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Graphical-Analytical Methods Usable for Predictive Reliability Study of Heat Pumps

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ABSTRACT

A short theoretical presentation of graphical-analytical methods used for the analysis of system predictive reliability is given. This includes a reliability equivalent diagram, fault tree, events tree, table of true, minimal cuts, and minimal success paths.. Case studies related to the assessment of predictive reliability of heat pumps using the above mentioned methods are also discussed.

1. INTRODUCTION

Predictive reliability has a prognosis character and represents the reliability of a system expressed through the indicators that resulted from calculations made by using the reliability of system component elements. Predictive reliability analyses are made both in the design stage of new installations and in the exploitation stage of certain systems in order to be able to forecast their availability (in time, power or energy).

Predictive reliability analysis of a system (based on its structure) may be characterized as being accurate enough only when the behavior of its component elements (from the reliability point of view) is known and also when the operation conditions are previously known. Reliability indicators of component elements are used in order to evaluate the predictive reliability indicators of a system; if their values don't correspond to reality, calculation results will not offer a correct image of the system's future reliability behavior. On the other hand, there are frequent situations when the system operation deviates from the standard conditions (for which the reliability indicators of the component elements are guaranteed), leading to a deviation of operational reliability indicators from the calculated values (previewed ones).

The necessity of studying the predictive reliability of technical systems leads to a diversified development of probabilistic models and methods of calculation, motivated by the nature and complexity of installations. In the following, a short overview of the main methods and models for reliability calculations in the energy engineering domain are made.

Predictive reliability analyses for any type of systems may be divided in two categories, (Pages and Gondran, 1980):

A. **Qualitative Analysis** aims to provide information regarding how component element failure affects the operation of the entire system. The following stages are present when a qualitative analysis is made:

- ♦ analysis of the failure manner and the effects of the failure (AFME), that allows the failures identification and the assessment of their consequences on system operation;
- ♦ organization and graphical representation of the information resulting from AFME as a logical scheme (equivalent reliability diagram, fault tree or events tree).

Objectives of the qualitative analysis are:

- weak points identification in early design stages, providing data for their removal actions;
- highlighting potential faults and identification of their significance;
- providing the necessary information for a quantitative reliability analysis.

B. **Quantitative Analysis** aims to quantify – as numerical indicators – the reliability level in order to:

- compare two or more solutions from a reliability performance point of view;
- verify how the reliability indicators fit certain required limits at the border with other systems;
- detect the weak link of the analyzed system;
- forecast some guarantee indicators that are included in the contracts between suppliers and beneficiary.

Calculation methods of systems reliability can be grouped either with respect to their nature, or to the nature of repartition functions of random variables, as it is shown in Table 1 (Nitu et al., 1993).

Table 1 – Classification of probabilistic methods for systems reliability calculation

By the nature of probabilistic methods		By the nature of the repartition functions	
Analytic	Graphical-analytical	Exponential	Non-exponential
• binomial; • Markov chains with continuous parameter; • Markov chains with discrete parameter; • semi-Markov chains with continuous parameter; • Monte-Carlo.	• event trees; • fault trees; • equivalent reliability diagrams; • Petri type networks.	• Markov chains with continuous parameter; • Markov chains with discrete parameter.	• binomial; • semi-Markov chains with continuous parameter; • Monte-Carlo; • events tree; • fault tree; • equivalent reliability diagrams; • Petri type networks.

2. GRAPHICAL-ANALYTICAL METHODS USABLE FOR PREDICTIVE RELIABILITY

Representing a system through a logical diagram means to represent all operating and failure stages of a system and the links between those stages. If the system accepts more levels of operation (success), there will be one logical diagram for each level.

All the methods presented below may be used for systems that have independent elements – from the reliability point of view – characterized by two stages: operation/failure.

2.1 Equivalent Reliability Diagram e.g. Pages and Gondran, (1980)

The equivalent reliability diagram (ERD) is a graphical representation, having an entrance (I) and an exit (E), and where the blocks represents the system's elements and the links between the blocks are the relations between the elements. Based on AFME, the success status of a system is determined and then the ERD is built. If the system accepts several success states for each of them a ERD will be built. The success is assured if there is a "success path" between the entrance and exit of ERD. The list of "success paths" allows the representation of all success stages of the system.

Knowing the reliability indicators of the component elements the ones of the entire system can be calculated, using the following relations:

- for series elements, the system reliability is:

$$R_S(t)=\prod_{i=1}^{n} R_i(t) \tag{1}$$

- for parallel elements, the system reliability is:

$$R_S(t)=1-\prod_{i=1}^{n}\left[1-R_i(t)\right]=1-\prod_{i=1}^{n} F_i(t) \tag{2}$$

where: $R_i(t)$, $F_i(t)$ are the reliability and non-reliability functions of the system component elements.

- if the elements are characterized by the exponential distribution law, knowing λ_i and μ_i, the failure and repair intensities will be:

$$\begin{cases} -\textit{for series elements} \\ \lambda_S=\sum_{i=1}^{n}\lambda_i ; \mu_S=\dfrac{\lambda_S}{\sum_{i=1}^{n}\dfrac{\lambda_i}{\mu_i}} \\ -\textit{for two elements in parallel} \\ \lambda_S=\dfrac{\lambda_1\cdot\lambda_2\cdot(\mu_1+\mu_2)}{\lambda_1\cdot\mu_2+\lambda_2\cdot\mu_1+\mu_1\cdot\mu_2} ; \mu_S=\mu_1+\mu_2 \end{cases} \tag{3}$$

2.2. Fault Tree , Pages and Gondran, (1980)

Fault tree (FT) is built with respect to a certain unsuccessful stage – unique and well defined – named as the unwanted event. FT is the graphical representation of combination of events that lead to the occurrence of unwanted event. FT is made by successive levels, so that each event (except the basic ones or the assumed basic ones) to be generated starting from the inferior level events through some operators, named logical gates. Symbols and terminology used for the building of a FT are shown in literature of Billigton and Allan (1987), Pages and Gondran, (1980), and Felea (1996).

For the quantitative assessment of a FT the probability of events occurrence (p_{Ei}) must be known. The probability of occurrence of the unwanted event is calculated through successive evaluations, using the following rules:

* for "AND" gate:

$$p(E_1 \cap E_2)=p_{E1}\cdot p_{E2} \tag{1}$$

* for "OR" gate:

$$p(E_1 \cup E_2)=p_{E1}+p_{E2}- \\ -p_{E1}\cdot p_{E2}\cong p_{E1}+p_{E2} \tag{2}$$

2.3. Events Tree

The events tree is a graphical representation - having an arborescent shape – of all events that may occur at the component elements level (Billigton and Allan, 1987). It is obtained through the combination of elements states (operation/failure – O/F) thus resulting all the states that system might be in. From the set of all these states the ones that insure the success (previously settled) respectively the refuse are identified.

Knowing the reliability function (R_i) and the non-reliability function (F_i) of the component elements, the entire system reliability, respectively non-reliability can be calculated:

$$R_S=\sum_{i\in\{S\}}\left(\prod_{j\in\{F_i\}}R_j\cdot\prod_{k\in\{D_i\}}F_k\right) \\ F_S=1-R_S \tag{3}$$

where:{S} is the set of success stages of the system; {F_i} is the set elements in operation that insure the "i" success stage; {D_i} is the set of fault elements that belong to "i' success stage.

2.4. Table of Truth

Table of truth is a representation model of the states of a system by marking all the possible combinations of components states (operation/failure – O/F) and highlighting the states that insure the success (S) of the system (Billigton and Allan, 1987).

For a quantitative assessment of system reliability, the above mentioned relations (6) are used.

2.5. Minimal Cuts

A cut is an assembly of elements whose failure leads to whole system failure. A minimal cut (MC) is a cut that hasn't got any elements belonging to other cut (the minimal set of elements that lead to system failure) (Pages and Gondran, 1980). Minimal cut set may be determined directly, by combining the faults of system elements (a difficult method, especially for systems that have a large number of elements), or starting from ERD or FT. It is recommended – for simplicity reasons – to settle the set of MC starting from ERD.

Interpretation of MC leads to several qualitative conclusions: weak points of the system, false redundancies, influence of a certain element on the entire system reliability etc. Furthermore, MC allow also the assessment of system reliability, if the reliability indicators of component elements [$R_i(t)$ şi $F_i(t)$] are known.

If $\overline{T}_j$ is minimal cut, the probability that, at time *t*, (i) elements of this MC fail, is:

$$P\left(\overline{T}_j\right)=\prod_{i\in T_j}F_i(t) \tag{4}$$

If the set of MC is $\left\{\overline{T}_1,\overline{T}_2,..,\overline{T}_m\right\}$, then the non-reliability of the system is expressed by the probability that at least one MC should in failure:

$$F_S(t)=\Pr ob\left(\overline{T}_1\cup\overline{T}_2\cup...\cup\overline{T}_m\right) \tag{5}$$

The system non-reliability will be evaluated based on the general relation (Poincaré formula):

$$\begin{aligned}F_S(t)&=\sum_{i=1}^{m}P\left(\overline{T}_i\right)-\sum_{j=2}^{m}\sum_{i=1}^{j-1}P\left(\overline{T}_j\cdot\overline{T}_i\right)+\\&+\sum_{k=3}^{m}\sum_{j=2}^{k-1}\sum_{i=1}^{j-1}P\left(\overline{T}_k\cdot\overline{T}_j\cdot\overline{T}_i\right)+...+\\&+(-1)^m\cdot P\left(\overline{T}_1\cdot\overline{T}_2\cdot...\cdot\overline{T}_m\right)\end{aligned} \tag{6}$$

Since the values of elements non-reliability (F_i) are relatively small, for quite precise evaluations a simplified expression is frequently used:

$$F_S(t)=\sum_{j=1}^{m}P\left(\overline{T}_j\right)=\sum_{j=1}^{m}\prod_{i\in T_j}F_i(t) \tag{7}$$

2.6. Minimal Success Paths, Pages and Gondran, (1980)

A success path is an assembly of elements whose operation leads to entire system operation. The system logic is represented by minimal success paths C_j, where $j=\overline{1,m}$.

System reliability $R_S(t)$ is defined as the probability that – at least – one of the minimal success paths is operational:

$$R_S(t)=\Pr ob\left(C_1\cup C_2\cup...\cup C_m\right) \tag{8}$$

Reliability is, therefore, given by Poincaré's formulation:

$$\begin{aligned}R_S(t)&=\sum_{i=1}^{m}P(C_i)-\sum_{j=2}^{m}\sum_{i=1}^{j-1}P\left(C_j\cdot C_i\right)+\\&+\sum_{k=3}^{m}\sum_{j=2}^{k-1}\sum_{i=1}^{j-1}P\left(C_k\cdot C_j\cdot C_i\right)+...+\\&+(-1)^m\cdot P\left(C_1\cdot C_2\cdot...\cdot C_m\right)\end{aligned} \tag{9}$$

where $P(C_j)$ represents the probability that (i) elements of the minimal success path are in operation at "t" moment, therefore:

$$P\left(C_j\right)=\prod_{i\in C_j}R_i(t) \tag{10}$$

As each element has a good reliability (R_i is close to 1), every term of equation (12) is close to 1 and, therefore, no simplifying assumptions can be made. That's the reason why, for systems having more elements, it is not recommended to apply the success path method, but minimal cuts method instead.

3. GRAPHICAL – ANALYTICAL METHODS APPLIED TO THE RELIABILITY STUDY OF HEAT PUMPS

In the following, in order to exemplify the application of the above mentioned methods, a simple case of a ground source heat pump system is considered. This system might be installed in any dwelling and its rated value for the heating power is S_n. To ensure the inhabitants a proper thermal comfort in case of heat pump (HP) system breakdown, another heating source is available (gas or electric boiler - B), 100 % redundant (dimensioned 100%), meaning that it has the same rated power as the ground source heat pump system S_n (Figure 1). Heat is supplied to the consumer by means of a circulating pump (P).

The heat pump system consists of several elements: the evaporator (V), the compressor (K), the condenser (C), automation and control system (SAC) and pipes and fitting system (SPF).

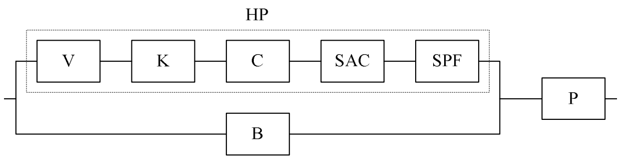

Figure 1 – Schematic diagram for the considered example

The reliability of this diagram will be analyzed in hypothesis that success means "the presence of heating fluid at the consumer".

For this hypothesis, it is easy to see that there are two levels of diagram success (100 %), either when ground source heat pump system is working, or when boiler does.

a). Equivalent Reliability Diagram

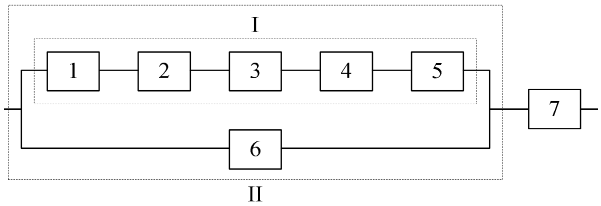

Figure 2 - ERD for the success state in the hypothesis of the heating fluid present at the consumer

The system reliability is:

$$R_S=\left[1-\left(1-R_1\cdot R_2\cdot R_3\cdot R_4\cdot R_5\right)\cdot\left(1-R_6\right)\right]\cdot R_7$$

The failure and repair intensities of the system are:

$$\lambda_I=\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5\,;$$

$$\mu_I=\frac{\lambda_I}{\dfrac{\lambda_1}{\mu_1}+\dfrac{\lambda_2}{\mu_2}+\dfrac{\lambda_3}{\mu_3}+\dfrac{\lambda_4}{\mu_4}+\dfrac{\lambda_5}{\mu_5}}$$

$$\lambda_{II} = \frac{\lambda_I \cdot \lambda_6 \cdot (\mu_I + \mu_6)}{\lambda_I \cdot \mu_6 + \lambda_6 \cdot \mu_I + \mu_I \cdot \mu_6}; \qquad \mu_{II} = \mu_I + \mu_6$$

$$\lambda_S = \lambda_{II} + \lambda_7; \qquad \mu_S = \frac{\lambda_S}{\frac{\lambda_{II}}{\mu_{II}} + \frac{\lambda_7}{\mu_7}}$$

Mean time between failures (MTBF) and mean time of repair (MTR) are:

$$MTBF_S = \frac{1}{\lambda_S}; \quad MTR_S = \frac{1}{\mu_S}$$

b). Fault Tree

The following seven basic events are considered:

* E_1 – evaporator breakdown;
* E_2 – compressor breakdown;
* E_3 – condenser breakdown;
* E_4 – automation and control system breakdown;
* E_5 – pipes and fittings system breakdown;
* E_6 - boiler breakdown;
* E_7 - circulating pump breakdown.

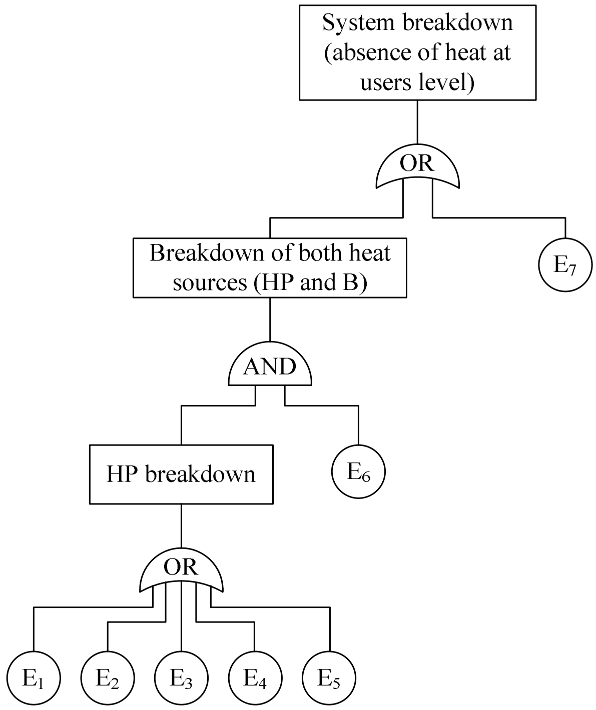

Figure 3 - FT for the unwanted event (UE) "absence of heating fluid at the consumer level"

The probability of occurrence of the unwanted event is:

$$p_{EN} = (p_{E1} + p_{E2} + p_{E3} + p_{E4} + p_{E5}) \cdot p_{E6} + p_{E7}$$

c).Events Tree

Figure 4 presents the simplified events tree, showing the operation (O) and failure (F) states of system components.

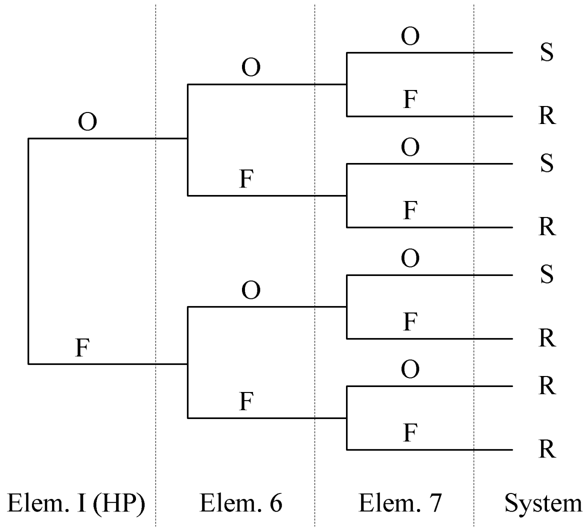

Figure 4 – Simplified ET (without highlighting the HP elements)

The system reliability and non-reliability are:

$$R_S = R_I \cdot R_6 \cdot R_7 + R_I \cdot F_6 \cdot R_7 + F_I \cdot R_6 \cdot R_7;$$

$$F_S = 1 - R_S$$

d). Table of truth

State	Elements			System
	I (HP)	6	7	
1	O	O	O	Success
2	O	O	F	Refuse
3	O	F	O	Success
4	O	F	F	Refuse
5	F	O	O	Success
6	F	O	F	Refuse
7	F	F	O	Refuse
8	F	F	F	Refuse

The evaluation relations are the following:

$$R_S = R_I \cdot R_6 \cdot R_7 + R_I \cdot F_6 \cdot R_7 + F_I \cdot R_6 \cdot R_7$$

$$F_S = 1 - R_S$$

e). Minimal Cuts

- For the hypothesis of "presence of heating fluid at consumer level", the set of MC is: {7}; {1, 6}; {2, 6}; {3, 6}; {4, 6}; {5, 6}, therefore the non-reliability of the system is:

$$F_S = F_7 + F_1 \cdot F_6 + F_2 \cdot F_6 + F_3 \cdot F_6 + F_4 \cdot F_6 + F_5 \cdot F_6$$

f). Minimal Success Paths

Based on the diagram (figure 2) the minimal success paths may be written as follows:

{1, 2, 3, 4, 5, 7} and {6, 7}

therefore, the system reliability is:

$$R_S = R_1 \cdot R_2 \cdot R_3 \cdot R_4 \cdot R_5 \cdot R_7 + R_6 \cdot R_7$$

4. CONCLUSIONS

Graphical-analytical methods used for the evaluation of systems' operational reliability are relatively simple and quick. They can be used by people that don't have thorough knowledge on reliability since for their application isn't needed a complicated mathematical apparatus. These methods can be applied to any system composed by repairable or non-repairable elements, characterized by any type of distribution function of random variables „time between failures" and „time to corrective maintenance" (exponential, Weibull, normal, lognormal etc.).

By applying these methods a relative small number of reliability indicators may be determined:

- Reliability function R(t);
- Non-reliability function F(t);
- Probability of success P_S;
- Probability of refuse P_R.

If more reliability indicators are needed (probable yearly average period of running/failure, probable average number of failures, mean time between failures (MTBF), mean time to repair (MTR), etc.), more complex analytical methods must be used (binomial, Markov chains, Monte Carlo simulation etc.)

The present study - made for a heating system that has as heat source a ground coupled heat pump - highlights the fact that these graphical-analytical methods can be successfully used and their implementation is relatively simple. For quantitative evaluations of reliability indicators, information are needed regarding the behavior – from the reliability point of view – of components of the analyzed heat pump system These information may be gathered by watching, monitoring, the system in operation. In order to obtain realistic values, a long term monitoring is needed (years), and the number of monitored systems to be as high as possible. Other possible source of finding the reliability indicators of system component elements is represented by the heat pumps manufacturer, but unfortunately they keep a strict confidentiality on these values, thing that makes practically impossible the quantitative assessment of operational reliability.

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