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Predictive Reliability Analysis of Ground Coupled Heat Pumps Using Markov Chains

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ABSTRACT

The method of Markov chains with continuous time is analytically presented for use in the study of system reliability. Then, this method is applied to the assessment of predictive reliability indicators of a geothermal heat pump system. The number and complexity of the necessary initial data, accuracy of the evaluation, number of reliability indicators that may be determined, and hierarchy of component sub-systems are given, considering their impact on the reliability of geothermal heat pump systems.

1. INTRODUCTION

For predictive reliability analysis (design stage and operation stage), it is very important to keep in mind that geothermal heat pump systems can be considered as complex technical systems that have restoration and consist of different sub-systems and elements. The sub-systems also have restoration, which is characteristic to most of the elements.

In order to estimate the predictive reliability indicators of a ground coupled heat pump system, the following issues must be known:

- the technological structure, detailed at least to the level at which the reliability analysis is done;
- the operating manner of the system, sub-systems and all its components, till the level of the reliability study is extended;
- reliability indicators of sub-systems and components directly involved in the equivalent reliability scheme;
- settlement of the analysis method and, consequently, the mathematical modeling according to the envisaged reliability indicators.

2. MARKOV CHAINS WITH CONTINOUS TIME

Modeling the operational behavior of a ground coupled heat pump system using a Markov chain with continuous time may be done according to the following hypothesis [1]:

- failure and repair of component elements is random, so the momentary state depends only on the previous state;
- repartition functions of the operational time and maintenance time of all elements are exponential, and therefore haveconstant time failure rate (λ) and respective repair rate (μ).

The time evolution of a heat pump system might be assimilated - through different states that can occur as a result of breakdown and repair of component elements – with a Markov process with continuous time and stationary transition probabilities. Therefore, state probabilities can be determined – for a given period of analysis – using the matrix system of differential equations:

$$[P'(t)] = [a_{ij}] \cdot [P(t)] \tag{1}$$

where [P(t)] is matrix of state probability at t moment and $[a_{ij}]$ is a transition matrix (matrix of intensity switches from one state to another), whose elements satisfy the following relations:

$$\begin{cases} \sum_{j=1}^{n} a_{ij} = 0,\ (\forall) i = \overline{1,n} \\ a_{ij} \geq 0,\ (\forall)\ i, j = \overline{1,n},\ i \neq j \\ a_{ii} \leq 0,\ (\forall) i = \overline{1,n} \end{cases} \tag{2}$$

where *n* is the number of possible states of the analyzed system.

Solving the system (1) through analytical methods leads to several difficulties. This is the reason why, for engineering assessments, there is the possibility of transforming it into an algebraic system of equations. In order to do this, it may be considered that for large times ($t \to \infty$), absolute probabilities P(t) are practically independent of the initial state, and therefore are considered constant, leading to $P'(t) \to 0$, and the system becomes:

$$[a_{ij}] \cdot [P] = 0 \tag{1}$$

Since matrix $[a_{ij}]$ is singular, the system is indeterminate and, when solving it, one of the equations is replaced by the following relation:

$$\sum_{i=1}^{n} P_i = 1 \tag{2}$$

where P_i are the elements of the states probability matrix [P].

The elements of the transition matrix are determined as follows:

* if the transition from *i* state into *j* is done when an element fails, then $a_{ij} = \lambda$, where (λ) is the failure rate;
* if the transition from *i* state into *j* is done when an element is repaired, then $a_{ij} = \mu$, where (μ) is the repair rate;

* if the transition from i into j is done when an element is replaced, then $a_{ij} = \nu$, where (ν) is the replacement rate;

* according to equations (2) we can write:

$$a_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij}, (\forall) i = \overline{1,n}$$

To simplify the form of the transition matrix, the states of the system and the transition between them can be illustrated by the so-called transition graph.

Solving the systems (3) and (4) allows us to determine the absolute state probabilities (P_1, P_2, ..., P_n), which serve to find reliability indicators of the heat pump system, using the following equations:

* The success probability of the heat pump system is given by:

$$P_S = \sum_{i \in S} P_i \tag{3}$$

where S is the set of success states.

* The refusal probability of the heat pump system is given by:

$$P_R = 1 - P_S = \sum_{i \in R} P_i \tag{4}$$

where R is the set of refusal (non-success) states.

* Mean total probable period of success of the system for the analyzed period T is given by:

$$M[\alpha(T)] = P_S \cdot T \tag{5}$$

* Mean total probable period of refusal of the system for the analyzed period T is given by:

$$M[\beta(T)] = P_R \cdot T \tag{6}$$

* Mean probable number of transitions from success states into refusal states (mean number of failures) for the analyzed period T:

$$M[\nu(T)] = \left[\sum_{i \in S} \left(P_i \cdot \sum_{j \in R} a_{ij} \right)\right] \cdot T \tag{7}$$

* Mean probable time of uninterrupted operation (between two failures) is given by:

$$MTBF = \frac{M[\alpha(T)]}{M[\nu(T)]} = \frac{1}{\lambda_S} \tag{8}$$

where λ_S is the equivalent failure rate at system level and MTBF is the mean time between failures.

* Mean probable time of maintenance is given by:

$$MTM = \frac{M[\beta(T)]}{M[\nu(T)]} = \frac{1}{\mu_S} \tag{9}$$

where μ_S is the equivalent repair rate at system level and MTM is the mean time of maintenance.

* Probability of uninterrupted operation for a given time interval (t, t+Δt) is given by:

$$R(t, t + \Delta t) = P_S \cdot e^{-\lambda_S \cdot \Delta t} \tag{10}$$

* Mean probable number of failures for the analyzed period T, that do not exceed a critical value t_{cr} is given by:

$$M[\nu_{tcr}(T)] = M[\nu(T)] \cdot e^{-\mu_S \cdot t_{cr}} \tag{11}$$

In cases of systems having a large number of elements, the state number is too big and solving the equation systems (3), (4) is quite difficult. To avoid this situation, the ***method of general solution*** [2] for determination of absolute state probability is used. This method allows determining the absolute probability, hence denominator addends from the expression of probability of the state with all elements in operation (P_0) show the occurrence order of system failures.

Probability of the state with all elements in operation is given by:

$$\begin{aligned} P_0^{-1} = 1 &+ \sum_{i \in S_1} \frac{\lambda_i}{\mu_i} + \sum_{i \in S_1} \left(\frac{\lambda_i}{\mu_i} \cdot \sum_{\substack{j \in S_2 \\ j \neq i}} \frac{\lambda_j}{\mu_i + \mu_j} \right) + \\ &+ \sum_{i \in S_1} \left[\frac{\lambda_i}{\mu_i} \cdot \sum_{\substack{j \in S_2 \\ j \neq i}} \left(\frac{\lambda_j}{\mu_i + \mu_j} \cdot \sum_{\substack{k \in S_3 \\ k \neq j \neq i}} \frac{\lambda_k}{\mu_i + \mu_j + \mu_k} \right) \right] + \ldots \end{aligned} \tag{14}$$

where (S_1, S_2, S_3, ...) is the set of states with (1, 2, 3, ...) broken elements.

The absolute probabilities of states having simple, double or triple failures are expressed as follows:

$$\begin{cases} P_i = \dfrac{\lambda_i}{\mu_i} \cdot P_0;\ i = \overline{1,n} \ \ \textit{simple failure} \\ P_{ij} = \dfrac{\lambda_i}{\mu_i} \cdot \dfrac{\lambda_j}{\mu_i + \mu_j} \cdot P_0;\ i, j = \overline{1,n}; \\ \qquad\qquad i \neq j - \textit{duble failure} \\ P_{ijk} = \dfrac{\lambda_i}{\mu_i} \cdot \dfrac{\lambda_j}{\mu_i + \mu_j} \cdot \dfrac{\lambda_k}{\mu_i + \mu_j + \mu_k} \cdot P_0; \\ i, j, k = \overline{1,n}; \quad i \neq j \neq k - \textit{triple failure} \end{cases} \tag{12}$$

This method avoids solving the matrix equation, but requires a thorough analysis when the terms corresponding to failures are written. Generally, for energy installations, and especially for heat pump systems, triple failures are most considered, and the occurrence probability of states has more than three broken elements being neglected.

In order to calculate the reliability indicators of complex systems and installations (that have a great number of elements), the ***method of equivalent elements*** [1] may be used. To simplify the calculations, this method allows the replacement of some group of elements with equivalent elements. The assimilation conditions require the equivalent element to get the same reliability indicators as for the group of assimilated elements. By doing this, only the values of random variable averages are kept, not their distributions and dispersions. The equivalent relations for different type of configurations are presented in the literature [1, 3].

3. RELIABILITY ASSESSMENT OF A GROUND COUPLED HEAT PUMP SYSTEM USING MARKOV CHAINS

From the reliability point of view, the ground coupled heat pump system consists of several sub-systems, each having one or more elements. The ground coupled heat exchanger sub-system may have either a horizontal heat exchanger (HHE) or a borehole heat exchanger (BHE). The heat pump sub-system consists of an evaporator (V), a compressor (K) and a condenser (C). The backup heating system is an electrical one (EHS). Also, there is the sub-system of auxiliary elements needed for the best operation of the system. It consists of a boiler (B), an automation and control system (ACS), a pipes and fittings system (PFS) and circulation pumps (P).

Based on the schematic diagram (Figure 1), the equivalent reliability diagram is made. The following notations are used: 1 – ground coupled heat exchanger (1.1 - BHE, 1.2 – HHE); 2 – heat pump (2.1 – V, 2.2 – K, 2.3 - C); 3 – backup heating system EHS; 4 – auxiliary elements of the heating system (4.1 – B, 4.2 – ACS, 4.3 – PFS, 4.4 - P).

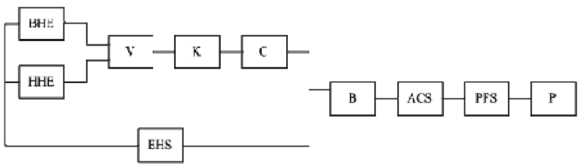

Figure 1: Schematic diagram of the analyzed heating system.

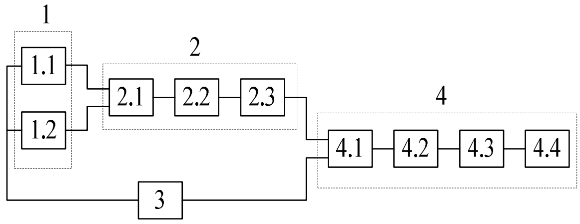

Figure 2: Equivalent reliability diagram of the system.

According to the equivalent reliability diagram and using the equivalent elements method, the system was reduced to 4 equivalent elements characterized by the following reliability indicators (failure rate λ and repair rate μ):

$$\begin{cases} \lambda_1 = \dfrac{\lambda_{1.1} \cdot \lambda_{1.2} \cdot (\mu_{1.1} + \mu_{1.2})}{\lambda_{1.1} \cdot \mu_{1.2} + \lambda_{1.2} \cdot \mu_{1.1} + \mu_{1.1} \cdot \mu_{1.2}} \\ \mu_1 = \mu_{1.1} + \mu_{1.2} \end{cases}$$

$$\begin{cases} \lambda_2 = \lambda_{2.1} + \lambda_{2.2} + \lambda_{2.3} \\ \mu_2 = \dfrac{\lambda_2}{\dfrac{\lambda_{2.1}}{\mu_{2.1}} + \dfrac{\lambda_{2.2}}{\mu_{2.2}} + \dfrac{\lambda_{2.3}}{\mu_{2.3}}} \end{cases} \tag{16}$$

$$\begin{cases} \lambda_4 = \lambda_{4.1} + \lambda_{4.2} + \lambda_{4.3} + \lambda_{4.4} \\ \mu_4 = \dfrac{\lambda_4}{\dfrac{\lambda_{4.1}}{\mu_{4.1}} + \dfrac{\lambda_{4.2}}{\mu_{4.2}} + \dfrac{\lambda_{4.3}}{\mu_{4.3}} + \dfrac{\lambda_{4.4}}{\mu_{4.4}}} \end{cases}$$

The states graph for the analyzed system is shown in Figure 3. Then, the transition matrix is written as follows:

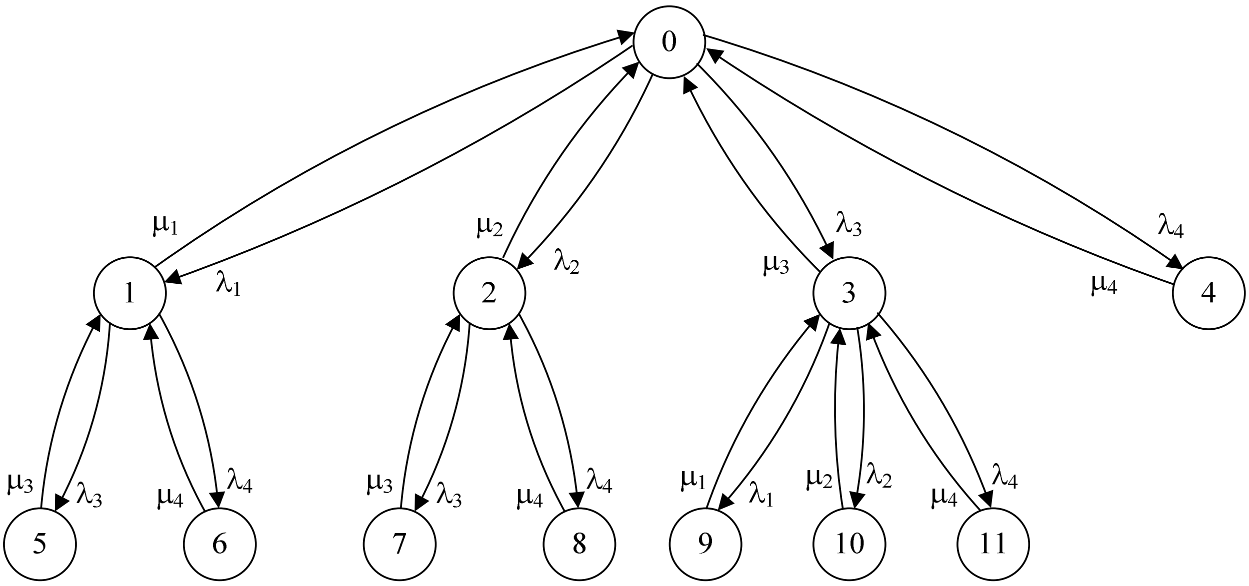

Figure 3: Graph of the system states.

$$[a_{ij}]=\begin{bmatrix}
-\sum_{i=1}^{4}\lambda_i & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda_1 & -(\mu_1+\lambda_3+\lambda_4) & 0 & 0 & 0 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 & 0\\
\lambda_2 & 0 & -(\mu_2+\lambda_3+\lambda_4) & 0 & 0 & 0 & 0 & \mu_3 & \mu_4 & 0 & 0 & 0\\
\lambda_3 & 0 & 0 & -(\mu_3+\lambda_1+\lambda_2+\lambda_4) & 0 & 0 & 0 & 0 & 0 & \mu_1 & \mu_2 & \mu_4\\
\lambda_4 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda_3 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda_4 & 0 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0\\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0\\
0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0\\
0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4
\end{bmatrix} \quad (17)$$

States probabilities obtained after solving the equation system (3, 4) are given by the following expressions:

$$P_0^{-1}=1+\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_1}{\mu_1}\cdot\left(\frac{\lambda_3}{\mu_1+\mu_3}+\frac{\lambda_4}{\mu_1+\mu_4}\right)+\frac{\lambda_2}{\mu_2}\cdot\left(\frac{\lambda_3}{\mu_2+\mu_3}+\frac{\lambda_4}{\mu_2+\mu_4}\right)+\frac{\lambda_3}{\mu_3}\cdot\left(\frac{\lambda_1}{\mu_3+\mu_1}+\frac{\lambda_2}{\mu_3+\mu_2}+\frac{\lambda_4}{\mu_3+\mu_4}\right)$$

$$P_1=\frac{\lambda_1}{\mu_1}\cdot P_0\,;\quad P_2=\frac{\lambda_2}{\mu_2}\cdot P_0\,;\quad P_3=\frac{\lambda_3}{\mu_3}\cdot P_0\,;\quad P_4=\frac{\lambda_4}{\mu_4}\cdot P_0\,; \quad (18)$$

$$P_5=\frac{\lambda_1}{\mu_1}\cdot\frac{\lambda_3}{\mu_1+\mu_3}\cdot P_0\,;\quad P_6=\frac{\lambda_1}{\mu_1}\cdot\frac{\lambda_4}{\mu_1+\mu_4}\cdot P_0\,;$$

$$P_7=\frac{\lambda_2}{\mu_2}\cdot\frac{\lambda_3}{\mu_2+\mu_3}\cdot P_0\,;\quad P_8=\frac{\lambda_2}{\mu_2}\cdot\frac{\lambda_4}{\mu_2+\mu_4}\cdot P_0\,;$$

$$P_9=\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_1}{\mu_3+\mu_1}\cdot P_0\,;\quad P_{10}=\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_2}{\mu_3+\mu_2}\cdot P_0\,;$$

$$P_{11}=\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_4}{\mu_3+\mu_4}\cdot P_0$$

Success states are {0, 1, 2, 3}; therefore, the success probability, respectively refusal probability will be:

$$P_S=P_0+P_1+P_2+P_3\,;\quad P_R=1-P_S=\sum_{i=4}^{11}P_i \quad (19)$$

Reliability indicators of the system are calculated using relations (7 ÷ 13).

4. NUMERICAL EXAMPLE

A calculation example for the determination of operational reliability indicators of a heating system, having the schematic diagram shown in Figure 1, is presented below.

The considered values for failure rate and respective repair rate are shown in Table 1.

Table 1: Failure and repair rate values for the component elements of the heating system that uses heat pump.

Element	λ [x10^{-4} h^{-1}]	μ [x10^{-4} h^{-1}]
BHE	0.04	40
HHE	0.04	40
V	0.1	500
K	0.3	400
C	0.08	500
EHS	0.1	600
B	0.05	700
ACS	0.2	300
PFS	0.1	200
P	0.25	500

Using relations (16 ÷ 19), the state probabilities of the system were calculated (Table 2).

Table 2: Probability values of the system states.

State	Probability of state occurence
0	0.996130476
1	$9.936\cdot10^{-7}$
2	0.001105788
3	0.000166022
4	0.001731397
5	$1.46\cdot10^{-10}$
6	$1.40\cdot10^{-9}$
7	$1.07\cdot10^{-7}$
8	$8.53\cdot10^{-7}$
9	$1.95\cdot10^{-11}$
10	$7.72\cdot10^{-8}$
11	$1.05\cdot10^{-7}$

Reliability indicators of the system (5 ÷ 11) have the following values indicated in Table 3.

Table 3: Reliability indicators of the heating system using heat pump.

Reliability indicator	Symbol	Measurement unit	Value
Probability of success	P_S	-	0.99740328
Probability of refusal	P_R	-	0.00259672
Mean probable period of success of the system for the analyzed period (one year)	$M[\alpha(T)]$	h/yr.	8,737.25
Mean probable period of refusal of the system for the analyzed period (one year)	$M[\beta(T)]$	h/yr.	22.75
Mean probable number of failures in analyzed period (one year)	$M[\nu(T)]$	failures/yr	0.5244
Mean time between failures	MTBF	h	16,661.4
Mean time to maintenance	MTM	h	43.38
Equivalent failure rate of the system	λ_S	h^{-1}	$0.6 \cdot 10^{-4}$
Equivalent repair rate of the system	μ_S	h^{-1}	$230.5 \cdot 10^{-4}$

5. CONCLUSIONS

The method of Markov chains with continuous parameters may be applied to systems that have independent elements, characterized by exponential distributions of random variables. The reliability of systems that are characterized by other distribution laws than the exponential one may be analyzed using the method of semi-Markov processes with continuous parameters. Application of these methods may be seen in [1, 4, 5, 6].

The operational reliability analysis of heating systems that have a ground coupled heat pump using the Markov chains method is slightly more difficult than the graphical-analytical methods, but offers the possibility to determine more reliability indicators ($M[\alpha(T)]$, $M[\beta(T)]$, $M[\nu(T)]$, MTBF, MTM, λ_S, μ_S). Based on the values of these indicators, a more detailed and precise image regarding the reliability performances of these systems may be determined.

The numerical example shows how this method is applied and also the obtained values of reliability indicators. In order to these values have a high rank of accuracy, the input data (failure rate λ and repair rate μ of the component elements of the heating system that have a ground coupled heat pump) must have credible values.

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