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The Doublet System Simulator

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ABSTRACT

A doublet system, consisting of a production well and an injection well, is an established setup for the engineering of groundwater hydraulics. In geothermal applications, fluid is extracted from a near-surface or deep subsurface formation in order to use its high heat content. The cooled fluid is usually re-injected into the same geological formation, which leads to a temperature decrease of in the aquifer. If several doublet systems are installed in the same formation, the systems may become competitive and influence each other. The productivity and lifetime of a single doublet may be drastically reduced should further installations be implemented in the vicinity.

The *Doublet System Simulator* software is designed to simulate and visualize the hydraulic interactions of wells in the described situation. The program is equipped with a graphical user interface (GUI) that allows user-friendly input of parameters and visual output of results. The software is aimed to assist in the planning of facilities before the installation and during operation of doublet systems. The tool can assist decision makers in the comparison of several alternative sites or the technician in finding the optimal positions of wells. The program can also be utilized for demonstrational and educational purposes. It is suitable to investigate and demonstrate the feasibility of a planned doublet system. For example, this software can be used in a first check to determine whether or not conditions at a specific site are appropriate.

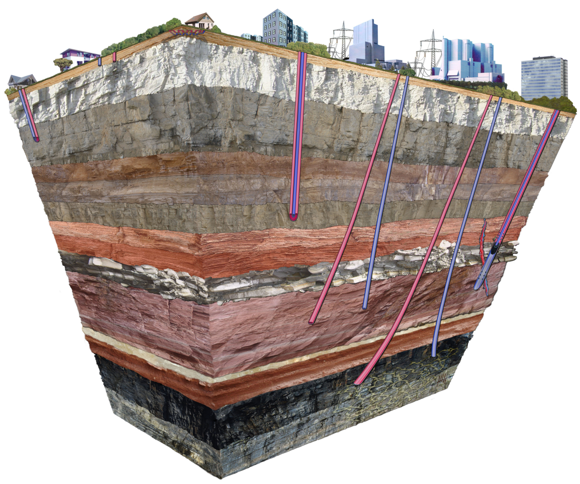

Figure 1: Doublet systems; source: Leibniz Institute for Applied Geophysics (LIAG).

1. INTRODUCTION

Geothermal facilities are increasingly utilized for energy production for heat and power supply. Geothermal energy is sustainable and renewable. A doublet system consisting of a production well and an injection well, is a well established setup which can be utilized for different aims. This setup is illustrated in Figure 1. Aside from geothermal applications, doublet systems are also used for ex-situ aquifer restoration, pit drainage, and mine drainage.

With the increased exploitation of heat stored in subsurface formations, the installed systems become competitive and influence each other. The productivity and lifetime of a single doublet may be drastically reduced if there are further installations in the vicinity. Thus, it is a severe challenge for decision makers and technicians to investigate hydraulic interactions of several systems in the planning phase. The *Doublet System Simulator* software is designed to simulate and visualize the hydraulic interactions of wells in the described situation.

The program is equipped with a graphical user interface (GUI) that allows user-friendly input of parameters and visual output of results. The main envisaged application of the program is as a stand-alone executable. It is based on analytical solutions of the governing differential equations in 2D. The resulting short execution time allows the user to make quick variations and explore the changes of hydraulic behavior easily.

The Doublet System Simulator software is designed to assist in the planning of facilities before installation and during the operation of doublet systems. The tool can serve the decision maker, who compares several alternative sites, or the technician in the field, who aims to find the optimal position of wells. The program can also be utilized for demonstrational and educational purposes. It is suitable as part of a demonstration task to explore the feasibility of a doublet system. Finally, it can be used as a quick check of whether or not conditions at a specific site are appropriate.

Using the Doublet System Simulator requires less skill than using a groundwater modeling program. For that reason, it can be applied not only by groundwater specialists, but also by technicians in the field and decision makers in non-technical departments of specialized companies.

2. ANALYTICAL SOLUTIONS

The mathematical algorithms used in the computations are implemented using analytical solutions based on potential theory. Darcy's Law is given in Equation 1

$$\mathbf{v} = -K \nabla h \qquad (1)$$

and is a well established empirical relationship for groundwater flow (Bear 1976). It states the proportionality between filtration (or Darcy-) velocity $\mathbf{v}$ and the gradient of dynamic pressure. The latter is here represented by hydraulic head h, which is allowed if density changes can be neglected. The proportionality constant is the material parameter K, or the so-called hydraulic conductivity.

Two types of aquifers can be distinguished concerning the upper bound of water bearing pore space. An aquifer is unconfined or phreatic if the pore space is free and confined if it is confined by an impermeable layer. The principle of mass conservation for these two types of aquifers is expressed in Equation 2:

$$\left.\begin{array}{r} \nabla \cdot H\mathbf{v} = 0 \\ \nabla \cdot h\mathbf{v} = 0 \end{array}\right\} \text{for} \left\{\begin{array}{l} \text{confined aquifers} \\ \text{unconfined aquifers} \end{array}\right. \qquad (2)$$

where H denotes the aquifer thickness of the confined aquifer (Strack 1989, Holzbecher 2007), and h denotes hydraulic head measured with respect to the aquifer basis.

Equations 1 and 2 can be combined to yield Equation 3:

$$\left.\begin{array}{r} \nabla \cdot HK\nabla h = 0 \\ \nabla \cdot hK\nabla h = 0 \end{array}\right\} \text{for} \left\{\begin{array}{l} \text{confined aquifers} \\ \text{unconfined aquifers} \end{array}\right. \qquad (3)$$

Aquifer modeling in 2D is usually based on these differential equations (Holzbecher 2002). The differential equation (3) is formulated in terms of hydraulic head. An alternative formulation based on the hydraulic potential φ is given in Equation 4:

$$\varphi = \left\{\begin{array}{c} K \cdot H \cdot h - \tfrac{1}{2} K \cdot H^2 + \varphi_0 \\ \tfrac{1}{2} K \cdot h^2 + \varphi_0 \end{array}\right\} \text{for} \left\{\begin{array}{l} \text{confined aquifers} \\ \text{unconfined aquifers} \end{array}\right. \qquad (4)$$

for which the classical *potential equation* or *Laplace equation* holds Strack 1989, Holzbecher 2007).

$$\nabla^2 \varphi = 0, \quad \text{in 2D: } \frac{\partial^2 \varphi(x,y)}{\partial x^2} + \frac{\partial^2 \varphi(x,y)}{\partial y^2} = 0 \qquad (5)$$

The potential φ has the physical unit of [m^3/s]. Usually, the constant φ_0 has to be chosen appropriately in order to fulfill a point condition for hydraulic head. Equation 4 is also valid for aquifers that are partially confined and partially unconfined. The head is continuous, as the condition for the transition from confined to unconfined conditions is given in Equation 6:

$$\varphi_{crit} = \tfrac{1}{2} KH^2 + \varphi_0 \qquad (6)$$

In the implementation of the program we utilize the complex potential, which is given in Equation 7:

$$\Phi = \varphi + i\psi \qquad (7)$$

with imaginary unit i and streamfunction ψ. This is an extended form of the real potential φ. The stream function is another function with several interesting features and is introduced in this extended formulation. The streamfunction is a classical concept that was exploited vastly even before the advent of computers (Lamb 1963). Concerning the visualization of flow patterns, the advantage of the streamfunction is that streamlines are identical to contour lines of ψ. Streamline patterns are visualized as contour lines of the streamfunction.

In the implementation, we also utilize that the complex potential is represented as a function defined on the complex plane by $z=x+iy$. We identify the model region with a part of the complex plane. Using these formulations, the analytical solutions for the complex potential can be given in a very compact form.

Two types of solutions of the potential Φ are implemented in the Doublet Systems Simulator. The analytic solution for a regional 1D flow field is given in Equation 8:

$$\Phi = \bar{Q}_0 z \qquad (8)$$

where Q_0 is the baseflow discharge vector given by a complex number. Real and imaginary parts represent the vector components of baseflow in the x and y directions, which have units of m^3/s. The overbar denotes a complex conjugate.

The solution for a well with a pumping rate Q_{well} at the position z_{well} is given in Equation 9:

$$\Phi = \frac{Q_{well}}{2\pi} \log\left(z - z_{well}\right) \qquad (9)$$

According to the principle of superposition, solutions for generic situations can be summed up as analytical elements to obtain a solution for a specific situation. In the MATLAB model, we take the baseflow element (8) and add a function of type (9) for each well. A single doublet in a constant flow field is thus given in Equation 10:

$$\Phi = \bar{Q}_0 z + \frac{Q_{well}}{2\pi}\left[\log\left(z - z_{well1}\right) - \log\left(z - z_{well2}\right)\right] \qquad (10)$$

3. THE GRAPHICAL USER INTERFACE

The software is implemented in MATLAB and can be used directly from the command window. The graphical user interface is divided into several panels, with the 'Aquifer', 'Well(s)', and 'Graphical Output' panels being the most important. The 'Well(s)' and 'Aquifer' input panels are shown in Figures 2 and 3, respectively.

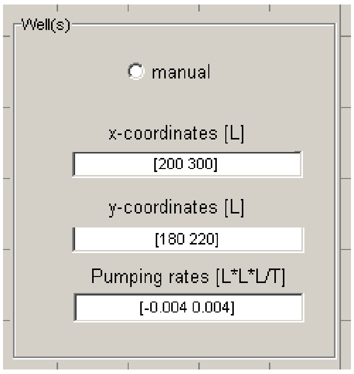

Figure 2: GUI panel for input of well positions and pumping/recharge rates.

There are two possibilities when setting the well field. Usually, the user will give the coordinates (x- and y-) and the pumping rates in the corresponding input fields of the GUI. Another option is manual set-up, in which the user can determine well positions by mouse-click. After the last position has been entered, an input field appears in which pumping rates must be specified. The manual option is similar to the one implemented in the Well Designer program (Holzbecher 2009, to appear).

Several options concerning the graphical output can be specified. There are options to visualize filled potential contours and streamlines. Various color maps can be chosen for the fill pattern. The user may also select line colors for the contour lines and streamlines. Line color 'none' can be used to disable line printing.

There are two input values for grid-spacing, and the user may alter grid-spacing in the *x*- and *y*-directions. Note that when using analytical formulas, the grid is used for visualization only, in contrast to numerical methods. Decreasing the grid-spacing results in smoother curves. If a grid is too coarse, streamlines and contour lines may show corners.

Finally, there is an input option to obtain the graphical output in a separate MATLAB window. When this is done, the user has the option to perform several post-processing fine-tunings of the graphical output. Lines can be re-colored in parts, if desired. Legends can be added, and the figure can be saved either in MATLAB or in bitmap formats.

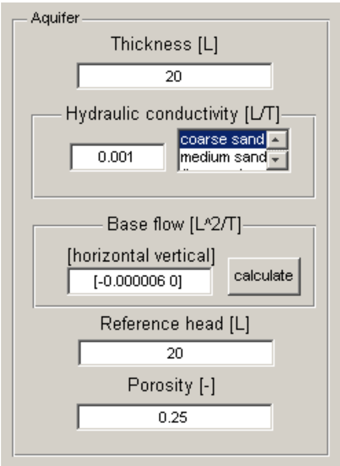

Figure 3: GUI panel for the input of aquifer data.

After the specification of input data, computations are initiated by clicking the 'Plot' button. The variables of the flow field are calculated at the grid points and visualized at the graphic panel.

4. THE CONTOURING PROBLEM

The contour lines of the streamfunction are called streamlines. Thus, streamlines can be displayed easily by contouring after the calculation of the complex potential, which includes the stream function (shown in Equation 7). An example for a single well is shown in Figure 4.

However, there is one difficulty in connection with the well solution (shown in Equation 9). The complex logarithm is not unique: it is a multi-valued complex function. The complex logarithm takes multiple (in fact, infinite) values at each location. Actually, 'log' denotes the principle value of the multi-valued function. It is only unique in *simply connected* regions (Howie 2004). More specifically, it is the stream function part of the complex potential, including the $\arctan(\mathrm{Im}(z)/\mathrm{Re}(z))$ term, which causes difficulties.

In the example with a single sink, the region has one singular point at the sink position and can be connected simply by a *cut* from the singular point to the boundary. In the mathematical literature, the technical term for this is *branch cut*. The branch cut makes the complex logarithm unique, but at the cut itself, the function shows a jump given by Equation 11:

$$\Delta\psi = Q_{well} \qquad (11)$$

With the use of the implemented arctan function (in MATLAB: atan2), the branch cut appears as a ray from the well position to the left.

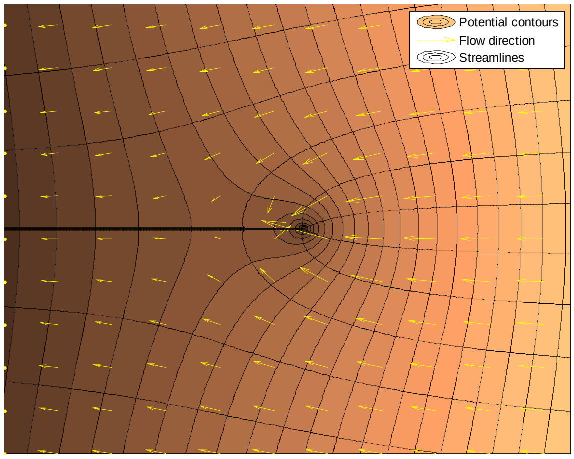

Figure 4: Single well in regional flow field (x-direction).

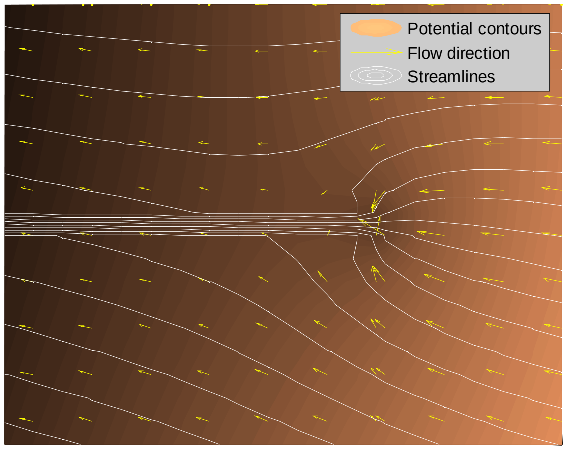

Figure 5: Single well in regional flow field; streamline visualization with simple contouring for streamfunction.

As the stream function is not continuous at the cut, the branch cut becomes visible in the contouring picture in case of crossing streamlines as shown in Figure 5.

In the left part of the figure, the streamlines obviously do not continue along the flowpaths. While the flow pattern is correct in the upper and lower parts, there are problems when the real axis is approached. The flowpaths make a sharp turn and connect to the left boundary in a horizontal line.

The ends of the streamlines from both sides should be connected. In order to achieve this, the streamfunction and the levels for contour lines have to be modified within the contouring procedure. The details of the algorithm that produces streamlines without visible branch cuts and represents the flow pattern in the entire domain is described elsewhere (Holzbecher 2009, submitted). The extended version of the contouring algorithm is used in the Doublet System Simulator. Thus, the figures for the application cases below are without branch cuts.

5. STAGNATION POINTS AND SEPARATRICES

For the situation of a single doublet in a baseflow field, the two stagnations points are given by Equation 12:

$$z_{1,2}^2 = x_0^2 + \frac{Qx_0}{Q_0\pi} \qquad (12)$$

Limit streamlines or separatrices can be obtained by using the contouring algorithm with the stagnation point as the starting position. Although this works numerically, the velocity is zero at the stagnation point. The limit streamlines provide a very good visualization of the flow topology. For example, the catchment of production wells and the zones of influence of injection wells can be identified directly. Note that the extended contouring procedure described in the previous subchapter has to be utilized for this visualization as well.

6. APPLICATIONS

6.1 Doublet and Baseflow Direction

DaCosta and Bennett (1960) explored the situation of a single doublet in regional groundwater flow fields that meet the doublet at different angles in relation to the connecting axis of the two wells. They showed that the hydraulic connection between the two wells depends significantly on the direction. This is visually explored here in Figure 6.

For this computation, we used dimensionless length and volumetric flux scales. Unit length x_0 is half of the well distance and unit flux is the product of baseflow Q_0 and x_0. In all figures of the series, the production well is located on the right, and the injection well is on the left. We show filled contours for the head values, the streamlines (in white colour), stagnation points and limit streamlines (in red color).

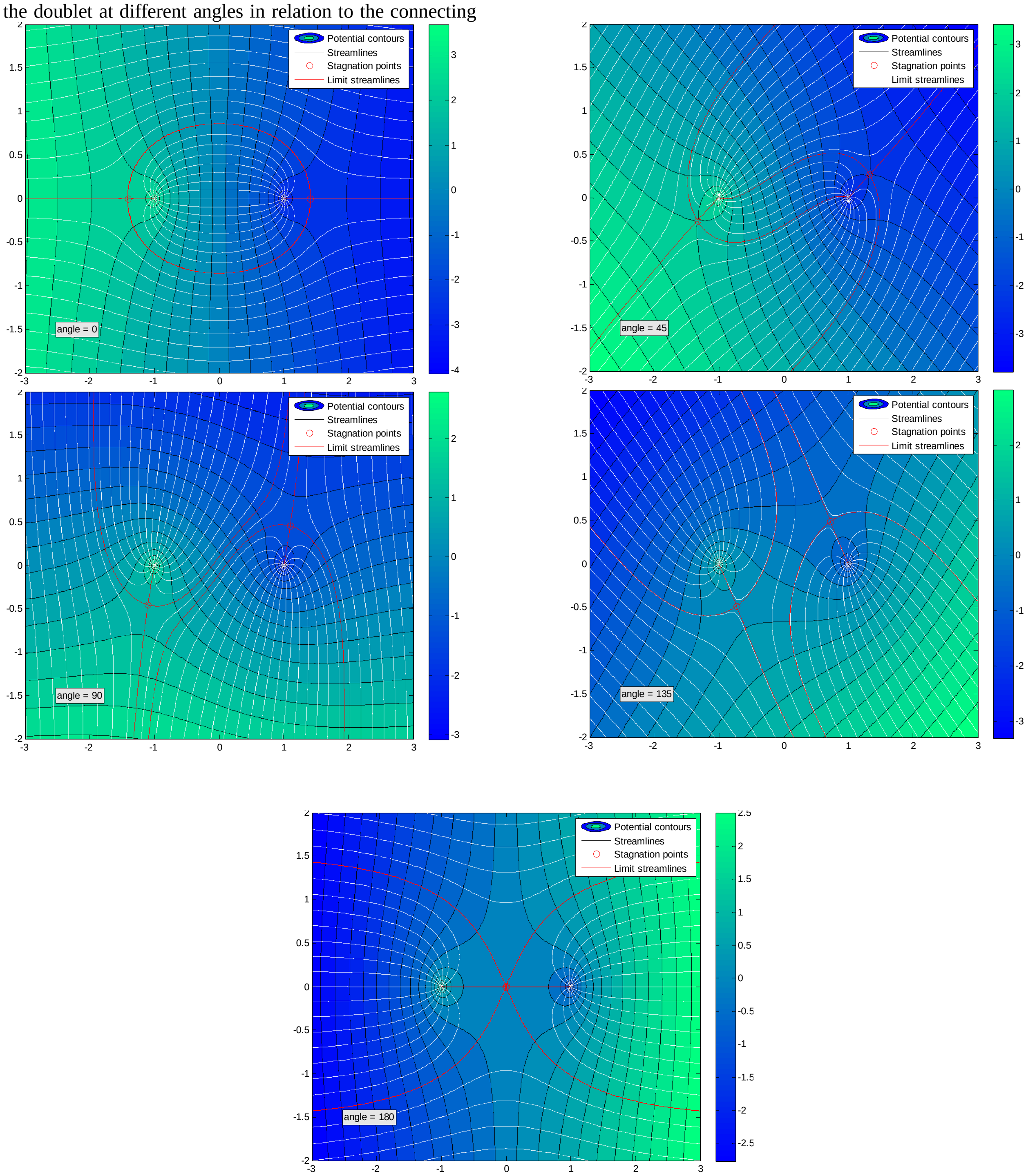

Figure 6: Flow pattern for a single doublet in base flow fields of different angles (0°, 45°, 90°, 135°, 180°).

The first picture of the series shows the situation for baseflow from the left. The production well is thus located downstream, and the injection well is upstream. One can clearly identify the two separate flow regimes: those of the production well and the injection well. The baseflow fluid is not involved in this circulation and flows around the doublet flow as if around an obstacle. Indeed, this solution is often used for the simulation of flow around circular obstacles (Strack 1989).

The second figure shows the flow field for baseflow, approaching with an angle of 45° (i.e. arriving from the bottom left corner). It can clearly be seen that there is a partial connection between the wells, but there is also a partial interaction with the baseflow for both wells. Under steady state conditions, the ratio of water pumped from the baseflow to water pumped from the injection well is approximately 6/10, which can also be ontained from streamline counting.

In the third figure, the situation with baseflow arriving from a 90° angle is depicted (i.e. regional flow is from bottom to top). It can clearly be observed that the wells are no longer connected. Pumped water is from baseflow only. A single streamline is visible, which passes between the catchment of the production well and the region influenced by the injection well. The gap between these two regions increases with the baseflow angle, as shown in the next figure of the series. Finally, for baseflow from right to left, we observe separated production and injection regimes, which have a contact in a single stagnation point.

6.2 Two Doublets

For the case of two doublets the influence of the distance between the doublets is demonstrated in Figure 7. All production and injection wells work at the same source and sink rate. There is no regional groundwater flow considered in this case.

The situation depicted in the first figure is (point- and mirror-) symmetric. The distance between all wells is the same. Thus, it is clear that the same amount of recharged water reaches each of the two production wells. This is confirmed by the symmetric flow pattern, as shown.

In the second figure, the distance between the doublets is double the distance between the wells of the single doublets. The resulting flow is still symmetrical. The ratio of pumped water from inside the doublet to that from outside can be read from the figure by counting streamlines and is approximately 11:4.

In the third figure, the distance between the doublets is doubled again. The ratio of pumped water from the same doublet to the water pumped from the distant doublet is approximately 13:2, as can be read from the figure.

6.3 Three Doublets

The flow field for an irregular constellation of three doublets in a baseflow field from right to left is shown in Figure 8.

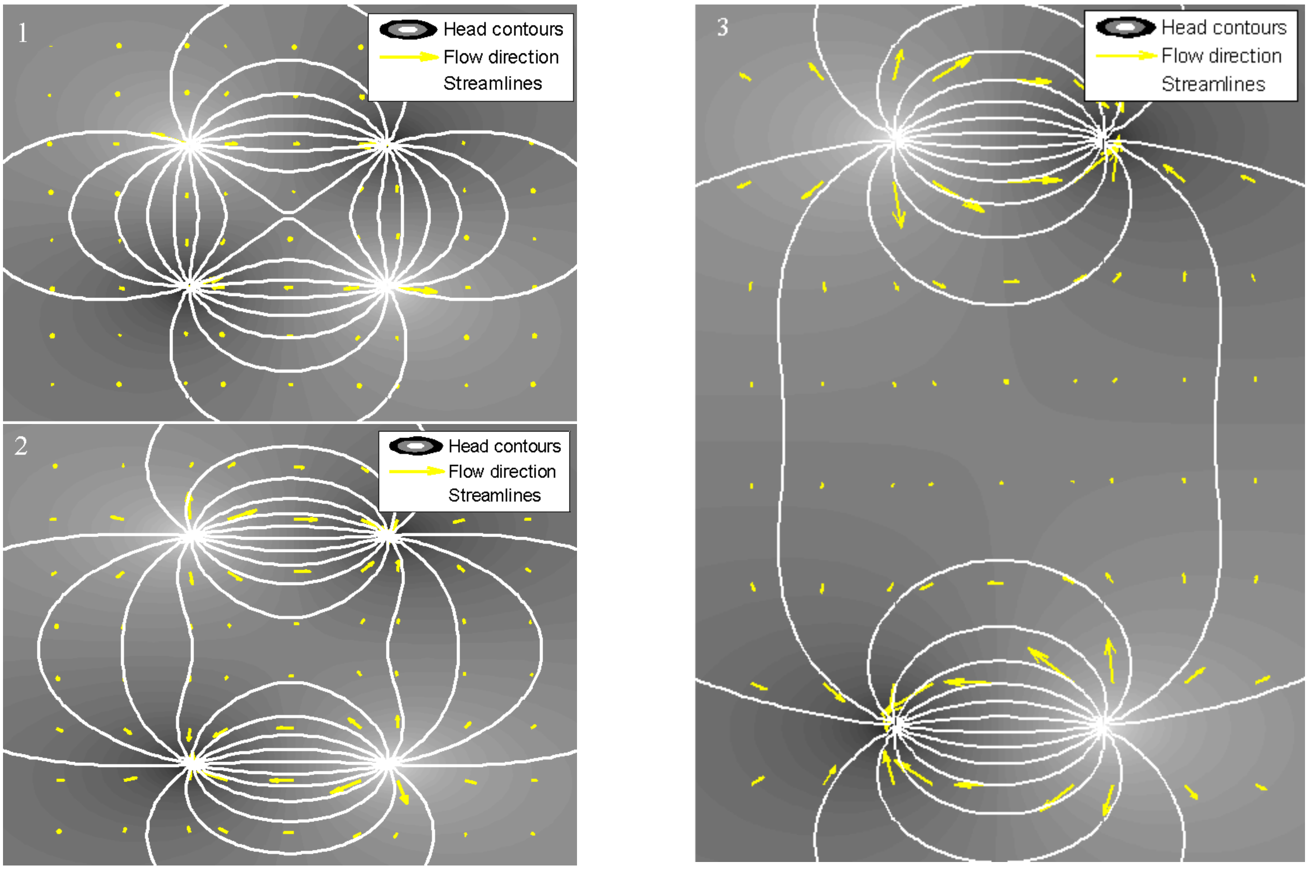

Figure 7: Flow pattern for two doublets in different distances.

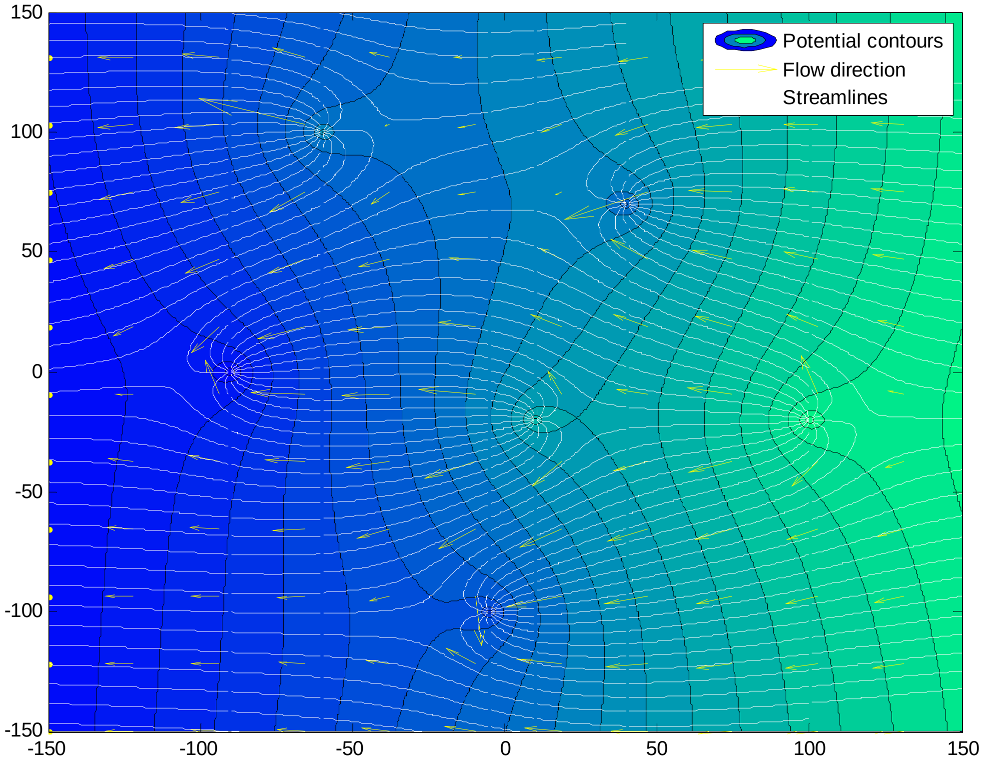

Figure 8: Flow pattern for an irregular setting with three doublets (filled contours for piezometric heads, streamlines and arrows, representing the velocity field).

7. CONCLUSIONS AND OUTLOOK

The Doublet System Designer is software that calculates and visualizes the flow field resulting from one or several doublets in a 1D baseflow field. Visualization includes the following options:

- filled contour map of piezometric heads for confined, unconfined and partially confined aquifers
- streamlines
- traveltime markers
- arrow fields, representing velocities
- stagnation points
- limit streamlines

At current state (May 2009), the software in still under development. Not all the mentioned options are implemented for the general state yet.

From visual output, it is easy to determine if there is a connection between an injection well and a production well. In case of connection, the ‘degree’ of connection can be evaluated graphically for a single doublet and other various doublet systems. This is an important feature, because in geothermal applications of doublets, the aim is a minimal connection between production and injection well. For a given flow field, the location of wells can be optimized already in the planning phase of another installation.

Also, the travel time can be determined using the Doublet Systems Simulator. This is an important feature, because for connected wells, the travel times indicate the lifetime of the doublet.

There are still features to be implemented and extensions to be made on the Doublet Systems Designer:

- coupling with a heat and mass transport model
- extension for unsteady state flow
- implementation in C/C++

These tasks are of various degrees of difficulty, and their implementation is still under discussion.

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