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Optimizing Revenue of a Geothermal System with Respect to Operation and Expansion

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ABSTRACT

The goal of this work is to conduct a decision-making tool that optimizes present value of profit when utilizing low temperature geothermal resources. The profit is measured as the difference between operating cost and price of energy. The usual approach in these studies, regarding the behaviour of the resource, is the lumped parameter model (LPM) which models pressure change in a geothermal reservoir with respect to given production history and drawdown. Here, a mixed integer linear programming (MILP) approach on the LPM will be used. The constraints are sustainability, demand and price of energy. A methodology for a quantitative definition of sustainability is established and coupled into the model. Demand and price of energy is simulated stochastically to account for uncertainty in future propagation using historical data. Results will include model comparison along with profit and lifetime under various conditions for different wells and values of parameters.

1. INTRODUCTION

The vast geothermal resources in Iceland have been utilized to a considerable extent and are primarily used for space heating, various industries, swimming pools and snow melting. Electricity generation using geothermal energy has also increased significantly in recent years and now accounts for over 20% of the total electricity generation in Iceland (Eggertsson, 2008).

Geothermal energy has generally been classified as a renewable energy source since it is believed to recharge at a similar rate as a normal production rate from the resource. This classification has occasionally been challenged, saying that the thermal depletion of geothermal systems requires such a long recovery time that strictly speaking it is not renewable on a human timescale. It has been found (Stefansson, 2000) that renewability of geothermal systems is ultimately determined by the transportation process of heat within the crust. All porous and hydrous geothermal systems can therefore be classified as renewable, with some reservations, while hot dry rock systems can hardly be classified as such (Stefansson, 2000).

It is important to make a clear distinction between renewability and sustainability. Renewability is a property of a resource where energy is continuously replaced at a similar rate as the extraction of energy.. Sustainability on the other hand refers to the exploitation of a resource where the production system applied is able to maintain production levels over long periods of time (Rybach & Mongillo,2006). Although many geothermal energy systems are renewable, the regeneration time of a geothermal reservoir may be quite long and it is unclear how to plan the operation strategy of power plants to ensure profitable and sustainable power production. The size and dynamic characteristics of geothermal reservoirs is often poorly understood when capital investments begin, since thorough exploration is quite costly. The economic horizon of large-scale investments such as power plants may be greater than the characteristic time for regeneration of the reservoir for a given system. Thus it is important to take into account the effects of long-term utilization of such fields, but they are generally not very well known. The demand for the geothermal resource may also vary, whether it is for pure generation of electricity or cogeneration of heat and power. A main concern of this work is to look at how to develop the resource in a sustainable manner in light of different constraints and uncertainty of production capacity, reservoir dynamics and market demand.

Several methods exist for reservoir assessment in geothermal systems. The most commonly used are volumetric methods involving conceptual modeling, detailed mathematical modeling and lumped parameter modeling. Volumetric methods are based on estimation of the total heat stored in a volume of rock but do not take into account the dynamic response of the system (Axelsson, 2008). Detailed mathematical and numerical modeling is the most powerful modeling method available for geothermal reservoirs (Lippmann, O´Sullivan & Pruess, 2001), which can simulate the structure, conditions and response of a geothermal system with reasonable accuracy. Numerical modeling can despite its advantages be very time consuming which makes it an unsuitable method in statistical analysis such as that which will be applied in this study.

In this work lumped parameter modeling (LPM) with a mixed integer linear programming (MILP) approach will be applied for resource assessment. It requires few and commonly available parameters for the physical modeling, but does not require much processing power and has an acceptable accuracy in modeling pressure change for isothermal low-temperature systems (Satman, Sarak & Onur, 2005). Those advantages do come at some cost since lumped parameter models usually do not take into account well spacing or well injection locations, nor do they consider fluid flow within the reservoir. They are also unable to match average enthalpy and the non-condensable gas content of the produced fluid and cannot simulate phase changes or thermal fronts (Pruess, Bodvarsson & Lippmann, 1986).

2. PREVIOUS WORK

Lumped parameter modeling has been successfully applied to geothermal fields around the world, including Iceland (Axelsson,1989, Bjornsson, Axelsson & Quijano, 2005, Axelsson, 1991, Axelsson, Hjartarson and Hauksdóttir , 2002), P.R. of China (Youshi, 2002), Turkey (Satman, Sarak & Onur, 2005), Central America (Bjornsson, Axelsson and Quijano, 2005) and at various other locations. In order to successfully model a geothermal field using lumped parameter modeling some production history must be available. The accuracy of the final model depends on the time span and resolution of available data, since the data is used to estimate model parameters.

Despite the inherent simplicity of lumped parameter modeling it has been shown to predict pressure changes in reservoirs with good accuracy, given sufficient data quality. The ultimate goal of such modeling is to predict the production capacity of the geothermal field. The model then serves as a useful tool in the decision making process with regards to exploitation rate, investment cost and sustainability considerations.

From an economical point of view, excessive production is beneficial, mainly due to the time value of money, where the annual revenue in the early years has the greatest effect upon the present value of the operation. It has been concluded (Lovekin, 2000) that a particular aggressive exploitation scenario resulted in a discounted return of investment and present worth almost three times more than a conservative use of the resource, despite higher costs of make-up wells at later stages in the operation. The main drawback of excessive production is that it can lead to resource deterioration or even depletion. In (Eugster, Ryback, 2000) it was for example shown that the time required for a thermal recovery in a heat pump-coupled well heat exchanger system was roughly equal to production time.

The increased use of geothermal resources has raised questions regarding their renewability and how the resource is harnessed in an optimal manner. It is currently unclear how to design optimal operation strategies of power plants that ensure profitable and sustainable power production. This is a complex problem which requires advanced modelling techniques to be combined with specific expertise in the problem domain. A model of the geothermal reservoir is required in a combination with a model of the operational and market environment including all constraints and objectives. To our knowledge integrated models of geothermal systems including market constraints for operation have not been implemented.

There is a dearth of research on the optimal utilisation of geothermal resources. Stefansson (2000) is one of few who have proposed this issue. Among other things he proposed a strategy where power plants are built in small phases (20-30 MW) and use the first wells to gain understanding of the geothermal system at the same time as generating heat and power. In that way companies can make some profit from geothermal resources at the same time as studying their characteristics and make better judged decisions about further investments (Stefansson, 2002). Prior to this work, wells had been drilled for experiments without using them also for generating heat and power.

Recently there has been an increasing interest and discussion on the sustainable harnessing of geothermal resources (Lovekin, 2000; Stefánsson, 2000; Flovenz, Axelsson & Armannsson, 2001; Rybach, 2006). These studies conclude that despite geothermal resources being generally considered as renewable they can be harnessed in an excessive manner which can lead to depletion or even deterioration if the removal rate of energy is greater than the rate of regeneration. Lovekin (Lovekin, 2000) investigated a certain case where the price of electricity is fixed, demand is unlimited and the production capacity of the geothermal resource is known. He concludes that the optimal harnessing strategy is to build as large a power plant as possible and exploit the resource in an excessive manner due to the time value of money, where the annual revenue in the early years has the greatest effect upon the net present value (NPV) of the operation. Gudni Axelson, Valgardur Stefansson, Grimur Bjornsson and Jiurong Liu (Bjornsson, Axelsson, Stefansson & Liu, 2005) have studied sustainable harnessing which they define as the possible continuous harnessing of the resource over an extended period (100-300 years). Among others they study the Nesjavellir area and their research indicates that it is currently not being harnessed in a sustainable manner and it will not be possible to maintain current removal rate for an extended period. The production will need to be reduced and probably shutdown in an attempt to let the geothermal resource regenerate.

The primary objective of this work is to develop new methods for creating strategies for harnessing geothermal resources that can ensure sustainable long-term utilization. The research is focused on optimal utilization of geothermal resources such that social and economic development objectives are fulfilled, in an environmentally benign way. In the following we use an innovative mathematical programming model to optimize the harnessing of geothermal resources and show how rules and regulations can be implemented with constraints in a simple manner.

3. METHODS AND MATERIALS

3.1 Model of a Geothermal Reservoir

The behavior of the geothermal reservoir plays a major part in any analysis of future cost and operational optimization of the system at hand. Therefore it is necessary to use a sufficiently accurate model to simulate this behavior, but it is also beneficial that the model is simple and can be run efficiently on a computer. The modeling in this study is based on a lumped parameter description of a water dominated geothermal reservoir, which is discretized in time for convenience in an operational optimization procedure.

3.1.1 Lumped Parameter Model

The lumped parameter model is based on three storage tanks, which represent the near neighbourhood of a geothermal well, a volume in some distance from the well and finally a large volume which covers the area of influence from the well utilization. These storage tanks are connected together so that fluid can flow between them and fluid can also flow from the tank nearest to the well to the surface. The state of the tanks is represented by pressure and the pressure difference along with connection resistances controls the actual flow between tanks. Figure 1 shows the connection between storage tanks.

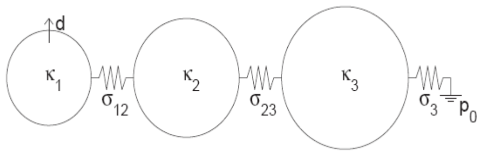

Figure 1: Storage tanks with connections.

In the description above, κ denotes the tank mass capacities, σ denotes resistance, p_0 is the external pressure of the environment and d is the flow through the well.

As mentioned before, the flow between the tanks is related to the actual pressure state in the tank. This relation can be written as three differential equations, one for each storage tank. This results in

$$\kappa_1 \frac{dp_1}{dt} = \sigma_{12}(p_1 - p_2) + d \tag{1}$$

$$\kappa_2 \frac{dp_2}{dt} = \sigma_{12}(p_1 - p_2) - \sigma_{23}(p_3 - p_2) \tag{2}$$

$$\kappa_3 \frac{dp_3}{dt} = \sigma_{23}(p_2 - p_3) - \sigma_3(p_0 - p_3) \tag{3}$$

which can be written in a more convenient matrix form (or state space form) as

$$K\dot{x} = Sx + u \tag{4}$$

where

$$K = \begin{bmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix} \quad x = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad u = \begin{bmatrix} d \\ 0 \\ \sigma_3 p_0 \end{bmatrix} \tag{5}$$

$$S = \begin{bmatrix} -\sigma_{12} & \sigma_{12} & 0 \\ \sigma_{12} & -\sigma_{12} - \sigma_{23} & \sigma_{23} \\ 0 & \sigma_{23} & -\sigma_{23} - \sigma_3 \end{bmatrix} \tag{6}$$

There are various ways to solve the system above, either analytically with integration or with numerical methods, which is the topic of next section.

3.1.2 Discrete Approximation of the Lumped Parameter Model

One way of solving the differential equations for the lumped parameter model is to use numerical integration in time. Generally, such methods are classified into implicit methods and explicit methods, where the explicit ones are easier to implement but have stability issues and criteria.

A relatively simple and accurate method to integrate the lumped equations is to use central finite difference approximation of the time derivative. If x_k denotes the current state at time step k and x_{k+1} denotes a new update at a future time step, the numerical approximation of the update is

$$K \frac{x_{k+1} - x_k}{\Delta t} = S \frac{x_{k+1} + x_k}{2} + \frac{u_{k+1} + u_k}{2} \tag{7}$$

which can be solved for x_{k+1}, resulting in

$$x_{k+1} = \left(K - \tfrac{\Delta t}{2} S\right)^{-1} \left(\left(K + \tfrac{\Delta t}{2} S\right) x_k + \frac{\Delta t}{2}(u_{k+1} + u_k) \right) \tag{8}$$

where Δt is the length of the time step used. The implicit approach results in a very accurate and stable method, but requires some simple matrix operations in each time step.

Another approach is to use an explicit method where the update to a new time step is based on the current time step only. The resulting equation is

$$K \frac{x_{k+1} - x_k}{\Delta t} = Sx_k + u_k \tag{9}$$

and the solution for the state update is

$$x_{k+1} = x_k + K^{-1} \Delta t (Sx_k + u_k) \tag{10}$$

Note that A is a diagonal matrix and therefore easily invertible. The explicit method requires that the time step Δt is chosen sufficiently small in order to ensure stability, but it is assumed that the reservoir conditions change slowly, so a time step of one month should be sufficiently small.

3.2 Sustainability Criteria

For each geothermal system, and for each mode of production, there exists a certain level of maximum energy production, E_0, below which it will be possible to maintain a constant energy production for a very long time (100–300 years). If the production rate is greater than E_0 it cannot be maintained for this length of time. Geothermal energy production below or equal to E_0, is termed sustainable production while production greater than E_0 is termed excessive production.

Sustainability can then be determined by the ratio between the power required to extract the water from the well and the specific energy contained in the water times the demand:

$$St = \frac{E_x d}{P} \tag{11}$$

If c $(J\,kg^{-1}K^{-1})$ is the heat capacity of water (at $25°\,C$), T_h (K) is the temperature of the heat source and T_0 (K) is the temperature of the heat sink the following approximation of the exergy per unit production can be made

$$E_x = c\left((T_h - T_0) - T_0 \ln \frac{T_h}{T_0} \right) \tag{12}$$

3.3 Operational Optimisation Model

To model the operation environment of the geothermal power plants mixed integer linear programming (MILP is a special type of mathematical programming techniques) as it is widely accepted that mathematical programming techniques offer appropriate methods to model and solve the complex constrained problems which arise in the planning and scheduling of complex production environments (Shah et al., 1999; Applequist et al.,1997; Engel et al.,2001). With the model we optimize the operation strategy of the geothermal power plant. Geothermal energy is classified as a renewable resource in the sense that water and heat flows into areas from which it has been removed. However, if water or heat is removed from the geothermal reservoir at greater rate than it is replenished, the time will come that plant operations are no longer profitable and the reservoir must be allowed to rest while the system heats up and / or fluid re-enters. The optimized operations strategy will determine initial investment, the rate of extraction and whether production should be continuous or intermittent.

To be able to provide decision support for creating sustainable harnessing strategies it is necessary to develop a model that mimics the feedback of the geothermal reservoir to harnessing. Due to the extensive calculations needed for optimising the operation strategies it is important that the reservoir model is efficient but at the same time it needs to be accurate enough. The lumped parameter model contains differential equations that make it more difficult to solve the MILP model. To make the model easily solvable we use a implicit discrete time scale and replace the differential equations with discrete constraints that take care of mass balance in the tanks.

3. RESULTS

3.1 Parameter Estimation

In order to estimate the accuracy of discrete lumped parameter models the model is fitted against data from Lauganes reservoir in SW-Iceland. Figure 3 shows the production history and the historical drawdown can be seen in figure4. The model parameters are estimated by minimizing the squares of the difference between theoretical drawdown and empirical observation such as

$$\min_{h_0,K,S} \frac{1}{2}\left\|h_e - h(h_0,K,S)\right\|_2^2 = \frac{1}{2}\sum_j (h_{e,j} - p(h_0,K,S,t_j))^2 \tag{13}$$

where $h_{e,j}$ is an empirical observation at time step j and $p(h_0,\kappa,\sigma,t_j)$ is the theoretical drawdown at the same time step. This returns the vectors of the values for the initial drawdown in each tank, h_0 the storage coefficients, κ (from K) and the conductance values σ (from S) which are the characteristic parameters for the particular geothermal system in question. This is solved by using lsqnonlin MATLAB function.

The discrete lumped parameter model follows the data quite accurately compared to the discrete approximation that has been done or root mean square (RMS) of 12.58 and standard deviation of 12.57. More accurate parameter estimation is obtained by solving the differential equation (see section 3.1.1) directly or RMS of 6.65 and standard deviation 6.74.

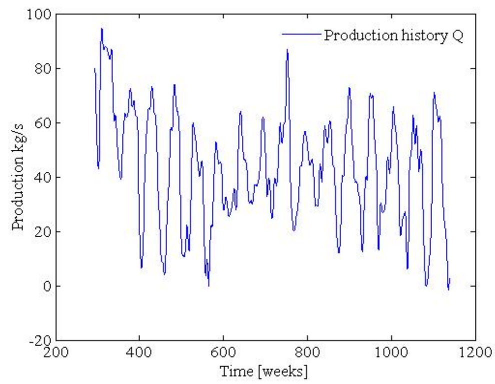

Figure 3: Production history in kg/s over 17 years.

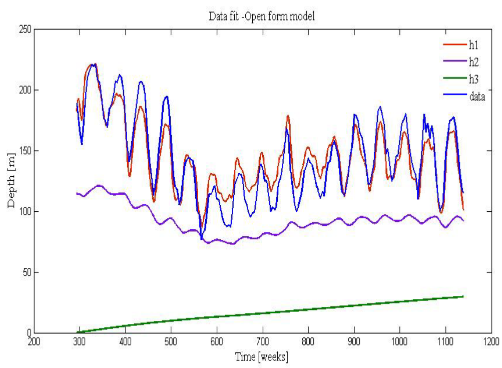

Figure 4: The drawdown data used to estimate model parameters versus the drawdown from the fitted model.

3.2 Behaviour of the Discrete Reservoir Model

After the parameters have been estimated optimization is tested. We use CVX, a Matlab-based modeling system for convex optimization. CVX turns Matlab into a modeling language, allowing constraints and objectives to be specified using standard Matlab expression syntax (CVX, 2009).

We compare the behaviour of the discrete reservoir optimization model to data from Lauganes reservoir. We use one reservoir and assume decision variable Q can not exceed historical production. We run the optimization for 3 years (150 weeks).

From figure 5 we see that h1 has the highest fluctuation of the 3 tanks and shows greatest response to variation in production rates; h2 shows a slightly decreasing drawdown and h3 slightly increasing drawdown. From figure 6 we see that the drawdown in tank one follows the data to some extent.

We observe the relative error from this optimization. The relative error increases in year two and decreases again in year 3 where the optimized drawdown goes below historical drawdown.

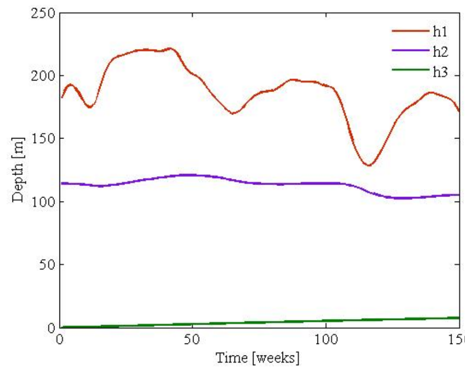

Figure 5: Results from optimization showing drawdown in tanks 1, 2 and 3.

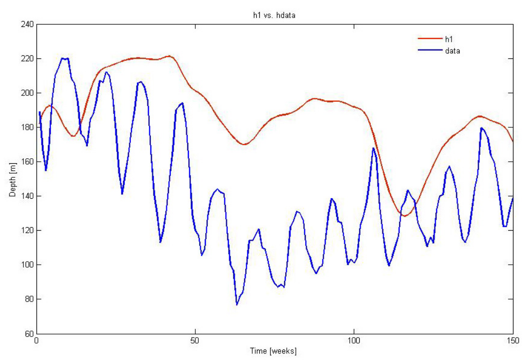

Figure 6: Historical drawdown compared with calculated drawdown from the simple model

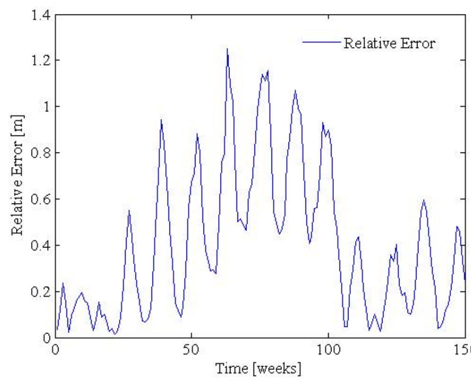

Figure 7: Relative error from optimization

Let's now assume that Q can exceed historical production by 20% and not go lower than 80% of the historical production. We run the optimization for 1 year (50 weeks) and obtain the results shown in figures 8 and 9.

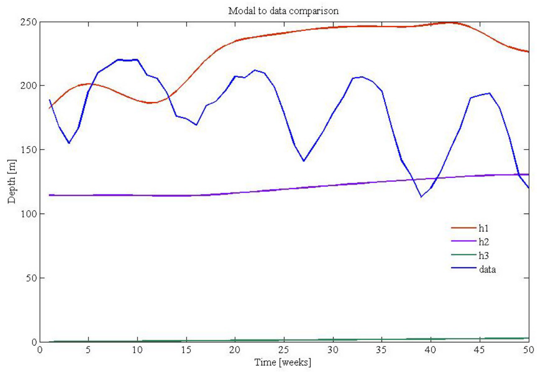

Figure 8: Result from optimization with more slack in production. Here the historical drawdown is compared with outcome of optimization model.

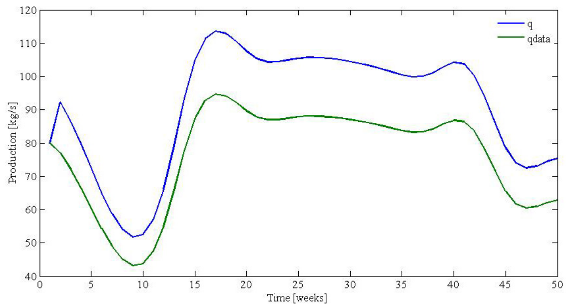

Figure 9: Result from optimization with increased slack in production. Here the historical production is compared with outcome of the optimization model.

The production is profitable with the data used for the Lauganes reservoir and we see that with increased slack in production the production increases to its maximum allowed limits and as a result the drawdown increases.

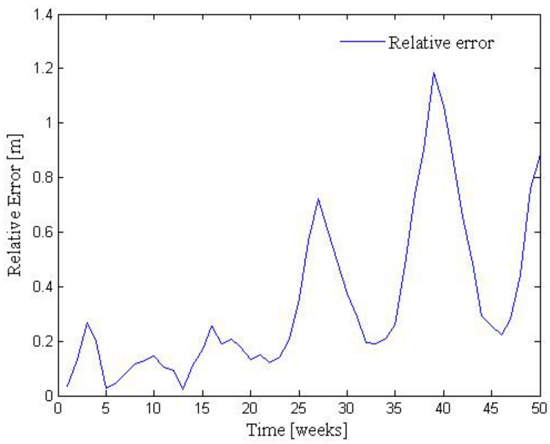

Figure 10: Relative Error with more slack in production

4. CONCLUSIONS AND FUTURE WORK

According to the parameter estimation implicit discrete approximation gives a relatively good estimate.

The optimization model indicates that such a model can be implied but relative error is still too large in our model.

Drawdown increases with more slack in the production constraint which is sensible since our revenue increases with more production. The interaction between the three tanks is also logical.

CVX in Matlab can only handle 3 years of optimization. With a stronger solver this could be applied for a longer period which would give better opportunities for validating the model behaviour and exploring the usability of the model.

Future work will focus on reducing the error of the model and solving it for extended time horizon and multiple reservoirs.

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APPENDIX

A1 Nomenclature

A1.1 Indexes

t	Index for time periods, t Є [1,2,....,T]
w	Index for wells, w Є [1,2,....,W]

A1.2 Variables

Decision variables

$Q_{t,w}$	Extraction from tank 1, well *w* in time period *t* [kg/s]
$Y_{t,w}$	Binary variable. $Y_{t,w}$=1 if we buy well/pump *w* in time period *t*, otherwise $Y_{t,w}$ =0

State variables

$h1_{t,w}$	Height in tank 1 of well *w* in time period *t*
$h2_{t,w}$	Height in tank 2 of well *w* in time period *t*
$h3_{t,w}$	Height in tank 3 of well *w* in time period *t*

Parameters

$\sigma_{1,2}$	The conductivity between tanks 1 and 2
$\sigma_{2,3}$	The conductivity between tanks 2 and 3
σ_3	The conductivity between tanks 3 and the external environment of the system
S	$\begin{bmatrix} -\sigma_{12} & \sigma_{12} & 0 \\ \sigma_{12} & -\sigma_{12}-\sigma_{23} & \sigma_{23} \\ 0 & \sigma_{23} & -\sigma_{23}-\sigma_3 \end{bmatrix}$
H_0	The external drawdown
$h1_{1,w}$	Drawdown in tank 1 of well *w*
$h2_{1,w}$	Drawdown in tank 2 of well *w*
$h3_{1,w}$	Drawdown in tank 3 of well *w*
$H1^{max}$	Maximum drawdown of tank 1 for sustainability constraint
QW^{max}	Maximum production capacity for each well
κ_1	Storage coefficient of tank 1
κ_2	Storage coefficient of tank 2
κ_3	Storage coefficient of tank 3
K	$\begin{bmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix}$
g	Gravity
dt	Length of each time period (in seconds)
ρ	Density of water at 25°C.
C_{elect}	Price of electricity [$/kWh]
C_{water}	Price of water [$/m^3]
C_{start}	Pricing of adding a new well/pump[$]

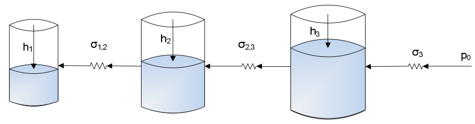

Figure 1: The three tank system.

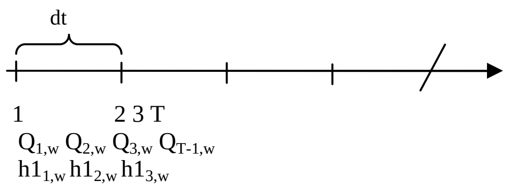

Figure 2: Explanation of the discrete time axis and examples of variables used in the model.

A1.3 Constraints

Mass balance equations on matrix form

$$h_{k+1,w} = \left(K - \tfrac{\Delta t}{2} S\right)^{-1} \left(\left(K + \tfrac{\Delta t}{2} S\right) h_{k,w} + \frac{\Delta t}{2} \left(Q_{k+1,w} + Q_{k,w}\right) \frac{1}{\rho g} + \frac{\Delta t}{2} + \Delta t \sigma_3 H_0 \right.$$

$\forall w \in W, t \in T$

Production capacity constraint for each well:

$$\sum_t Y_{t,w} = 1 \quad \forall w \in W$$

$$Q_{t,w} \le \sum_0^t Y_{t,w} \cdot QW^{\max} \quad \forall w \in W, t \in T$$

Sustainability constraint:

$$h1_{t,w} \le H1^{\max} \quad \forall w \in W, t \in T$$

Setup-cost constraint

$$Q_{t,w} \le y_w \cdot M \quad \forall w \in W, t \in T$$

Demand constraint:

$$Q_{t,w} \le D_{\max}$$

$$Q_{t,w} \ge D_{\min}$$

A1.4 Objective Function

$$\text{Total Revenue} = \sum_{t,w} \frac{TL \cdot Q_{t,w} \cdot C_{water}}{(1+r)^t}$$

$$\text{Production Cost} = \sum_{t,w} \frac{TL \cdot (\rho \cdot g \cdot h1_{t,w}) \cdot C_{elect}}{(1+r)^t} Q_{t,w}$$

$$\text{Well Start Cost} = \sum_{t,w} \frac{C_{water} \cdot y_{w,t}}{(1+r)^t}$$

Total Cost = Production Cost + Well Start Cost

Max (Total Revenue – Total Cost)