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Dynamic Response Simulations of Circulating Fluid and a Borehole Heat Exchanger

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ABSTRACT

A three-dimensional numerical model has been developed to simulate transient fluid transport and heat transfer in and around Borehole Heat Exchangers (BHEs). Common approaches to the simulation of BHEs assume heat transfer in circulating fluid and grout to be in a quasi-steady state and ignore fluctuations in fluid temperature due to transport of fluid through the loop. However, the dynamic response of the circulating fluid has important implications for system design and operation under some circumstances. By applying the numerical model, this paper examines the dynamics of fluid transport and the transient response of a BHE at short timescales in detail by considering the variations in outlet temperature in response to the variations of inlet temperature. The response at short timescales has been characterised in terms of response to periodic variation in inlet temperature at a range of frequencies. This frequency response is compared to that of related two-dimensional models and the transient behaviours of the BHE which can only be captured by the three-dimensional model are highlighted. The model is being used to develop improved simplified models of BHEs.

1. INTRODUCTION

Borehole Heat Exchanger (BHEs) are probably the commonest form of ground heat exchanger found in Ground Source Heat Pump (GSHP) systems. Their careful design is not only critical to the long term performance but also the short term performance of the heat pump systems. A horizontal cross-section of half of a typical BHE is shown in Figure 1. Pipes are formed in a 'U' loop and then grouted into vertical boreholes, which normally range between 50 to 100 meters deep and typically are around 100 mm to 150 mm in diameter. In this work we consider the most common arrangement for ground source heat pumps which is a single U-tube and fully grouted borehole.

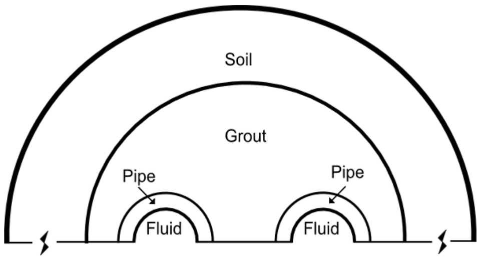

Figure 1: A half cross-section of a Borehole Heat Exchanger.

BHEs of this type are not only used in conventional building heating and cooling systems but also in large thermal storage schemes. BHEs can not be designed on the basis of steady-state calculations but require application of dynamic thermal models that are able to take account of the heat transfer inside the borehole as well as the surrounding soil/rock formation. The purpose of the model developments discussed here is to:

- Investigate the effects of the dynamics of the fluid transport along the pipe loop;
- Investigate the three-dimensional characteristics of heat transfer in and around the borehole;
- Develop insight into the limitations of two-dimensional models and suggest ways in which they can be improved.

2. BOREHOLE HEAT EXCHANGER MODELS

Models of BHEs have three principle applications:

1. Design of BHEs. This means sizing the borehole depth, number of boreholes etc.
2. Analysis of in-situ ground thermal conductivity test data.
3. Integration with building system simulation i.e. with the model coupled to HVAC system and building heat transfer models to study overall performance.

Analytical models have been developed by making a number of simplifying assumptions and applied to both the design of BHEs and analysis of in-situ test data. The analytical Cylinder Source solution presented by Carslaw and Jaeger (1947) has been applied by treating the two pipes as one pipe coaxial with the borehole. Further simplifying the pipe and the borehole as an infinitely long line source, the line source solution (Ingersoll, 1954) can also be used and is commonly done so in the analysis of in-situ thermal conductivity test data.

Although analytical solutions require less computing effort, they are less suited to design and simulation tasks where one would like to take account of time varying heat transfer rates and the influence of surrounding boreholes on both long and short time scales. A number of approaches that have combined analytical and numerical methods have been developed with design tasks in mind. Eskilson (1987), and later Hellstrom (1991) developed a response factor approach – using so called g-function - to design BHEs for thermal storage applications. Response to heat flux over timescales of approximately one month to more than ten years can be derived from application of two-dimensional numerical models and integration of the response according to the number of boreholes and the relationship to those neighbouring. The response can be normalised and so applied to ranges of BHE configurations.

Models such as the line source approach and also the g-function models make simplifications about the grout and pipes within the borehole. The common assumption is that this relationship between the fluid and the borehole wall can be defined by a thermal resistance i.e. a coaxial pipe without thermal mass. The fluid temperature applied in the model is one representative of the loop inlet and outlet temperatures (often their average). The borehole thermal resistance then becomes an important quantity for design purposes. Adopting this representation of the borehole ignores the dynamic effects of fluid flow around the loop but also the dynamic response of the material surrounding the pipes inside the borehole (grout, backfill or water). This approximation is reasonable where the model is applied to borehole field design where the response of the ground surrounding the BHE over long timescales is the main interest.

Application of models for system simulation tasks – unlike design tasks – requires the ability to operate at shorter timescales – possibly less than one minute. The dynamic response of the grout material inside the borehole should then be considered. Yavuzturk and Spitler (1999) extended the g-function model to short time steps to be considered by applying the finite difference method on a two-dimensional radial-axial coordinate system. The model was applied and used to extend the g-function representation of the response to shorter timescales. This short time step g-function model has been implemented in EnergyPlus and validated by Fisher et al. (2006). Hellstrom developed the DST model (1991) to simulate BHEs using a one-dimensional radial mesh to calculate the thermal resistance of a borehole by approximating the steady-state heat transfer in a borehole. Likewise, the DST model has been implemented in TRNSYS (SEL, 1997).

Two-dimensional numerical models such as that of that of Yavuzturk (1999) can be used to calculate the dynamic behaviour of grout and pipes explicitly. The relationship between the pipe temperature and the circulating fluid is then just dependent on a convection coefficient. The thermal mass of the fluid and the dynamics of its circulation around the loop is ignored however. Young (2004) used such a model but also defined additional cells in the mesh on the inside of the pipe to represent the fluid more explicitly. This allows the thermal mass of the fluid to be taken into account in a simplified manner. However, as variation in fluid temperature with depth can not be considered explicitly, some assumptions have to be made – as it does with simpler models – about the fluid temperatures in the two pipes and the associated boundary conditions. These assumptions can be avoided in a three-dimensional numerical model. We discuss this in more detail later in this paper.

The delayed response to transient variations in inlet temperature is of significance in that GSHP system designs (i.e. choice of borehole depth) are sometimes constrained by peak load conditions. In these cases, selection of too small a BHE could result in fluid temperatures close to or outside the operating range of the heat pump for short periods. Two-dimensional models (numerical or analytical) are not able to consider the effects of fluid transport in the pipe.

A three-dimensional finite volume model has been developed in this study to simulate BHEs. Several three-dimensional models have been developed to simulate BHEs (Zeng, 2003; Lee, 2008; Bandyopadhyay, 2008). The advantages of a three-dimensional model include:

- Fluid transport along the pipe loop and the dynamics of the fluid can be represented;
- Fluid, borehole and ground temperature variation along the borehole depth can be modelled;
- Different layers of rock and soil can be explicitly represented;
- Climate dependent boundary conditions at the surface can be applied;
- Heat transfer below the borehole can be explicitly considered;
- Initial vertical ground temperature gradients can be applied.

Two-dimensional models may now be computationally efficient enough for practical design and simulation purposes. Three-dimensional models offer most generality and most accurate representation of heat transfer and so are useful for detailed studies like that presented here but are not yet suited to practical simulation of annual or super annual performance.

3. MODEL DEVELOPMENT

3.1 Three-Dimensional Model

A dynamic three-dimensional BHE numerical model has been developed that is built upon a finite volume solver known as GEMS3D (General Elliptical Multi-block Solver in 3 Dimensions). This has been developed to simulate the dynamic response of the circulating fluid and transient heat transfer in and around BHEs. The GEMS3D model applies the finite volume method to solve the partial differential equation for heat transfer on three-dimensional boundary fitted grids, which can be written as:

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\frac{k}{\rho c_p}\frac{\partial T}{\partial x}\right) + S \qquad (1)$$

where k is thermal conductivity, ρc_p is volumetric heat capacity, and S is source term. u_j is velocity and $|u_j| > 0$ in fluid cells and $|u_j| = 0$ elsewhere.

Subdividing the solution domain into a finite number of small control volumes, and then integrating the partial differential equation to form an algebraic equation in terms of fluxes at the boundaries of the control volume allows the temperatures and heat fluxes to be calculated. The approach to dealing with non-orthogonal cells and discretisation of the diffusion fluxes is similar to that described by Ferziger and Peric (2002). The Strongly Implicit Procedure (also known as Stone’s method) has been employed for solving the equation system arising from discretisation of the partial differential equation for heat transfer. The hybrid differencing scheme has been applied to discretise the convection term of the equation i.e. the term used to represent fluid motion in the pipe. Diffusion fluxes are dealt with using central differencing and the temporal term is discretised using a first order implicit method.

The multi-block structured mesh that is used in this model allows the complex geometries around the pipes in BHEs to be represented exactly and allows close control of the mesh cell distribution (Figure 2). The multi-block mesh is also a convenient means of defining zones with different physical properties i.e the pipe material and grout. The mesh is essentially extruded in the third dimension in the three-dimensional model and further blocks are added to represent the U fitting and the region below the borehole.

The ability to model the heat transfer below the borehole is an advantage of a three-dimensional model compared to a two-dimensional model in the horizontal plane. Figure 3 shows a temperature distribution diagram of the bottom of the BHE and the ground underneath it.

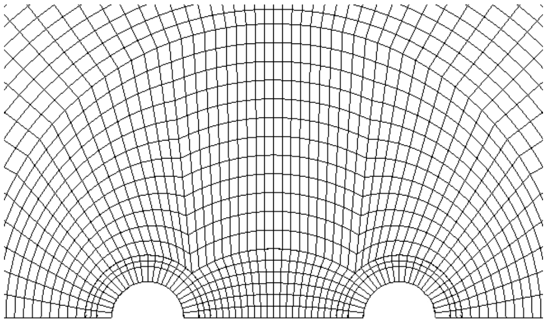

Figure 2: A horizontal cross-section of boundary fitted grid showing the pipe and grout region (it is only necessary to model half the borehole).

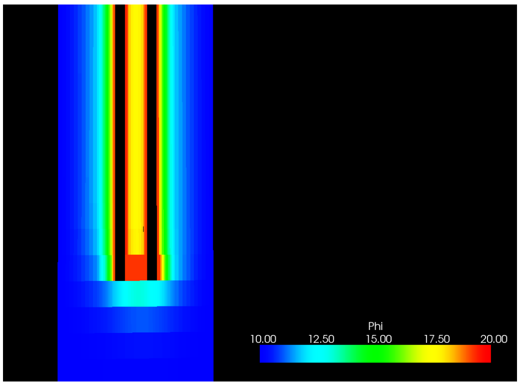

Figure 3: Temperature distribution at of the bottom of the borehole.

3.2 Two-Dimensional Implementation

Precisely the same numerical method has been implemented in a two-dimensional version of the code (GEMS2D). A two-dimensional implementation of the model (equivalent to the three dimensional model of one metre depth) has been used here to highlight the differences between model predictions that are due solely to three-dimensional effects and dynamic fluid transport. The two-dimensional model necessarily employs some of the simplifications of other existing models. One important issue in defining a two-dimensional model is relating the model boundary conditions to the inlet and outlet fluid temperatures. In reality, and in the three dimensional numerical model, the tube nearer the inlet has a higher temperature than that of the outlet. The temperature differences between the neighbouring tubes causes some heat to flow between the tubes rather than from the tubes to the ground. Preliminary calculations suggest this is approximately 5% of the total BHE heat transfer rate (He et al., 2009). A three dimensional model can account for the variations in this inter-tube heat flux along the length of the borehole but some assumptions about the differences in pipe temperature have to be made in any two dimensional model. Two different approaches have been implemented in this study and compared to the results of the three-dimensional model.

In the first approach, one pipe of the model is assumed to have a temperature the same as the inlet. The second pipe in the model is assumed to have the same temperature as the outlet. The temperature boundary condition applied to the second pipe is calculated in an iterative manner so that the fluid heat balance is consistent with the heat flux. In this case the pipe temperatures in the model really reflect conditions near the top of the borehole where the difference in temperature between the pipes is greatest. This can be expected to result in over estimation of the heat flux between the tubes, and therefore GEMS2D exaggerates the heat flux between the two pipes. The iteration method is illustrated in Figure 4.

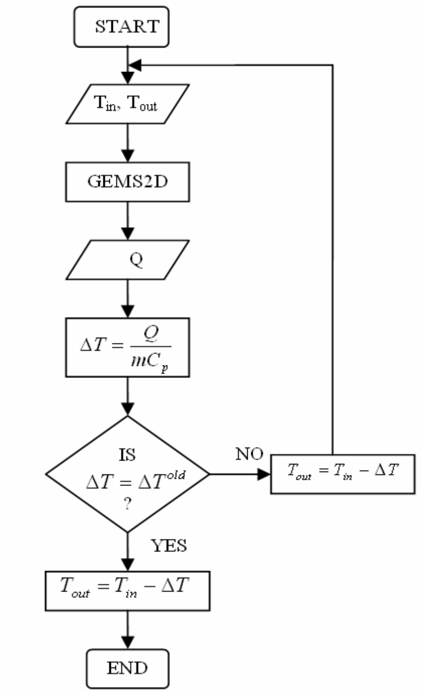

Figure 4: An illustration of the first iterative procedure used to calculate the outlet temperature of a two-dimensional BHE model (GEMS2D).

In the second form of two-dimensional model (denoted as GEMS2D-b), the temperature boundary condition applied to both pipes is assumed to be a mean temperature of the inlet and outlet. This mean temperature is akin to the borehole temperature (T_b) in other models, i.e. the g-function model. An iterative procedure is still necessary to ensure that the pipe temperature and the outlet temperature are consistent with the heat flux calculated by the finite volume model. As the temperatures of the two pipes are the same, there is no inter-tube heat flux (much like at the bottom of the borehole) and so the effect of this flux is always underestimated. The iterative procedure is illustrated in Figure 5.

It is important to point out that these iterative procedures are not guaranteed to be numerically stable. These formulations do not guarantee that the outlet temperature is consistent with the second law of thermodynamics – at least during the iteration process if not the end. This is not generally a problem when the time steps are large. However, with short time steps, and when there is a sudden change in inlet temperature, the model tends to become unstable. This is problematic if the interest is in control system interaction for example. This issue is common to all models that take the borehole temperature boundary condition to be the average of the inlet and outlet temperatures and try to calculate the outlet temperature (this is not usually an issue with such models in design tool applications but is an issue in models used to simulate system operation).

One of our aims is to develop a two-dimensional model that avoids these numerical issues and is well behaved in terms of properly bounded predictions of outlet temperature.

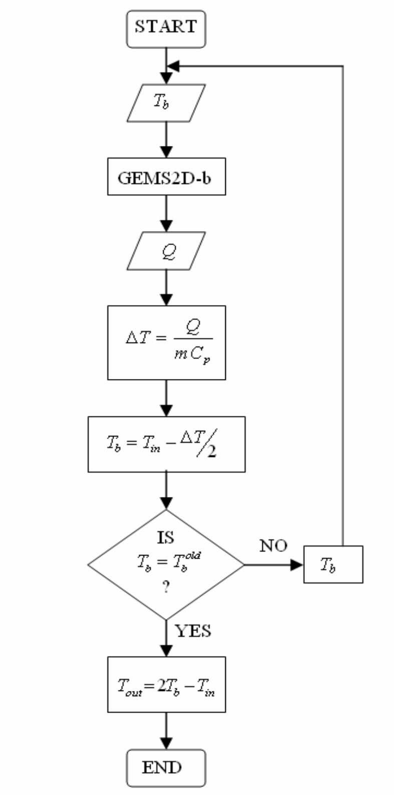

Figure 5: An illustration of the iterative procedure used to calculate the outlet temperature (GEMS2D-b). In this procedure the pipe boundary condition is the average of the inlet and outlet temperatures.

4. MODEL VALIDATION

4.1 Borehole Thermal Resistance

There is no analytical solution for three-dimensional heat transfer in a borehole geometry that can be applied to try to validate the numerical model. It is useful however to show some validation using a two-dimensional calculation of borehole thermal resistance. Numerical values can be compared with the multi-pole analytical solution method (Bennet, 1987). This is done by making a two-dimensional steady-state calculation of the heat flux for a given fluid and far field temperature. Assuming the heat transfer of BHEs is in steady-state, the borehole thermal resistance (R) can be found from the total heat flux calculated by the finite volume model (Q) and the difference between the fluid temperature (T_f) and the borehole temperature ($T_{borehole}$) The later is the average temperature at the interface between the grout and the ground. The resistance is simply,

$$R_b = \frac{T_f - T_{borehole}}{Q} \tag{2}$$

The borehole resistance defined here includes the convective resistance between the fluid and the inner side of the pipe, the conductive resistance of the pipe, and the conductive resistance of the grout.

A single BHE has been configured to examine the borehole thermal resistance and used in the validation study with two levels of thermal conductivity. Variation of the calculated thermal resistance with mesh density has also been tested. The model can be seen to be capable of matching analytical values extremely closely. Table 1 shows the variation of the calculated borehole thermal resistance is less than 0.4% by variation of mesh density (the mesh shown in Figure 2 is in the middle of this range). In practice, calculation using courser meshes to reduce computation times would be reasonable.

Table 1: Borehole thermal resistance by GEMS2D and Multipole models.

Number of Cells		Borehole Thermal Resistance R_b (mK/W)	
		GEMS2D	Multipole
864	k_{grout} = 0.75 (W/mK)	0.1826	0.1823
1450		0.1824	
1944		0.1823	
2448		0.1821	
3652		0.1821	
5118		0.1820	
Number of Cells		Borehole Thermal Resistance (mK/W)	
		GEMS2D	Multipole
864	k_{grout} = 1.5 (W/mK)	0.1160	0.1158
1450		0.1159	
1944		0.1159	
2448		0.1158	
3652		0.1157	
5118		0.1157	

4.2 Fluid Transport Model

Fluctuations in fluid temperature due to transport of the fluid through the loop are usually ignored in common approaches to modelling BHEs. In situations where the heat pump and circulating pumps switch on and off during a given hour, and in situations where the building loads have noticeable peaks, the dynamic response of the circulating fluid is of great importance (Kummert and Bernier, 2008). The effect of the thermal mass of the circulating fluid and the dynamics of fluid transport through the loop is to damp out fluctuations in the outlet temperature of BHEs, which has important implications for system design.

Using a layer of cells inside the pipe in the three dimensional model allows the fluid to be discretised along the length of the borehole. Fluid velocity is imposed in these cells and the transport of heat from one cell to the next along the pipe is then represented by a convection term in the temperature differential equation being solved (Equation 1). The fluid cells in the finite volume model can be considered similar to a Compartments-In-Series model (Wen and Fan, 1975) as illustrated in Figure 6.

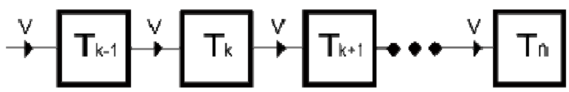

Figure 6: Diagram of the Compartments-In-Series representation of the fluid flow along a pipe.

Fluid transport models of this type have been widely used in process engineering and their characteristics are well known. Analytical solutions to the problem of transport of fluid along a pipe exist (e.g. Bosworth, 1949) and have been used to validate this aspect of the model. Predicted outlet temperatures can be compared to an analytical solution for response to step changes in inlet temperature. The finite volume model can also be verified by comparison with Compartments-In-Series model and other pipe fluid transport models.

The transport characteristics of flow in pipes can be thought of in terms of Residence Time Distribution (RTD). The RTD is considered as the fraction of fluid, which undergoes a step change at the inlet, for example, a step change of the colour, or a step change of the temperature, that appears in the outgoing fluid at time t. It is represented by the function *F(t)*, and illustrated in a *F*-Diagram. The analysis is simplified by using dimensionless time given by,

$$\tau = \frac{\dot{V}t}{V} \tag{3}$$

where $\dot{V}$ is volume flow rate and V is system volume, in this case is the volume of the two pipes. The actual shape of the *F*-Diagram depends primarily on the velocity profile, in which case the faster-moving elements near the centreline will arrive at the end of the pipe more quickly than the average. Fluid undergoes a diffusion process as it travels along the pipe (by virtue of the non-uniform velocity profile) so that step changes in inlet condition are smoothed. Hanby et al. (2002) examined the Compartments-in-Series model and found it performed well but the solution was not independent of the number of compartments. They made comparisons with an analytical solution for the RTD in turbulent flow. The results indicated that the optimum number of nodes is 46, but given computational constraints, 20 nodes give a reasonable approximation.

Figure 7 shows the *F*-Diagram generated by GEMS3D using 20, 40 and 80 cells compared with the analytical solution (Bosworth, 1949) and the results indicate the dynamics of fluid transport predicted by the GEMS3D model satisfactorily matches the analytical solution.

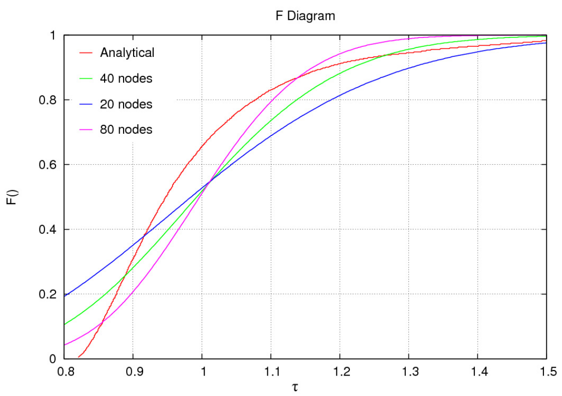

Figure 7: F-Diagrams by GEMS3D model with different cells compared with the analytical solution.

5. RESULTS AND DISCUSSION

We have attempted to characterise the behaviour of the two and three-dimensional models in three ways.

- Steady-state heat transfer rates
- Response to a single step change in inlet temperature
- Response to periodic step changes in inlet temperature
- Response to sinusoidal variations in inlet temperature of different frequencies

These analyses are reported in sections 5.1–5.4 respectively. These analyses have been done with a view to characterise the behaviour of the models in a generic manner. This allows meaningful comparison between the two and three dimensional models. We take the two-dimensional models, which share many characteristics with other models that ignore the dynamics of the fluid transport in the pipe, as comparison. We believe the three-dimensional model can be used as a reference model that also gives insight into the real behaviour of boreholes. Although the model consists of a single borehole, this arrangement can be used to examine the behaviour of whole borehole fields when the period of the temperature fluctuations is short. In this case heat transfer fluctuations occur only in the region very close to the borehole.

A single borehole with a diameter of 150 mm and a depth of 100 m has been simulated by GEMS3D to examine the dynamic response of the circulating fluid and the transient heat transfer of the BHE. Fluid circulates within the pipes at a velocity of 1m/s and is in the fully developed turbulent flow regime (Re = 24,300). The nominal transit time for the fluid flowing along the whole U-tube pipe is correspondingly 200 seconds. The configurations and thermal properties of the borehole are shown in Table 2. The comparison of calculated fluid outlet temperatures by two-dimensional (GEMS2D, GEMS2D-b) and three-dimensional (GEMS3D) models has been made by setting the same fluid inlet temperature in these models and then comparing the fluid outlet temperatures that calculated by these models.

Table 2: BHE configurations and thermal properties.

Borehole Diameter		D	150	mm
Borehole Depth		L	100	m
Pipe Inner Diameter		D_{in}	27	mm
Pipe Outer Diameter		D_{out}	33.4	mm
Spacing between pipes		L_s	28	mm
Water	Conductivity	k_{water}	0.6	W/mK
	Thermal Capacity	ρc_p	4.2	MJ/m^{3}K
Pipe	Conductivity	k_{pipe}	0.39	W/mK
	Thermal Capacity	ρc_p	1.77	MJ/m^{3}K
Grout	Conductivity	k_{grout}	0.75	W/mK
	Thermal Capacity	ρc_p	3.9	MJ/m^{3}K
Soil	Conductivity	k_{soil}	2.5	W/mK
	Thermal Capacity	ρc_p	2.5	MJ/m^{3}K

5.1 Steady-State Heat Transfer Rates

Steady-state simulations of a single BHE have been carried out using GEMS3D, GEMS2D and GEMS2D-b. The three dimensional model naturally predicts a variation in fluid temperature and heat flux along its length. In transient conditions the net effect of this variation in heat flux with depth is certainly not equivalent to that of any two-dimensional model (He et al., 2009). This temperature variation with depth is not necessarily equivalent to that of a two dimensional model in steady-state either (we hope to improve this in future work) and it is worthwhile making a steady-state comparison of predicted BHE overall heat transfer rates. In these steady-state calculations we fix the temperature at the far field boundary at 10°C, while the inlet fluid temperature is 20°C. The results of the steady-state simulations are shown in Table 3.

Table 3: Steady-State simulation results

	GEMS3D	GEMS2D	GEMS2D-b
Inlet Temp (°C)	20.00	20.00	20.00
Outlet Temp (°C)	18.51	18.44	18.44
Borehole Temp (°C)	11.54	11.56	11.56
Inter-Tube Heat Flux (W)	31.19	71.09	1.17
Total Heat Flux (W)	1742.0	1759.8	1761.3

The outlet fluid temperatures and total heat flux predicted by GEMS2D and GEMS2D-b are the same, even though the inter-tube heat flux by GEMS2D is much higher than that by GEMS2D-b. However, the outlet fluid temperature predicted by GEMS3D is slightly higher than those by GEMS2D and GEMS2D-b, while the inter-tube heat flux is in-between those by GEMS2D and GEMS2D-b.

Figure 8 shows the fluid temperature along the borehole depth by numerical model (GEMS3D). Also, the fluid temperature distribution based on the assumption that $T_m = (T_{in}+T_{out})/2$ (as in GEMS2D-b) is shown in Figure 8. In this case, with the fluid flow rate of 0.54 l/s (1 m/s), the estimation reasonably fits the numerical result.

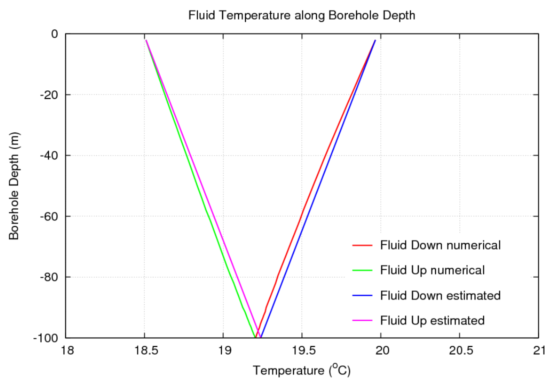

Figure 8: Fluid temperature along borehole depth by numerical model compared with estimation.

5.2 Single Step Response

A single step of heat flux has been applied as the inlet temperature to test the dynamic response in the circulating fluid. The response can be normalised by considering the differences between the fluid temperatures and the initial temperature. The inlet and outlet temperature differences can be normalised with respect to the initial temperature as follows:

$$A^* = \frac{T_{out} - T_{initial}}{T_{in} - T_{initial}} \quad (4)$$

where T_{in} is inlet fluid temperature, T_{out} is outlet fluid temperature, and $T_{initial}$ is initial ground temperature. The responses to a single step change in inlet temperature predicted by the two and three-dimensional models are shown in Figure 9. In the case of the three dimensional model the nominal fluid transit time is 200 second. The responses are shown on a logarithmic horizontal scale where the simulation has been carried out over duration of 120 minute.

The differences in response predicted by the two-dimensional model (GEMS2D) and three-dimensional model (GEMS3D) is occurs in the first 10 minutes. The temperature response that is predicted by GEMS3D is much slower than that is predicted by GEMS2D in this period. The response of the GEMS2D model is governed entirely by the transient conduction of heat through the pipe wall and grout. In the GEMS3D model, not only are the thermal mass of the pipes and grout taken into account but also the effect of the transient fluid flow. This is a question of both the thermal mass of the fluid itself as well as its transport along the pipe and heat loss through the pipe wall. The response is not just delayed according to the notional transit time (200s) but can be seen to be significant for several times this period – up to approximately 10 minutes in this case. Beyond this timescale the response predicted by the GEMS2D model is very similar to that of the GEMS3D model.

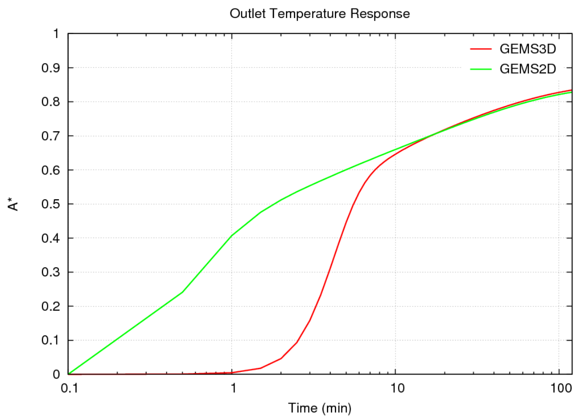

Figure 9: Outlet fluid temperature responses by GEMS3D and GEMS2D.

5.3 Periodic Step Responses

The practical significance of the transient fluid transport can be investigated by applying step changes in borehole inlet temperature. This has been done using step changes that might be typical of a domestic GSHP system and the on-and-off operating intervals of the heat pump operation. The heat pump cycles twice per hour with the inlet temperatures changing between 25°C and 15°C. The far-field ground temperature is 10°C. The simulation has been started from the steady-state condition i.e. predicts the response after many years of excitation. The predicted outlet temperatures are shown in Figure 10. The outlet temperature predicted by the two-dimensional model necessarily shows an instant response to changes in inlet temperature. This is representative of all models that are formulated on a one or two-dimensional basis. On the other hand, the outlet temperature predicted by the three dimensional model shows a delay in response to the step change, which could significantly change overall system behaviour when interaction with heat pump control system (i.e. cycling) is considered (Kummert and Bernier, 2008). The dynamic response of the circulating fluid also depends on its flow rate. Figure 11 shows the fluid temperature variations that are predicted by GEMS3D according to different flow rates. The dynamics of the circulating fluid varies if the flow rate is different.

5.4 Periodic Response

BHE arrays experience periodic variations in system temperatures on a wide range of time scales varying from annual, daily and sub-minutely. Response is often analysed as that of a superposition of steps of different magnitude and duration. It is equally valid to think of the overall response in terms of a superposition of sinusoidal variations. A useful way to characterise the response of the BHE and the models is, accordingly, to impose a periodic excitation of the inlet temperature and compare this with that of the outlet over a range of frequencies. In this case a sinusoidal variation in inlet temperature with the same amplitude (A_{in}) but different frequencies (f) (or periods P) have been applied. The dynamic responses of the BHE model can then be characterised by examining the amplitude and phase (θ) of the predicted outlet fluid temperature.

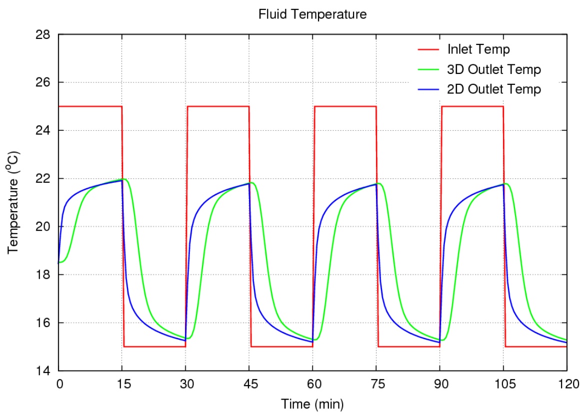

Figure 10: Step function test fluid temperature.

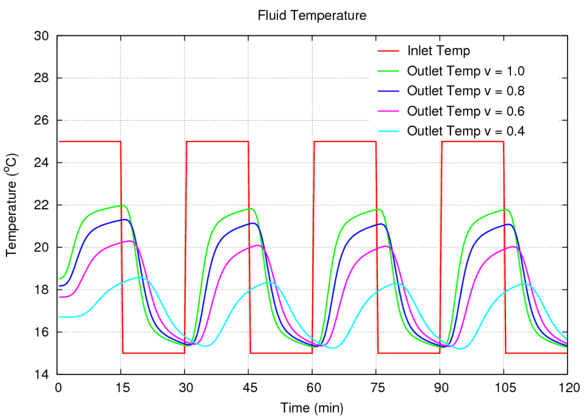

Figure 11: Fluid temperatures by GEMS3D at different flow rates.

The variation of the inlet fluid temperature can be expressed as:

$$T_{in} = A_{in} \sin(\frac{2\pi}{P}t) = A_{in} \sin(2\pi f t) \tag{5}$$

and the variation of the outlet fluid temperature can be expressed as:

$$T_{out} = A_{out} \sin(\frac{2\pi}{P}t + \frac{2\pi}{\Delta P}) = A_{out} \sin(2\pi f t + \theta) \tag{6}$$

where ΔP is the time shift or delay between peak inlet and outlet temperatures. This can also be expressed as a phase shift θ.

All the simulations are started from the steady-state condition (i.e. representative of operation over a long period) with an initial temperature of 10°C and a constant inlet fluid temperature of 20°C to enable a consistent and generic comparison. Figure 12 shows the outlet fluid temperature variations by GEMS3D and GEMS2D in response to the sine wave inlet fluid temperature with the amplitude of A_{in} = 5 K and the frequency of 1/600. At this particular frequency, the response calculated by GEMS3D, shows A_{out} = 2.325 K and ΔP = 225 s (θ = 0.75π).

The amplitude of the outlet fluid temperature predicted by GEMS2D is similar to the one predicted by GEMS3D at this particular frequency although the phase angle is much smaller. A range of frequencies have been applied to the inlet fluid temperature in order to obtain the full frequency response, and part of the results of the corresponding outlet fluid temperatures by GEMS3D and GEMS2D are shown from Figure 13 to Figure 17.

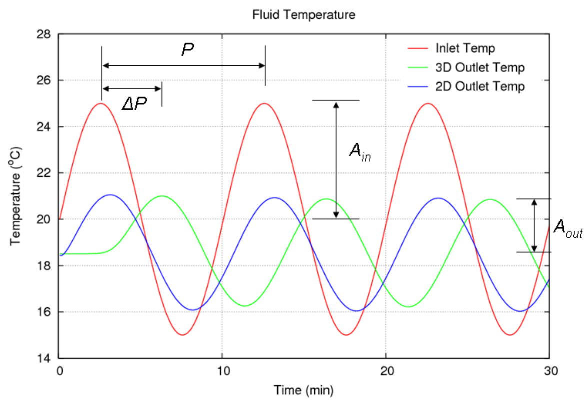

Figure 12: Frequency responses by GEMS3D and GEMS2D at *f* = *1/600*.

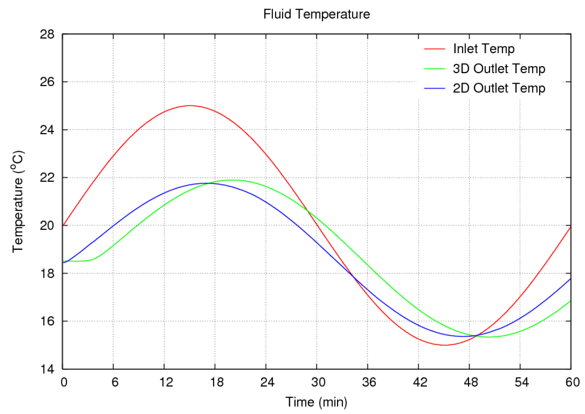

Figure 13: Frequency responses by GEMS3D and GEMS2D at *f* = *1/3600*.

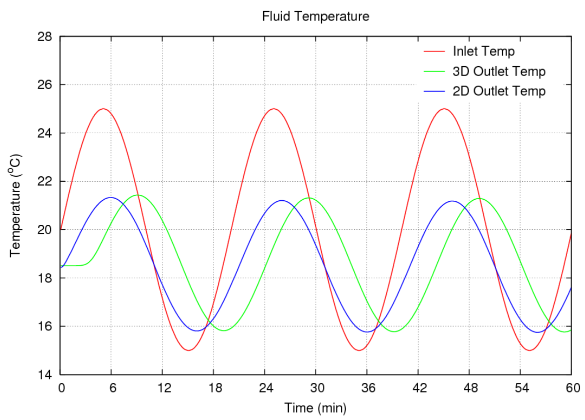

Figure 14: Frequency responses by GEMS3D and GEMS2D at *f* = *1/1800*.

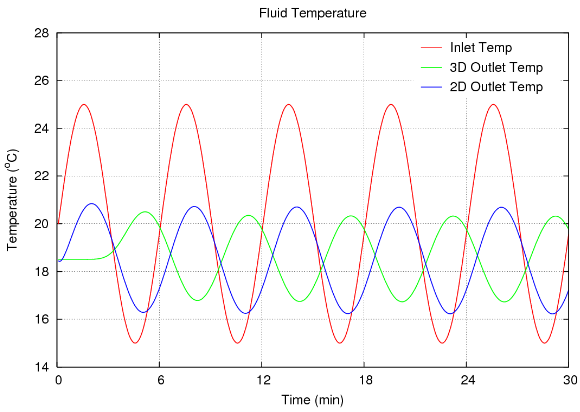

Figure 15: Frequency responses by GEMS3D and GEMS2D at *f = 1/360*.

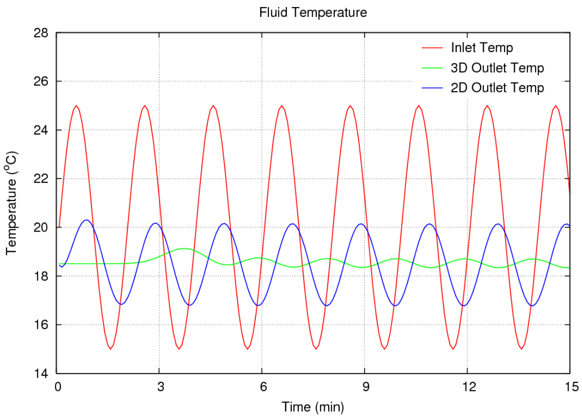

Figure 16: Frequency responses by GEMS3D and GEMS2D at *f = 1/120*.

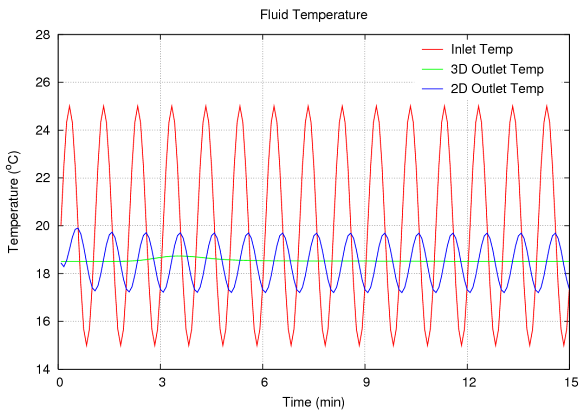

Figure 17: Frequency responses by GEMS3D and GEMS2D at *f = 1/60*.

Figure 18 shows amplitude ratio (A_{in}/A_{out}) calculated by GEMS3D and GEMS2D on a logarithmic frequency scale. The amplitude ratio calculated by GEMS2D decreases almost linearly with frequency. In this model the damping is only due to diffusion of heat into and out of the inlet pipe and adjacent grout. However, GEMS3D predicts a much reduced response when the frequency is higher than 0.00028. In the GEMS3D model the response is damped by the moving fluid as well as heat transfer into and out of the fluid. At high frequencies the variation in inlet temperature results in no change in outlet temperature suggesting that the fluctuations are damped out by heat exchange relatively close to the pipe inlet.

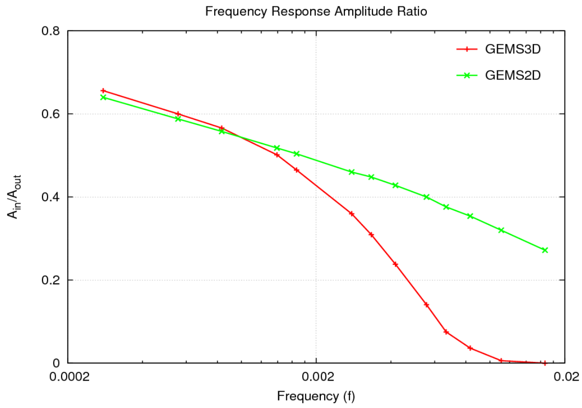

Figure 18: Variation of Amplitude ratio (A_{in}/A_{out}) according to frequency predicted by the GEMS3D and GEMS2D models.

Figure 19 shows the variation of phase shift according to frequency for the three-dimensional model GEMS3D. The phase can be more than 2π, and in this case the outlet fluid temperature comes to the peak after the second peak of the inlet fluid temperature occurs (Figure 16). When the frequency gets higher, the phase $\rightarrow \infty$, and this means the outlet fluid temperature become constant (Figure 17). Therefore, all the fluctuation in the inlet fluid temperature will not have any effect on the outlet fluid temperature.

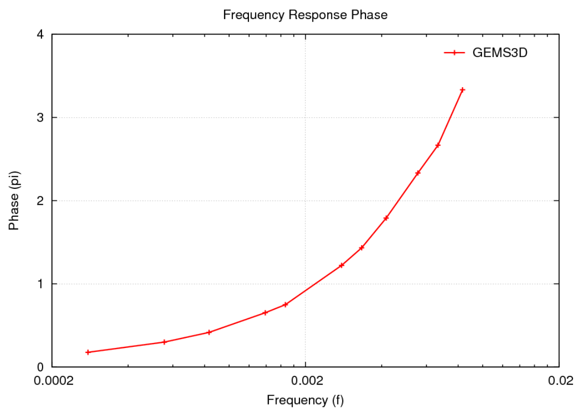

Figure 19: Phase in frequency response by GEMS3D.

6. CONCLUSIONS AND FUTURE WORK

A three-dimensional numerical model (GEMS3D) for Borehole Heat Exchangers (BHEs) has been developed to simulate the fluid transport along the pipe loop as well as heat transfer with the ground. The model applies the finite volume method to solve the partial differential heat transfer equation on three-dimensional boundary fitted grids, which enable the method to be applied to complex geometries of BHEs. The model has been validated by reference to analytical models of borehole thermal resistance and also the fluid transport inside the pipe.

Both the steady-state and the dynamic responses of a BHE by the three-dimensional model have been investigated and compared with those responses by the two-dimensional models. The results show similar fluid temperatures and steady state heat transfer rates. However, delayed response associated with the transit of fluid along the pipe loop, is of some significance at short time scales (10 minutes) but much longer than the transition time of the fluid along the pipe loop (200 seconds). This effect, both in terms of response, control operation and effective heat transfer rate maybe significant where there is on/off control or where peak loads of buildings are important.

Study of BHE characteristics using this detailed three-dimensional model gives insights into the different behaviours of the three-dimensional and two-dimensional models, as well as the limitations of two-dimensional models. Though the three-dimensional model can simulate more realistic dynamic responses of a BHE, it is still not practical for system annual simulation due to the computation power it requires. A more practical two-dimensional model which can capture the same dynamics response as the three-dimensional model is being developed.

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REFERENCES

Bandyopadhyay, G., Gosnold, W., and Mann, M.: Analytical and Semi-Analytical Solutions for Short-time Transient response of ground heat exchangers, *Energy and Buildings*, **40**, (2008), 1816-1824.

Bennet, J., Claesson, J., and Hellstrom, G.: Multipole Method to Compute the Conductive Heat Flows To and Between Pipes In a Composite Cylinder, University of Lund, Sweden (1987).

Bosworth, R. C. L.: *Phil. Mag.*, **39**, (1949), 847-850.

Carslaw, H. S., and Jaeger, J. C.: *Conduction of Heat in Solid*, Claremore Press, Oxford (1947).

Eskilson, P.: Thermal Analysis of Heat Extraction Boreholes, University of Lund, Lund (1987).

Ferziger, J. H., and Peric, M.: *Computational Methods for Fluid Dynamics*, Springer, Berlin (2002).

Fisher, D. E., Rees, S. J., Padhmanabhan, S. K., and Murugappan, A.: Implementation and Validation of Ground-Source Heat Pump System Models in an Integrated Building and System Simulation Environment, *HVAC&R Research Special Issue*, **12(3a)**, (2006), 693-710.

Hanby, V. I., Wright, J. A., Fletcher, D. W., and Jones, D. N. T.: Modelling the Dynamic Response of Conduits, *HVAC&R Research*, **8(1)**, (2002), 1-12.

Hellstrom, G.: Ground Heat Storage: Thermal Analysis of Duct Storage Systems: Part I Theory, University of Lund, Lund (1991).

He, M., Rees, S. J., and Shao, L.: Applications of a Dynamic Three-Dimensional Numerical Model for Borehole Heat Exchangers, *Effstock Conference Proceedings*, Stockholm (2009).

Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C.: *Heat Conduction with Engineering, Geological, and Other Applications*, McGraw-Hill, New York (1954).

Kummert, M., and Bernier, M.: Sub-hourly Simulation of Residential Ground Coupled Heat Pump Systems, *Building Serv. Eng. Res. Technol.*, **29(1)**, (2008), 27-44.

Lee, C. K., and Lam, H. N.: Computer Simulation of Borehole Ground Heat Exchangers for Geothermal Heat Pump Systems, *Renewable Energy*, **33**, (2008), 1286-1296.

Marcotte, D. and Pasquier, P.: On the Estimation of Thermal Resistance in Borehole Thermal Conductivity Test, *Renewable Energy*, **33**, (2008), 2407-2415.

SEL.: A Transient Systems Simulation Program, User's Manual, Version 14.2, Solar Energy Laboratory, University of Wisconsin-Madison (1997).

Wen, C. Y., and Fan, L. T.: *Models for Flow Systems and Chemical Reactors*, Marcel Dekker, Inc, New York (1975).

Yavuzturk, C., and Spitler, J. D.: A Short Time Step Response Factor Model for Vertical Ground Loop Heat Exchangers, *ASHRAE Transactions*, **105(2)**, (1999), 475-485.

Young, T. R.: Development, Verification, and Design Analysis of the Borehole Fluid Thermal Mass Model for Approximating Short Term Borehole Thermal Responser, Oklahoma State University, Stillwater (2004).

Zeng, H., Diao, N., and Fang, Z.: Heat Transfer Analysis of Boreholes in Vertical Ground Heat Exchangers, *Internation Journal of Heat and Mass Transfer*, **46**, (2003), 4467-4481.