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Development of Updating Reservoir Simulation Model

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ABSTRACT

History matching is a process of incorporating the observed data into the reservoir simulation models. The important properties of the reservoir are submitted to a reservoir simulator and the objective of the model is to reproduce the observed data. In the past, history matching has been implemented manually by interference of the reservoir expert. This approach is a time consuming task, and many automatic history matching methodologies have been developed. The Kalman filter has gained popularity as a methodology for history matching and online updating of reservoir simulation models. The objective of this paper is to explore the potential of the Kalman filter technique in the history matching process. The filter is used to update reservoir simulation models. The simulations processes are done using a standard simulator based on finite difference. The observed data and reservoir properties are integrated using Kalman filter methodology. Some examples of reservoir simulation models will be discussed.

1 INTRODUCTION

An interference test is a test of pressure interrelationship between wells serving the same formation. The experiment consists of two wells: the injection well and the observational well. The pressure change at the observational well allows estimation of the formation permeability. The radial flow behavior is assumed during the injection process. Kutasov and Eppelbaum (2008) developed a technique for determination of formation permeability, hydraulic diffusivity and porosity compressibility from an interference test in a geothermal reservoir. The Newton's method was used for solving the coefficient of exponential integral. Fan et al, (2005), used linear flow model in obtaining an improved estimation of fluid reserves. The interference test data can be analyzed by means of regression technique. El-Khatib (1987) used the derivative of line-source solution of the diffusion equation and regression methodology to estimate the transmissivity and storativity. Reservoir simulation plays an important role in reservoir performance studies. A numerical simulator solves the equations of fluid flow in the reservoir approximately, based on partitioning into a set of grid blocks, where each grid is assumed to be homogeneous. A parameter estimation in reservoir simulation, called history matching, aims to find model parameters that result in simulations which better match the observations (production history). In the past, history matching has been considered as a manual process where the parameters are tuned and the results are examined through a reservoir simulator. There has been a growing interest in a stochastic approach for reservoir history matching. The ensemble (sample) Kalman Filter (EnKF) has become a popular method for doing history matching of reservoir simulation (Naevdal et al. 2007). EnKF is a product of Kalman filtering (KF) which is a set of equations to forecast the state of process and to update the estimate as a new observation arrives. KF is an interpretation of recursive Bayesian estimator. The methodology consists of two main steps: 1. Forecast the current state to obtain prior estimate. 2. Update the prior estimate, an improved posterior estimate, using the new observation. The KF ideas apply to reservoir monitoring using porosity and permeability as parameters and production as observations. The forecast step is carried out by flow simulator to obtain some forecast of future state of reservoir given current data; prior estimate. Update the prior using new observation. This procedure simulates the behavior of the reservoir. Due to the nature of fluid flow modeling, the simulators are highly nonlinear, and the KF will break down. The EnKF has proved significant contribution in tackling these issues.

2. METHODOLOGY

Bayesian statistics are used to combine a model forecast of a state at a given time with a set of measurements. The error of the model forecast and measurements are characterized by covariances. Evensen (2003, 2007) and Burgers et al. (1998) derived the optimal linear unbiased estimator for a scalar state combined with a single observation. Consider a nonlinear line source model:

$$\mathbf{X}_t = f\left(\mathbf{X}_{t-1}\right) + \boldsymbol{\varepsilon}_t^m \tag{1}$$

$$\mathbf{Y}_t = \mathbf{H}\mathbf{X}_t + \boldsymbol{\varepsilon}_t^0 \tag{2}$$

where f is a nonlinear function of the state $\mathbf{X}$, $\mathbf{X}$ is the state**.** Given the nonlinear character of the dynamics, the mean and variance of the true state $\mathbf{X}$ are difficult to find. The difficulties can be solved using simulation: start with a sample (ensemble) of model states, evaluate each realization of the model state through the model dynamics (1) and obtain a new sample; prior sample, and information can be extracted from usual estimation procedure. Using all members in the (state) samples, the update (assimilation) step is done with all members in the sample of observation taking as a mean the true observation. Each of prior sample members is updated to reflect the simulated observations.

The true state is approximated by the average of members of the sample. The notation and formula used for the EnKF are as follows: $\mathbf{X}=(\mathbf{X}_1,\ldots,\mathbf{X}_p)$ is the matrix $(m\times p)$ of sample members, m is the sample (ensemble) size, $\bar{\mathbf{X}} = \mathbf{X1}^t$ is the vector $(m\times 1)$ of sample average, $\mathbf{y}(n\times 1)$ is the observation, $\mathbf{P_e} = \frac{(\mathbf{X}-\bar{\mathbf{X}})(\mathbf{X}-\bar{\mathbf{X}})^{\mathbf{t}}}{m-1}$ is the $(m\times m)$ sample covariance matrix, $\mathbf{Y} = \mathbf{y}+\boldsymbol{\varepsilon}$ is $(n\times m)$ perturbed observation, $\boldsymbol{\varepsilon}=(\boldsymbol{\varepsilon}_1,\ldots,\boldsymbol{\varepsilon}_m)$ is the $(n\times m)$ perturbation, $\boldsymbol{\varepsilon}_i \sim N(0,\boldsymbol{\Sigma}_i^e)$, $\mathbf{R}_e = \frac{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathbf{t}}}{m-1}$

The EnKF is a Monte Carlo Kalman filter. States are simulated, advanced in time, and updated for each time an observation arrives. The mean and variance of the forecast step are not available explicitly as in linear case, it is estimated from the sample; $\bar{\mathbf{X}}$ and $\mathbf{P_e}$. The algorithm is as follows:

1. Initialization: Create an initial sample $\mathbf{X_0} \sim N(\boldsymbol{\mu}_0,\boldsymbol{\Sigma}_0)$ 2. Forecasting: Forecast in time sample $\mathbf{X}$:

$$\mathbf{X}^f = f(\mathbf{X}) + \boldsymbol{\varepsilon}^m \tag{3}$$

where the model error $\boldsymbol{\varepsilon}^m = \left(\boldsymbol{\varepsilon}_i^m \sim N(0,\Sigma_i^m)\right)$. Repeat this step until reaching an observation at t_{obs}. 3. Updating: Update the estimate $\mathbf{X}^f$ using equation (4)

$$\mathbf{X^a} = \mathbf{X^f} + \mathbf{P_e H^t}\left(\mathbf{HP_e H^t} + \mathbf{R}_e\right)^{-1}\left(\mathbf{Y} - \mathbf{HX}^f\right) \tag{4}$$

where $\mathbf{Y}$ is the perturbed actual observation **y.** Go to the forecasting step using equation (3).

Consider a synthetic history matching, where the simulator is a radial flow model. The solution is known as a line source solution. The equation of flow is given by the diffusion process

$$\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P(r,t)}{\partial r}\right) = \frac{\phi\mu c}{k}\frac{\partial P(r,t)}{\partial r}\\
&P(r,0) = P_i, P(\infty,t) = P_{initial},\\
&\frac{2\pi kh}{\mu}\lim_{r\to r_w} r\frac{\partial P}{\partial r} = q
\end{aligned} \tag{5}$$

where P(r,t) is the pressure (psi) at time t, r is the radial distance (ft), k is the permeability (md), μ is the viscosity (cp), ϕ is the porosity (%), c is the compressibility (1/psi). The line source solution is given by the expression

$$P(r,t) = P_i - \frac{q\mu}{4\pi kh}\mathrm{Ei}\left(\frac{\phi\mu c r^2}{4kt}\right) \tag{6}$$

where h is the thickness of the reservoir and Ei is the exponential integral

$$\mathrm{Ei}(x) = \int_x^\infty \frac{e^{-s}}{s}ds \approx .5772 + \ln x - x + \frac{1}{2}x^2 + O(x^3) \tag{7}$$

The focus of the study is on estimating the permeability based on the pressure P. The state consists of the static permeability k and the dynamic pressure P. The simulated pressure, P_{sim}, is obtained from (6) at the well radius r_w. The observation of the observed pressure P_{obs}:

$$\mathbf{X} = (k\ P_{sim})^t,\ Y{=}P_{obs} \tag{8}$$

The initial permeability k_0 is drawn from a normal distribution with a prior mean μ_0 and a variance σ_0^2. Static condition for permeability is modeled as $k_t = k_{t-1}$. The simulator of pressure provides the model for the simulated pressure

$$P_{sim,t} = P(r_w,t,k_t) + \varepsilon_t \tag{9}$$

The formula gives a dependence on the permeability only. The observations are

$$y_t = \mathbf{H}\mathbf{X}_t + \varepsilon_t^o \tag{10}$$

where **H** is the projection; $\mathbf{H} = (0\ 1)$.

Let $t = 0, T, \Delta t$ be the time interval from t = 0 until T with time step Δt. Inside this interval, a set of observations are created using (6) given k_{true}. The aim is to show that EnKF will converge to the true permeability after a number of iterations.

El-Khatib (1987) developed a linear regression model to estimate the parameters of the line source solution of the diffusivity equation. The derivative of the line source solution is

$$\frac{dP(r,t)}{dt} = -Ae^{-b/t}$$

Multiplying by –t and taking logarithms

$$\ln\left(t\frac{dP}{dt}\right) = \ln A - b\times\frac{1}{t}$$

where $T = \frac{kh}{\mu} = \frac{70.6qB}{A}, S = h\phi c = \frac{Tb}{948r^2}$. The term $t\frac{dP}{dt}$ is approximated by $\Delta P/\Delta \ln t$. Least squares technique was used to estimate the parameters A and B.

3 TEST EXAMPLE

In order to estimate the energy reserves, it is essential to estimate the invisible unknown reservoir properties, such as permeability, porosity and subsurface structures. Reservoir simulator is often used to predict the future reservoir performance. The purpose of history matching is to estimate the model parameters that better match the production history. Well tests have been used to assess well condition and reservoir parameters, and remain an important component for reservoir characterization. The most commonly used analytical solution for interpreting an interference test is the line source solution which corresponds to an infinite acting isotropic reservoir, single phase, and constant production rate. The diffusivity equation is used to describe the flow in well tests and interference tests. This section aims to present the source of data used on the application of EnKF in a well test and interference test. In the well test experiment, the solution of the diffusivity equation is given by the line source solution. The EnKF is used to estimate the permeability. The experiment is set up for T = 2000 hours with reservoir properties $r_w = 0.1$ ft, $\mu = 1.5$ cp, $Q = 300$ STB/day, $B = 1.25$ bbl/STB, $\phi = 15\%$, $P_i = 4000$ psi, $h = 15$ ft, $c = 12\times10^{-6}$ psi^{-1}. The time step is half of an hour, the number observations is 40 equally spaced; i. e, the pressure is measured every 50 hours. The initial ensemble for permeability was sampled from normal distribution with a mean of 80. The true permeability is 60. Define a time interval from t = 0 to T = 2000 hours with time step $\Delta t = 0.5$. Inside this interval, a set of equally spaced observations; $t_{obs} = 50,100,150,\ldots$, were generated using line source solution given $k_{true} = 60$, perturb the observations with normal distribution. The pressure data prior to the observation time was generated using line source solution which depends only on the permeability; i.e, the generated permeability ensemble was used to calculate the pressure data prior to observations. For interference test, the following parameters are used: r = 495 ft, $q = 1.25\times10^5$ (STB/d), $c = 23.45$ psi^{-1}, $B = 5.63\times10^{-3}$ (res vol/stock-tank vol)), h = 12 ft, $\mu = 0.0203$ cp, $\phi = 17(\%)$

4. RESULTS AND DISCUSSION

Following El-Khatib (1987), Figure 1 shows the regression of $\ln(-\Delta P/\Delta \ln t)$ on a geometric time average based on the data $(t.P_w)$: (0, 2715) (48, 2712) (72, 2706) (96, 2700) (120, 2693) (144, 2687) (168, 2680) (336, 2640) (504, 2615) yields the tramsmissivity T = 6249.2 md-ft/cp and storativity S = .00277 $\text{ft}-\text{psi}^{-1}$.

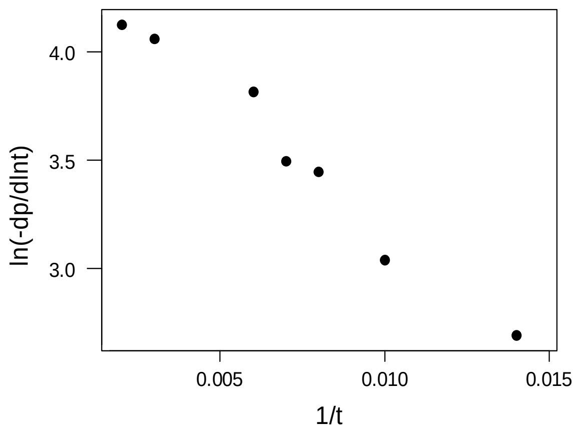

Figure 1. Plot of $\ln(-dP/d\ln t)$ versus 1/t. The regression equation is $\ln(-dP/d\ln t) = 79.5058 - 102.867\ \frac{1}{t}$.

Figure 2 and Figure 3 are intended to show how the EnKF may be used to recover the true parameter values. Figure 2 concerns with well testing to estimate the average well formation permeability using line source solution. The study is set up for T = 2000 hours with time step $\frac{1}{2}$ hour, and the parameters are: $r_w = .1$, $\mu = 1.5$, q = 375, $\phi = 15\%$, $P_i = 4000$, h = 15, $c = 12\times10^{-6}$. The initial state for permeability is sampled from $N(\mu_0 = 80, \sigma_0^2 = 1)$. The true permeability is 60, and the ensemble (Monte Carlo sample) size was m = 100. The number of observation is 40 equally spaced and represented by circles in Figure 2. Inside observations interval a set of equally spaced simulated pressures was greeted using line source solution. The observations updated the permeability values, reduced the standard deviation and converge towards to the true permeability. As expected, for each state updating, a reduction of the standard deviation of the ensemble is shown. Figure 3 shows the results for estimating storativity and transmissivity. The experiment was run for time period T = 5000 hours. The observations correct the parameters values, and a significant reduction in standard deviation. The iteration for transmissivity converges after 500 hours, the iteration for storativity takes longer iterations to converge. The experiment shows that the method works properly in well testing and interference analysis using line source solution.

The line source solution of the diffusivity equation can be arranged so that a linear relationship between pressure P and log t is obtained. A plot of P vs log t yields a straight line whose slope is used to estimate the permeability, transmissivity and storativity. The application of least squares to the observations (Figure 4) yields an estimate 74.724 for permeability and 747.242 for transmissivity. Comparison with results obtain by EnKF indicates a comparable results indicating the nature of the EnKF methodology. The least squares approach is limited to a small number of grid, EnKF combined with spatial correlation and reservoir simulator offer flexibility in terms of number of grids, multi well system, and multi phase reservoir model.

4. CONCLUSIONS

In order to optimize the reservoir production, it is essential to estimate the reservoir properties. The EnKF methodology is proposed to estimate the parameter for well and interference test using synthetic data. In the forecasting step, the line source solution was used. In a real field application, the line source is replaced by reservoir simulator. The observations were used to adjust the parameter estimate. The method works successfully once the initial distribution has been correctly chosen.

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NOMENCLATURE

B = formation volume factor (bbl/STB, well field barrels/stock tank barrels), Q = oil flow (STB/day), P(r,t) = pressure (psi), r = radius (ft), k = permeability (md), μ = viscosity (cp), ϕ = porosity(%), c = total compressibility (psi^{-1}), r_w = well radius (ft), h = thickness of reservoir (ft), $T = kh/\mu$ = transmissivity (md-ft/cp), $S = h\phi c$ = storativity (ft-psi^{-1})

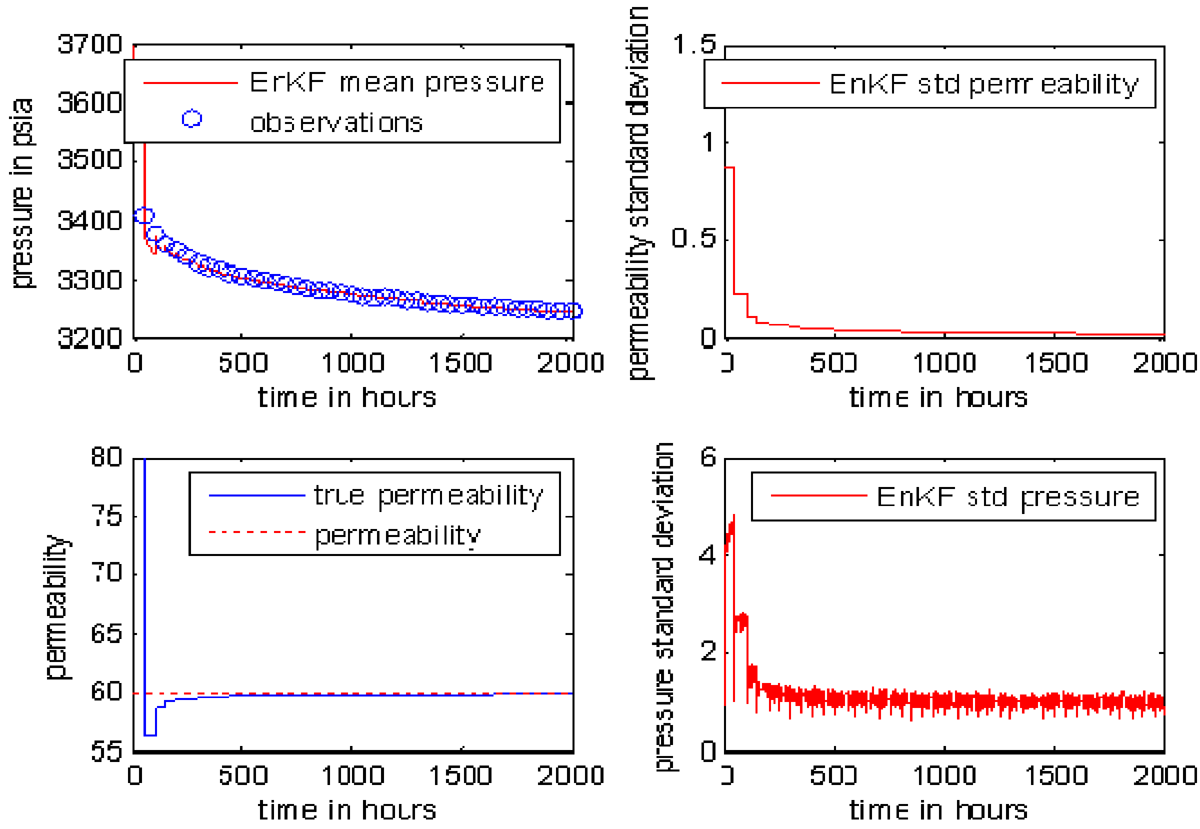

Figure 2: EnKF for well testing, the updated formation permeability converge to the true permeability 60, the standard error of ensemble of permeability converge to zero , period T = 2000 hours, $\Delta t = 0.5$ hours , $t_{obs} = 50,100,150,\ldots,2000$

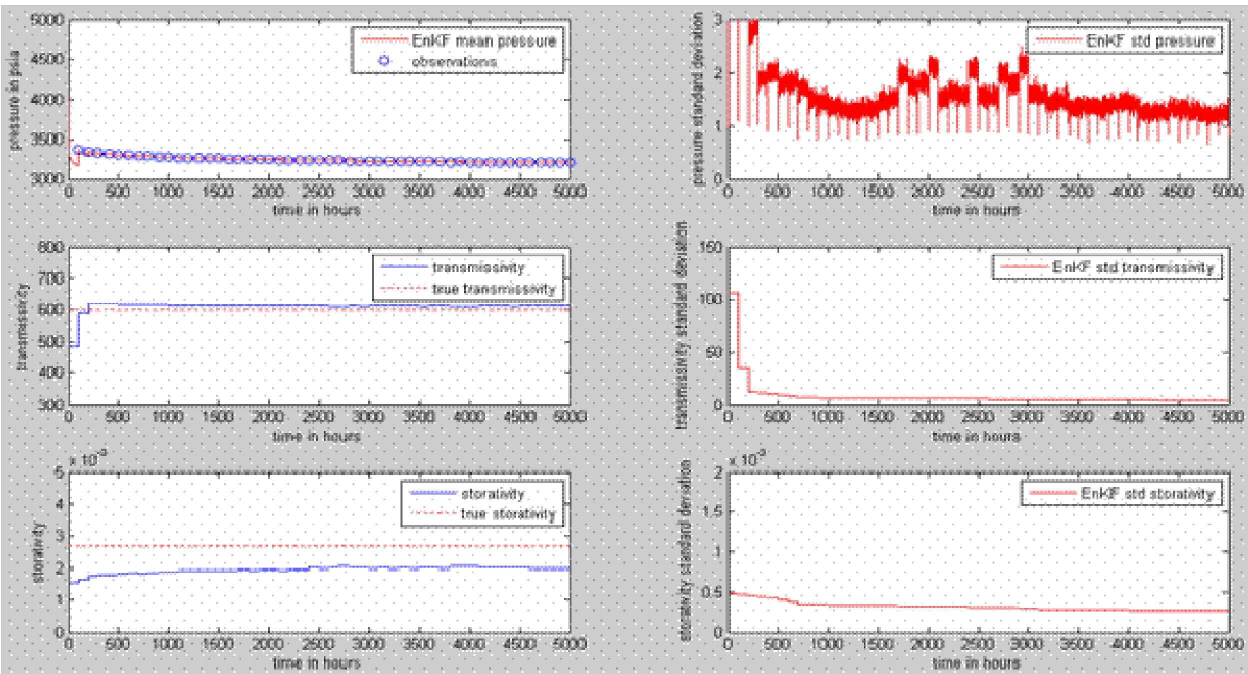

Figure 3: EnKF for interference test based on line source solution. The transmissivity and storativity converge to the true values.

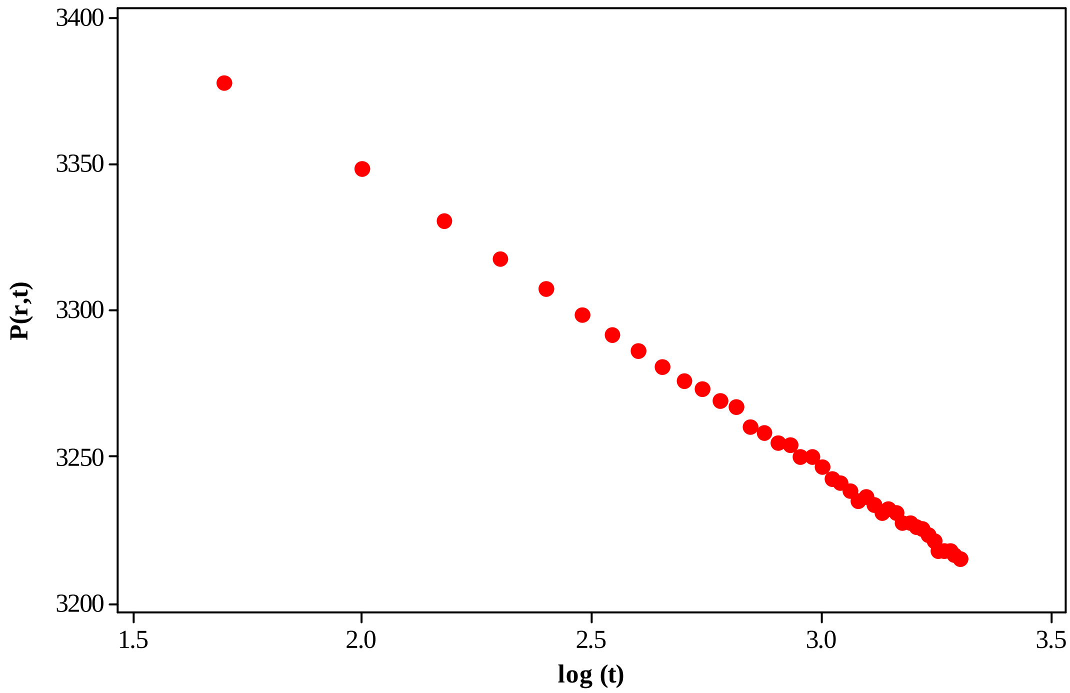

Figure 4. Plot pressure P(r,t) versus log t. The linear regression was used to estimate the permeability, transmissivity, and storativity

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