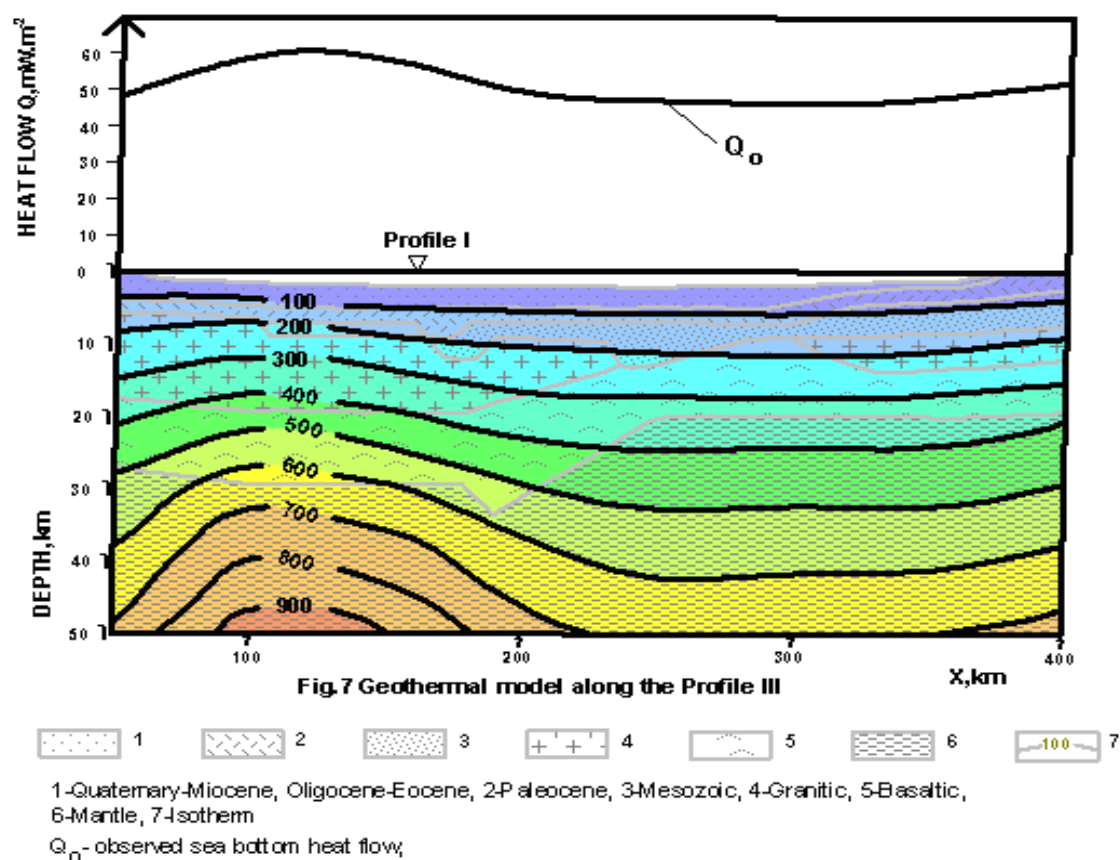
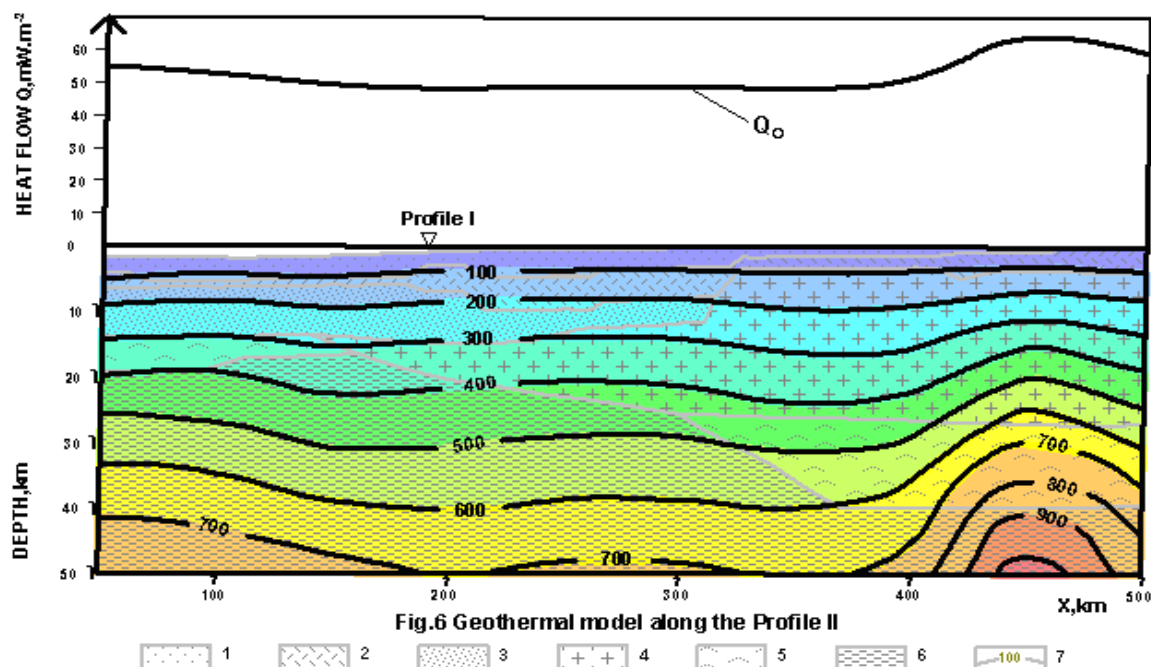
H C		Balkanides	West Black Sea depression	Central Black Sea uplift	East Black Sea depression	Shatski dome	Great Caucasus
10	T	207-218	188-207	173-201	156-178	156-179	154-179
	Q	49.9-51.3	40.7-49.1	37.8-43.0	37.5-38.6	36.8-39.5	35.4-36.8
20	T	396-410	338-388	305-352	298-327	298-303	266-303
	Q	44.1-45.6	31.9-44.0	31.0-35.7	32.6-34.3	27.1-32.6	25.3-27.1
30	T	575-589	436-568	428-483	422-439	412-422	350-412
	Q	41.2-42.5	31.3-41.3	27.6-34.1	29.6-33.0	24.0-29.6	18.6-24.0
40	T	708-754	530-701	515-588	516-543	505-517	421-505
	Q	39.7-40.5	30.8-40.6	26.3-34.4	29.0-32.7	21.3-29.0	15.2-21.3



The Curie isotherm is observed between 25 and 40 km. The highest temperatures are recorded in the west part of the depression, as well as in the area of the Central Black Sea uplift. The high local geothermal anomalies that are noticeable in the Balkanides sector and in the East Black Sea depression are likely to have originated as a result of the rising of the asthenosphere. This issue, however, is not a subject of discussion of the current article.

The results of the depth temperature modelling along the profile I (Varna-Sukhumi) are illustrated in Fig. 3, 4 and 5. The depth temperature along profiles II and III are illustrated in Fig. 6 and 7. For greater clarity, the calculated temperatures and heat flow values in depth are examined according to the geological structures and are presented in Table 2. In the table, H is the depth in kilometres, C is structural units, T is temperature in degrees Celcius ($^{\circ}\text{C}$), and Q is heat flow in mW/m^2 .





CONCLUSIONS

One characteristic feature of the Black Sea depression is the low heat flows measured at the sea bottom. In the western part where the thickness of the Meso-Cenozoic sediments amount to 16-18 km, the heat flows do not exceed 25-40 mW/m². In the eastern part, on the other hand, where the thickness of sediments is 10-12 km, the heat flow range from 40 to 50 mW/m². On the outskirts of the depression, a number of anomaly zones are observed that, in most cases, extend to the continents. This is a proof of the genetic unity of the anomalies along the coastal zones in sea and on land.

Of all geological and geographic factors deforming temperature and heat flows at the bottom of the Black Sea, the ones causing maximum field deformations are the following:

1. The penetration of warm and salty waters from the Mediterranean;
2. The deposition of Neogene-Quaternary sediments. It is proved that the heat flows that have not been disturbed by sedimentation amount to 45-60 mW/m².

Based on the heat field numerical modelling, the components of the heat flow are determined that are caused by the mantle and by the Earth's crust, respectively. Those from the mantle vary from 20 to 40 mW/m², and those from the Earth's crust reach from 17 to 30 mW/m².

ACKNOWLEDGEMENTS

This article is initiated by the fact that between 1st January 2009 and 12th December 2011, Project № 226592, entitled "UP-GRADE BLACK SEA SCIENTIFIC NETWORK", was worked out as part of the Seventh Framework Program (FP7). A team from the University of Mining and Geology, Sofia, took part in the project developing a geothermal database for the Black Sea basin. Part of the data was employed for the modeling of then geothermal field along the Varna-Sukhumi Profile. A catalogue is being prepared that is going to comprise all geothermal data of the Black Sea that are available so far and that amount more than 750 at present.

The authors wish to thank the Project Management for the provided opportunity to work on this problem. The numerical modelling the analysis and interpretation of geothermal data will contribute to the study of the geological evolution of the lithosphere of the Black Sea depression.

REFERENCES

- [1] Vasilev, A. D., V.A. Tzviashenko – Geothermal field in Bulgarian Black Sea. BAS, Sofia, 1990. (Russian)
- [2] Gobarenko, V.S., T.P. Egorova- Structure of the Lithosphere of West and East Black Sea. Physics of the Earth, №6, 2010. (Russian)
- [3] Galushkin U.I., A.A. Shreider – Heat Flow of Lithosphere of Black Sea, Oceanology, №2, 2006. (Russian)
- [4] Duchkov, A. D., C.A. Kazanchev – Heat Flow of the West Part of Black Sea. Geology and Geophysics. №8, 1985. (Russian)
- [5] Zolotarev, V.G., V.M. Kobzar – New Data of the Heat Flow of Black Sea. Oceanology, т. XX, 1980. (Russian)
- [6] Zolotarev, V. G., V. V. Sochelnikov, I. P. Malovitzki – Heat Flow in Black Sea and Mediterranean Sea. Oceanology, 6, 1979. (Russian)
- [7] Kutas R. I., V. A. Tzviashenko, I. N. Korchagin. Modelling of Geothermal Fields of the Lithosphere. Kiev, N. Dumka, 1989. (Russian)
- [8] Kutas R.I., V.P. Kobolev, V.A. Tzviashenko, M.I. Bevzuk, O.P. Kravchuk – Heat Flow in N-W Black Sea. Geophysical Journal, №2, 1999. (Russian)
- [9] Kutas R.I., V.P. Kobolev, M.I. Bevzuk, O.P. Kravchuk – New Data of Heat Flow in N-E Part of Black Sea. Geophysical Journal, №2, 2003. (Russian)
- [10] Kutas, R.I., O.P. Kravchuk, M.I. Bevzuk, L.I. Stahova – Geothermal Investigations in N-Part of Black Sea. Geophysical Journal, №4, 2007. (Russian)
- [11] Kutas, R.I., V.A. Tzviashenko, V.P. Kobolev, O.P. Kravchuk, M.I. Bevzuk – Geothermal Aspects of Gashydrates in Black Sea. Geophysical Journal, №3, 1996. (Russian)
- [12] Kutas, R.I., V.A. Tzviashenko - Influence of Sedimentation on the Heat Flow in Black Sea. Geophysical Journal, №1, 1993. (Russian)
- [13] Kobzor V. M. – Heat Flow and Construction of Lithosphere of Black Sea. Geophysical Journal, №4, 1987. (Russian)
- [14] Kondurin A. V., V. V. Sochelnikov – Heat Flow of the West Part of Black Sea, XXIII, 1983. (Russian)
- [15] Moisenko G. I., A. A. Smislov - The temperature of the Earth, Leningrad, Nedra, 1986. (Russian)
- [16] Chereminski G. P. – Geothermics. L., Nedra, 1972. (Russian)
- [17] Dobrev T., S. Dimovski, S. Kostyanov, V. Stoyanov, G. Alexsiev - Characteristics of the surface heat flow density on the territory of Bulgaria and the neighbouring Black Sea shelf. BAS, Problems of Geography, 1-2, Sofia, 2006.
- [18] Kutas R. I., V. P. Kobolev, V. A. Tzviashenko - Heat flow and geothermal model of the Black Sea depression. Tectonophysics. 291, 1998.
- [19] Kostyanov S. - Mathematical Modeling of Geophysical Fields in Gradient Media (Monograph). Sofia, 200 p. 1999.
- [20] Kostyanov S. - Mathematical Modeling of Some Geothermal Processes in Gradient Media. (Monograph) (in print)
- [21] Kostyanov S. - Geothermal Problems. Methods and Algorithms. Moscow University, 1983.
- [22] Kostyanov S. - Mathematical Models of Geothermal Fields. XXI Intern. Congress of Hydrothermal Technics, 1985, Varna.
- [23] Kostyanov S. - Mathematical Modeling of Geothermal Fields in Boreholes. Intern. Symposium, 1985, Sukhumi.
- [24] Kostyanov S. - Mathematical Models of Stationary Geothermal Fields in Gradient Media. Geothermal Investigations, 1989, Tbilisi.
- [25] Cermak, V., L. Bodri - 2D Temperature Modeling Along Five East European Geotraverses. Journal of Geodynamics, 5, 1986.