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A Four-Dimensional Formulation of Geothermal Poroelasticity

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ABSTRACT

Rocks in geothermal systems are porous, compressible, and elastic. The presence of a moving fluid in the porous rock modifies its mechanical response. Its elasticity is evidenced by the compression that results from the decline of the fluid pressure, which can shorten the pore volume. This reduction of the pore volume can be the principal source of fluid released from storage. Poroelasticity explains how the fluid inside the pores bears a portion of the total load supported by the porous rock. The skeleton supports the remaining part of the load. The skeleton is treated as an elastic solid with a laminar flow of pore fluid coupled to the framework by equilibrium and continuity conditions.

This paper introduces an original tensorial formulation of Biot's isothermal theory of linear porous rocks and of its extension to thermoporoelastic processes. By defining a total stress tensor in four dimensions and three basic poroelastic coefficients, it is possible to deduce a system of equations coupling two tensors, one for the bulk rock and one for the fluid. The inclusion of the fourth dimension is necessary to extend the theory of solid linear elasticity to thermoporoelastic rocks, taking into account the effect of both, the fluid phase and the temperature. In linear thermo-poroelasticity, we need five poroelastic modules to describe the relations between strains and stresses. Introducing three volumetric thermal dilation coefficients, one for the fluid and two for the skeleton, a complete set of parameters for geothermal poroelastic rocks are obtained.

This formulation makes more comprehensible the linear theory. The Finite Element Method is very convenient to solve the resulting equations. To illustrate the practical use of this tensor four-dimensional formulation three applications are outlined: a) Full deduction of the classical Biot's theory coupled to thermal stresses; b) deformation of an aquifer under cold isothermal conditions and c) simulation of the same aquifer when its temperature changes to a geothermal state.

1. INTRODUCTION

Several factors affect the geomechanical behavior of porous crustal rocks containing fluids: porosity, temperature, and pressure, characteristics of the fluids, fissures, and faults. In classic elastic solids only the two Lamé moduli, (λ, G) or Young's elastic coefficient and Poisson's ratio (E, ν), are sufficient to describe the relations between strains and stresses. In poroelasticity, we need five poroelastic moduli for the same relationships, but only three of these parameters are independent. The Biot's field variables for an isotropic porous rock are the stress σ acting in the rock, the bulk volumetric strain ε_B, the pore pressure p_f and the variation of fluid mass content ζ. The linear relationships among these variables are the experimental foundations of Biot's poroelastic theory (Biot & Willis, 1957; Wang, 2000).

In matrix form these relationships are:

$$\begin{pmatrix} \varepsilon_B \\ \zeta \end{pmatrix} = \begin{pmatrix} C_B & H^{-1} \\ H^{-1} & R^{-1} \end{pmatrix} \cdot \begin{pmatrix} \sigma \\ p_f \end{pmatrix} \tag{1}$$

Where K_B, H and R are poroelastic coefficients that are experimentally measured as follows (Wang, 2000):

$$\begin{aligned} &\varepsilon_B = \frac{\Delta V_B}{V_B}, \quad C_B = \left(\frac{\Delta \varepsilon_B}{\Delta \sigma}\right)_{p_f} = \frac{1}{K_B} \\ &\zeta = \frac{\Delta m_f}{\rho_0}, \quad \frac{1}{H} = \left(\frac{\Delta \zeta}{\Delta \sigma}\right)_{p_f}, \quad \frac{1}{R} = \left(\frac{\Delta \zeta}{\Delta p_f}\right)_{\sigma} \end{aligned} \tag{2}$$

Here V_B is the bulk volume, consisting of the rock skeleton formed by the union of the volume of the pores V_Φ and the volume of the solid matrix V_S. The control volume is ΔV_B; m_f is the fluid mass content per unit volume V_B, and ρ_0 is the initial fluid density. The drained coefficients K_B and C_B are the bulk modulus and the bulk compressibility of the rock, respectively; $1/H$ is a poroelastic expansion coefficient, which describes how much ζ changes when σ changes when p_f remains constant. Finally $1/R$ is an unconstrained specific storage coefficient, which represents the changes of ζ when p_f changes. This parameter is measured when the applied stress σ remains constant. The value of 1/R is determined by the compressibilities of the frame, the pores, the fluid and the solid grains (Wang, 2000). The poroelastic coefficients given by equation (2) entirely describe the poroelastic response of the rock for isothermal, isotropic processes. Inverting the matrix equation (1) and replacing the value of σ in ζ we obtain:

$$\sigma = K_B \varepsilon_B - \frac{K_B}{H} p_f \;\Rightarrow\; \zeta = \frac{K_B}{H} \varepsilon_B + \left(\frac{1}{R} - \frac{K_B}{H^2}\right) p_f \tag{3}$$

The sign conventions are: stress $\sigma > 0$ in tension and $\sigma < 0$ in compression; the volumetric strain $\varepsilon_B > 0$ in expansion and $\varepsilon_B < 0$ in contraction; $\zeta > 0$ if fluid is added to the control volume ΔV_B and $\zeta < 0$ if fluid is extracted from ΔV_B; the pore pressure $p_f > 0$ if it is larger than the atmospheric pressure. Biot (1941) and (Biot & Willis, 1957) introduced three additional poroelastic parameters, b, M and C, that are fundamental for the tensorial formulation herein presented. $1/M$ is called the constrained specific storage, which is equal to the change of ζ when p_f changes measured at constant strain. Both parameters M and C are dependent and expressed in terms of the three fundamental ones defined in equation (2):

$$\frac{1}{M}=\left(\frac{\Delta\zeta}{\Delta p_f}\right)_{\varepsilon_B}=\frac{1}{R}-\frac{K_B}{H^2}\Rightarrow$$
$$M=\frac{RH^2}{H^2-K_BR};\quad C=\frac{K_B}{H}M \tag{4}$$

Let $C_S = 1/K_S$ be the compressibility of the solid matrix. The Biot-Willis coefficient b is defined as the change of confining pressure p_k with respect to the fluid pressure change when the total volumetric strain remains constant:

$$b=\left(\frac{\partial p_k}{\partial p_f}\right)_{\varepsilon}=1-\frac{K_B}{K_S}=\frac{C}{M}=\frac{K_B}{H} \tag{5}$$

The coefficient C represents the coupling of deformations between the solid grains and the fluid. The coefficient M is the inverse of the constrained specific storage, measured at constant strain (Wang, 2000); this parameter characterizes the elastic properties of the fluid because it measures how the fluid pressure changes when ζ changes. These three parameters b, M and C are at the core of the poroelastic equations (Bundschuh and Suárez, 2009).

2. RELATIONSHIPS WITH OTHER POROELASTIC PARAMETERS UNDER DIFFERENT CONDITIONS

In the tests carried out to measure any poroelastic parameter, there are two type of experimental situations:

A) under drained conditions, the rock is confined and subjected to support an external hydrostatic pressure σ_H; the fluid in the pores is allowed to escape. Biot (1962) called these conditions, “an open system”. The total stress is entirely supported by the rock skeleton and the deformations are achieved at constant pore fluid pressure p_f.

B) For the undrained conditions test, the deformations are measured at constant fluid mass content ($\Delta\zeta = 0$). The rock is entirely submerged in a fluid in such a way that the external hydrostatic pressure is balanced by the pore pressure $\sigma_H = -p_f$. The fluid in the pores remains constant; no fluid is allowed to move into or out of the control volume. The fluid remains trapped in the skeleton. For this reason, Biot (1962) called this test a “closed system”.

2.1 The Skempton Coefficient and the Undrained Bulk Modulus

The Skempton coefficient B is an additional parameter, which represents the change in pore pressure when the applied stress changes for undrained conditions:

$$B=-\left(\frac{\Delta p_f}{\Delta\sigma}\right)_{\zeta}=\frac{R}{H} \tag{5}$$

The undrained bulk compressibility C_U is defined as the undrained deformation obtained when the rock is subjected to compressive stresses in all directions and the fluid contained in the pores remains constant:

$$C_U=-\frac{1}{V_B}\left(\frac{\partial V_B}{\partial p_k}\right)_{\zeta}=\frac{1}{K_U}\Rightarrow K_U=-\left(\frac{\Delta p_k}{\Delta\varepsilon_B}\right)_{\zeta} \tag{6}$$

The undrained bulk modulus is related to the previous defined coefficients:

$$K_U=\frac{K_B}{1-bB}\text{ , and }\; B=\frac{K_U-K_B}{b\,K_U}=\frac{bM}{K_U} \tag{7a}$$

Consequently:

$$K_S=\frac{K_U-b^2M}{1-b}=\frac{M\,K_U-C^2}{M-C} \tag{7b}$$

2.2 Rigidity, Lamé and Poisson Moduli

Other elastic parameters, such as the Lame moduli λ and G should be measured under the same conditions. Since there are no shear stresses in the fluid, the shear coefficient must be the same in both situations drained and undrained. Thus, $G = G_U = G_B$ (Biot, 1962; Wang, 2000). For the undrained Lamé modulus λ_U, we have:

$$\begin{aligned}&\frac{2}{3}G=K_U-\lambda_U=K_B-\lambda\\&\Rightarrow\;\lambda_U=K_U-K_B+\lambda\\&K_U-K_B=bC=b^2M\\&\Rightarrow\;\lambda_U=\lambda+bC\end{aligned} \tag{8}$$

The undrained Poisson’s coefficient is deduced in a similar way:

$$\nu_U=\frac{3\nu+bB(1-2\nu)}{3-bB(1-2\nu)} \tag{9}$$

2.3 The Three Classic Biot’s Coefficients

Introducing the bulk modulus of the fluid K_f, the Biot’s coefficients b, C and M can be expressed in terms of the other moduli and the porosity (Bundschuh & Suárez, 2009):

$$M=\frac{K_f\,K_S}{\varphi K_S+(b-\varphi)K_f},\quad C=\frac{\dfrac{K_f}{\varphi}\left(1-\dfrac{K_B}{K_S}\right)}{1-\dfrac{K_f}{K_S}+\dfrac{K_f}{\varphi K_S}\left(1-\dfrac{K_B}{K_S}\right)} \tag{20}$$

Using these expressions, the tangent modulus N (Biot, 1962) is defined as the pressure variation with respect to the porosity variation when both strain and temperature are held constant (Coussy, 1995):

$$\frac{1}{N}=\frac{1}{M}-\frac{\varphi}{K_f}=\frac{b}{C}-\frac{\varphi}{K_f}=\frac{b-\varphi}{K_S} \tag{13}$$

2.4 The Gassmann-Biot Formula and the Porosity

The Gassmann-Biot equation is easily derived from equations (7) and (11):

$$K_U=K_B+\frac{b^2}{\dfrac{\varphi}{K_f}+\dfrac{b-\varphi}{K_S}} \tag{42}$$

From this equation, a useful formula for the porosity is obtained, which includes the Biot’s coefficients and the Skempton parameter:

$$\varphi=\frac{K_f}{K_S-K_f}\left(\frac{K_S}{M\left(1-bB\right)}-\frac{b^2}{\left(1-b\right)}-b\right) \tag{53}$$

The experimental limits of poroelasticity can be deduced from the Gassmann-Biot equation (12). Numerical values of all the introduced poroelastic parameters are shown in Table 1. Both limit values of the Biot-Willis coefficient $0 \le b \le 1$ are also included and explained.

2.5 Experimental Values of the Poroelastic Constants

To construct Table 1 the basic experimental data set used was $\{E, G, \varphi, K_S$ and $K_f\}$. Where φ is porosity and E is the drained Young's elastic modulus. The rock types are:

[0].- Clay, (K_f = 3.9 GPa).
[1].- Boise sandstone, (K_f = 2.0 GPa).
[2].- Berea sandstone (K_f = 3.5 GPa);
[3].- Indiana limestone, (K_f = 3.1 GPa).
[4].- Andesite, (K_f = 2.1 GPa, p_k = 250 bar, T = 25°C).
[5].- Tennessee marble, (K_f = 3.5 GPa).
[6].- Rock with celestite, (K_f = 2.0 GPa).

Table 1. Poroelastic Parameters of Different Rocks.

Rock Type	φ (%)	E (GPa)	ν	ν_U	G (GPa)	λ (GPa)	λ_U (GPa)
[0]	*65.0*	*0.18*	*0.01*	*0.49*	*0.09*	*.001*	*6.1*
[1]	26.0	*9.7*	0.15	0.31	4.2	*1.8*	*7.3*
[2]	19.0	*13.1*	0.17	0.30	5.6	*2.9*	*10.9*
[3]	13.0	*30.5*	0.26	0.32	12.1	*13.1*	*23.2*
[4]	8.5	35.8	0.31	0.32	13.7	21.7	24.8
[5]	2.0	*60.0*	0.25	0.27	24.0	*24.2*	*28.3*
[6]	0.5	*42.2*	0.42	0.42	14.9	*73.9*	*74.0*

Rock Type	K_B (GPa)	K_U (GPa)	K_S (GPa)	B	b	M (GPa)	R (GPa)
[0]	0.062	6.2	*10^5*	0.99	1.0	*6.1*	*0.06*
[1]	4.6	*10.1*	42.0	*0.61*	*0.89*	*6.9*	*3.2*
[2]	6.6	*14.6*	28.9	*0.71*	0.77	*13.4*	*6.1*
[3]	21.2	31.2	72.6	0.46	0.71	*20.0*	*13.6*
[4]	30.8	33.9	49.7	0.24	0.38	21.6	19.6
[5]	40.0	44.3	50.0	0.49	0.19	*107.3*	*97.0*
[6]	83.8	83.9	85.0	0.06	0.01	*384.3*	*384.0*

References used: Rocks [0], [1], [2] and [3], Wang (2000); [4], Bundschuh and Suárez (2009); [5], Detournay and Cheng (1993); [6], Mavko (*et al.*, 2003). The figures in italics were estimated using the poroelastic formulas (4-13). Rocks of type [0] and [6] are the two limit cases of linear poroelastic theory, for $b \approx 1$ and $b \approx 0$, respectively.

For the case of a hard volcanic rock with very low porosity (Table 1):

$$\begin{gathered} b \approx 0 \;\Rightarrow\; K_U \approx K_B \approx K_S >> K_f \;\Rightarrow \\ C \approx \frac{K_f}{\varphi}\left(1-\frac{K_B}{K_S}\right) \text{ and } M \approx \frac{K_f}{\varphi} \end{gathered} \tag{64}$$

The parameter C depends on both, the fluid properties and the bulk rock properties. Whereas coefficient M depends only on the porosity and on the fluid bulk modulus. This is the case of a rigid frame.

The other limit case is for sedimentary rocks with high porosity, for example sandstones (Table 1). In this case equations (7a,b) and (12) become:

$$b \approx 1,\; K_U \approx K_B + \frac{K_f}{\varphi},\; K_B << K_S \;\Rightarrow \tag{75a}$$

$$\frac{1}{K_U} \approx \frac{\varphi}{K_f} + \frac{\varphi_S}{K_S}\;,\; C \approx M \approx \frac{K_f}{\varphi} \tag{85b}$$

We need many poroelastic coefficients to support poroelastic theory, but only three of these parameters are actually independent. The basic parameters selected to constitute an experimental reference set can be the drained bulk compressibility $C_B = 1/K_B$, the expansion coefficient $1/H$ and the unconstrained specific storage $1/R$ (Wang, 2000). Other three possible basic coefficients are the drained bulk modulus K_B, the undrained bulk modulus K_U, and the Biot coefficient b (Detournay and Cheng, 1993). However, it is impossible to compute all the coefficients of the theory using only the classical elastic constants; at least five mixed coefficients are necessary for the poroelastic coupling. It is proved in a recent publication (Bundschuh and Suárez, 2009) that a sufficient set of measured parameters is for example $\{E, G, \varphi, K_S, K_f\}$; with these coefficients, we can compute the full set of 16 poroelastic constants (C and H are not included in Table 1). To describe the saturating fluid, we need its pressure and its temperature. It is possible to make other choices of the basic experimental parameters using the relationships developed in this section.

3. FOUR DIMENSIONAL-TENSOR FORMULATION OF BIOT'S POROELASTIC THEORY

Let $\boldsymbol{u}_s$ and $\boldsymbol{u}_f$ be the displacements of the solid and fluid particles; let $\boldsymbol{u} = \boldsymbol{u}_f - \boldsymbol{u}_s$ be the displacement of the fluid phase relative to the solid matrix respectively. Let ε_s, ε_f, φ_s, φ, V_s and V_f be the volumetric dilatations, porosities and volumes of each phase; $-\varepsilon_V$ is the volumetric deformation of the fluid phase relative to the solid phase. The mathematical expressions of these variables are:

$$\begin{gathered} \frac{\Delta V_S}{V_S} = \varepsilon_S = \vec{\nabla}\cdot\vec{u}_S;\; \frac{\Delta V_f}{V_f} = \varepsilon_f = \vec{\nabla}\cdot\vec{u}_f \\ \varepsilon_V = \varepsilon_S - \varepsilon_f \;;\; \vec{u} = \vec{u}_f - \vec{u}_S \;\Rightarrow \\ -\varepsilon_V = -\vec{\nabla}\cdot\left(\vec{u}_S - \vec{u}_f\right) = \vec{\nabla}\cdot\vec{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \end{gathered} \tag{96}$$

Biot and Willis (1957) introduced the strain variable $\zeta\,(u, t)$, defined in equation (3), to describe the volumetric deformation of the fluid relative to the deformation of the solid with homogeneous porosity:

$$\zeta(\vec{u},t) = \varphi\vec{\nabla}\cdot\left(\vec{u}_s - \vec{u}_f\right) = \varphi\varepsilon_s - \varphi\varepsilon_f = \varphi\varepsilon_V \tag{107}$$

The function ζ represents the variation of fluid content in the pore during a poroelastic deformation. The total applied stresses in the porous rock are similar to the equations of classic elasticity. However, we need to couple the effect of the fluid in the pores. The linear components of the global stresses, deduced experimentally by Biot, (Biot, 1941; Biot and Willis, 1957; Biot, 1962; Wang, 2000) are:

$$\sigma_{ij} = \lambda_U \, \varepsilon_B \, \delta_{ij} + 2G\varepsilon_{ij} - C\,\zeta\,\delta_{ij}$$

$$\lambda_U = \lambda + Cb\,; \text{ for } i,\ j = x, y, z \tag{118}$$

The fluid stress is equal to the fluid pressure and is deduced from equation (3):

$$p_f = \frac{K_B R H^2}{H^2 - K_B R}\left[\frac{\zeta}{K_B} - \frac{\varepsilon_B}{H}\right]$$

$$\Rightarrow \quad \sigma_f = p_f = M\,\zeta - C\varepsilon_B \tag{129}$$

3.1 The 4D – Tensorial Equation for Stresses

We define a two-order tensor $\boldsymbol{\sigma_T} = (\sigma_{ij})$ which includes the bulk stress tensor $\boldsymbol{\sigma_B}$ acting in the porous rock and the fluid stress $\boldsymbol{\sigma_F}$ acting in the fluid filling up the pores, positive in compression. This global tensor is in four dimensions, because it is represented by a (4 4) matrix related to a linear map in 4D. Both parts of the tensor are influencing the bulk rock deformation in the following way:

$$\boldsymbol{\sigma}_T = \boldsymbol{\sigma}_B + \boldsymbol{\sigma}_F = \left(\sigma_{ij}\right) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} & 0 \\ 0 & 0 & 0 & \sigma_f \end{pmatrix} =$$

$$\underbrace{\varepsilon_B \begin{pmatrix} \lambda_U & 0 & 0 & 0 \\ 0 & \lambda_U & 0 & 0 \\ 0 & 0 & \lambda_U & 0 \\ 0 & 0 & 0 & -C \end{pmatrix} + 2G \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} & 0 \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\leftarrow \qquad \boldsymbol{\sigma}_B \qquad \rightarrow} +$$

$$\underbrace{-\,\zeta \begin{pmatrix} C & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & -M \end{pmatrix}}_{\leftarrow \qquad \boldsymbol{\sigma}_F \qquad \rightarrow} \tag{20}$$

This tensorial equation become identical to the equation of Hookean solids, when the rock has zero porosity and $b = 0$. The inclusion of the fourth dimension is necessary to extend the classical theory of solid elasticity to linear poroelastic rocks, taking into account the effect of the fluid phase.

3.2 The Terzaghi Effective Stresses

From equations (8) and (19), we deduce that:

$$\zeta = \frac{C}{M}\varepsilon_B + \frac{p_f}{M} \Rightarrow$$

$$\sigma_{ij} = \left(\varepsilon_B\left(\lambda_U - Cb\right) - b\,p_f\right)\delta_{ij} + 2G\,\varepsilon_{ij} \tag{213}$$

$$\text{thus, } \sigma_{ij} = \left(\lambda\,\varepsilon_B\,\delta_{ij} + 2G\,\varepsilon_{ij}\right) - b\,p_f\,\delta_{ij}$$

Therefore:

$$\tau_{ij} = \lambda\, e_B\, \delta_{ij} + 2G\varepsilon_{ij} \quad \Rightarrow \quad \boxed{\sigma_{ij} = \tau_{ij} - b\,p_f\,\delta_{ij}} \tag{22}$$

Tensor τ_{ij} is called the Terzaghi effective stresses that act only in the solid matrix; bp_f is the pore-fluid pressure. For this reason, b is called the effective stress coefficient. Since there are no shear tensions in the fluid, the pore fluid pressure affects only the normal tensions σ_i ($i = x, y, z$). The functions σ_{ij} are the applied stresses acting in the porous rock saturated with fluid. They express that the poroelastic response is controlled by the difference between effective stress and pore pressure. This is the Law of Effective Stress (Wang, 2000; Terzaghi, 1943). Equation (22) illustrates the mechanism of the poroelastic coupling. The solid matrix (τ_{ij}) supports one portion of the total applied tensions in the rock and the fluid in the pores (bp_f) supports the other part. This is a maximum for soils, when $b \approx 1$ and is minimum for rocks with very low porosity where $b \approx 0$ (Table 1).

The matrix form of equation (22) becomes:

$$\boldsymbol{\sigma}_T = \boldsymbol{\sigma}_B + \boldsymbol{\sigma}_F = \left(\sigma_{ij}\right) = \begin{pmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_f \end{pmatrix} =$$

$$= \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - p_f \begin{pmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \boldsymbol{\tau} - \mathbf{b}\,p_f \tag{23}$$

The simplicity of the 4D poroelastic formulation becomes clear and evident with this tensor formula.

3.3 The 4D – Tensorial Equation for Strains

Inverting the matrices of equation (20), we arrive to the following tensorial form of the poroelastic strains:

$$\varepsilon_{ii} = \frac{\sigma_{ii}}{2G} - \frac{3\nu}{E}\sigma_M + \frac{p_f}{3H}\,;\ \varepsilon_{ij} = \frac{\sigma_{ij}}{2G}$$

$$\zeta = \frac{\sigma_M}{H} + \frac{p_f}{R} = \frac{C\,\sigma_M + K_U\,p_f}{M\,K_U - C^2}$$

$$\sigma_M = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = K_B\,\varepsilon_B - bM\,\zeta \tag{24}$$

$$K_B = \lambda + \frac{2}{3}G\,;\quad i, j = x, y, z$$

Note that both tensorial equations (20) and (24) only need four basic poroelastic constants. The presence of fluid in the pores adds an extra tension due to the hydrostatic pressure, which is identified with the pore pressure, because it is supposed that all the pores are interconnected. This linear theory is appropriate for isothermal, homogeneous and isotropic porous rocks.

4. THE GIBBS POTENTIAL IN THE 4D-TENSORIAL FORM OF NON-ISOTHERMAL POROELASTICITY

The basic equations of state in a saturated porous rock are the internal energy and the specific enthalpy for the fluid phase. For the skeleton, the appropriate function is the free enthalpy or Gibbs potential (Coussy, 1995). This potential, completely describe the exchanges of energy in a porous medium. The Gibbs free internal enthalpy G_S (subindex S means skeleton), is useful to derive the equations of the rock skeleton in non-isothermal, reversible processes. If we assume that the internal energy dissipation is zero and that the processes involved are reversible, then the heat exchanged is equal to the product of the absolute temperature T [K] and the entropy S_S [J/K] of the skeleton. The function $G_S = G_S\,(\varepsilon_{ij}, p, T)$ establishes mathematical relationships among heat, strains, stresses and pore pressure. This potential is equal to the algebraic difference of the enthalpy H_S minus the heat exchanged $Q_S = TS_S$. It is more convenient to formulate this function per unit volume of rock:

$$g_S = \frac{G_S}{V_B} = \frac{H_S}{V_B} - T\frac{S_S}{V_B} \quad \Rightarrow$$
$$g_S = h_S - Ts_S \;\rightarrow\; \left[\frac{\mathrm{J}}{\mathrm{m}^3} = \frac{\mathrm{N\cdot m}}{\mathrm{m}^3}\right] = \left[\mathrm{Pa}\right] \qquad (25)$$

The functions g_S, h_S, and s_S, represent the volumetric Gibbs potential, the volumetric enthalpy and the volumetric entropy of the skeleton respectively. The total differential of $g_S\,(\varepsilon_{ij}, p, T)$ is by definition:

$$d\,g_S = \sigma_{ij}\,d\varepsilon_{ij} - \varphi dp - S_S dT \quad \Rightarrow$$
$$\sigma_{ij} = \frac{\partial g_S}{\partial \varepsilon_{ij}},\;\; \varphi = -\frac{\partial g_S}{\partial p},\;\; S_S = -\frac{\partial g_S}{\partial T} \qquad (26)$$

These partial differential equations are integrated between an initial state $g_S^0(\varepsilon_{ij}\,(0) = 0, p_f = p_0, T = T_0)$ when the strain was zero, and a final state $g_S\,(\varepsilon_{ij}, p, T)$. In this case, an initial reference temperature T_0 and an initial pore pressure p_0 are necessary because both thermodynamic variables T and p are going to change in non-isothermal processes occurring in the geothermal reservoir.

4.1 The Skeleton Energy Dissipation Function. Analytic definition of the Gibbs Potential

The potential $g_S\,(\varepsilon_{ij}, p, T)$ is the thermoporoelastic available enthalpy per unit volume [J/m^3]. Coussy (2004) deduced the energy dissipation function Ψ_S of the skeleton, using this volumetric Gibbs potential, in terms of the stresses, the porosity, the pore pressure and the density of entropy per unit volume of porous rock S_S [J/m^3/K] as follows:

$$\frac{d\,\Psi_s}{dt} = \sigma_{ij}\frac{d\,\varepsilon_{ij}}{dt} - S_s\frac{d\,T}{dt} - \varphi\frac{dp_f}{dt} - \frac{dg_s}{dt} \;\geq\; 0 \qquad (27)$$

If there is no energy dissipation ($d\Psi_S/dt = 0$) and for small changes in ΔV_B, the Gibbs' potential describes the behavior of the skeleton. Its analytical expression is obtained by integrating all the equations (26) (Coussy, 1995):

$$\text{Initial State: } g_S^0 = g_S\left(\varepsilon_{ij}=0, p = p_0, T = T_0\right)$$
$$\text{Final State: } g_S = g_S\left(\varepsilon_{ij}, p, T\right) \;\Rightarrow$$
$$\begin{aligned} g_S &= \sigma_M^0\,\varepsilon_B + \left(\sigma_{ij}^0 - \sigma_M^0\delta_{ij}\right)\left(\varepsilon_{ij} - \frac{\varepsilon_B}{3}\delta_{ij}\right) + \\ &- \varphi_0 p - S_S^0 T + \frac{K_B}{2}\left(\varepsilon_B\right)^2 - b\left(p - p_0\right)\varepsilon_B + \\ &+ \gamma_\varphi\left(p - p_0\right)\left(T - T_0\right) - K_B\,\gamma_B\left(T - T_0\right)\varepsilon_B + \\ &- \frac{\left(p - p_0\right)^2}{2N} - \frac{C_V\left(T - T_0\right)^2}{2T_0} + G\left(\varepsilon_{ij} - \frac{\varepsilon_B}{3}\delta_{ij}\right)^2 \end{aligned} \qquad (28)$$

The parameters γ_B and γ_φ [1/K] are the volumetric thermal dilatation coefficients. The first one γ_B measures the dilatation of the skeleton, while γ_φ is related to the porosity. C_V is the skeleton volumetric heat capacity coefficient under constant deformations (strains and pore pressure are held constants). Their mathematical definitions are:

$$\gamma_B = \frac{1}{V_B}\left(\frac{\partial V_B}{\partial T}\right)_{p_k} = -\frac{1}{\rho_S}\left(\frac{\partial \rho_S}{\partial T}\right)_{p_k} \qquad \left[\frac{1}{\mathrm{K}}\right] \qquad (29a)$$

$$\gamma_\varphi = \frac{1}{V_\varphi}\left(\frac{\partial V_\varphi}{\partial T}\right)_p = \frac{1}{\varphi V_B}\left(\frac{\partial\left(\varphi V_B\right)}{\partial T}\right)_p$$
$$\Rightarrow \quad \gamma_\varphi = \frac{1}{\varphi}\left(\frac{\partial \varphi}{\partial T}\right)_p \qquad \left[\frac{1}{\mathrm{K}}\right] \qquad (29b)$$

4.2 The Thermo-Poroelastic Equations in Geothermal Reservoirs

Using the differential relationships of equations (26) we obtain:

$$\begin{aligned} \sigma_{ij} &= \frac{\partial g_S}{\partial \varepsilon_{ij}} = \sigma_{ij}^0 + \lambda\,\varepsilon_B\,\delta_{ij} + 2G\,\varepsilon_{ij} + \\ &- b\left(p - p_0\right)\delta_{ij} - K_B\,\gamma_B\left(T - T_0\right)\delta_{ij} \end{aligned} \qquad (30)$$

$$\varphi = -\frac{\partial g_S}{\partial p} = \varphi_0 + b\,\varepsilon_B - \gamma_\varphi\left(T - T_0\right) + \frac{\left(p - p_0\right)}{N} \qquad (314)$$

$$\zeta = \zeta_0 + \frac{b\,\sigma_M}{K_B} + \frac{p - p_0}{M} + \varphi\left(\gamma_\varphi - \gamma_f\right)\left(T - T_0\right) \qquad (32)$$

$$S_S = -\frac{\partial g_S}{\partial T} = S_S^0 + \gamma_B\,K_B\,\varepsilon_B - \gamma_\varphi\left(p - p_0\right) + C_V\,\frac{\left(T - T_0\right)}{T_0} \qquad (33)$$

The bulk modulus K_f and the thermal expansivity γ_f [1/K] of the fluid are defined as follows:

$$\frac{1}{K_f} = C_f = \frac{1}{\rho_f}\left(\frac{\partial \rho_f}{\partial p}\right)_T$$
$$\gamma_f = \frac{1}{V_f}\left(\frac{\partial V_f}{\partial T}\right)_{p_f} = \frac{-1}{\rho_f}\left(\frac{\partial \rho_f}{\partial T}\right)_{p_f} \qquad (34)$$

Note that for isothermal processes in rocks with zero porosity $\varphi = 0$, $T = T_0$ and $p = p_0$, the system of equations (30), (31), (32) and (33), is reduced to Hooke's Law, for linear elastic solids. If only the temperature is constant, $T = T_0$, the system of poroelastic equations (18) is recovered. The pore pressure is equal to the fluid pressure ($p = p_f$), which can be measured in the field or calculated using the equation of state for the fluid.

4.3 The Heat Equation in Thermoporoelasticity

Using the classic relationship between heat and entropy for reversible processes:

$$\frac{\delta Q}{\delta t} = T_0\frac{\partial s_S}{\partial t} = -\vec{\nabla}\cdot\vec{q}_T$$
$$\vec{q}_T = -k_T\cdot\vec{\nabla}T \qquad (35)$$

Where $\boldsymbol{q}_T$ is the heat flow of the Fourier's law of conduction and k_T [W/m/K] is the average rock thermal conductivity.

Developing this expression in terms of the entropy function $s_S\,(\varepsilon_{ij}, T)$:

$$-\vec{\nabla}\cdot\vec{q}_T = T_0\frac{\partial s_s}{\partial t} = T_0\left(\frac{\partial s_s}{\partial \varepsilon_{ij}}\frac{\partial \varepsilon_{ij}}{\partial t}+\frac{\partial s_s}{\partial T}\frac{\partial T}{\partial t}\right) = \\ T_0\left(K_B\gamma_B\,\delta_{ij}\frac{\partial \varepsilon_{ij}}{\partial t}+\frac{C_V}{T_0}\frac{\partial T}{\partial t}\right) = K_B\gamma_B\,T_0\frac{\partial \varepsilon_B}{\partial t}+C_V\frac{\partial T}{\partial t} \tag{36}$$

Introducing the expression for the volumetric heat capacity in terms of the average specific isobaric heat capacity c_p and the fluid-solid density $\rho = \varphi\,\rho_f + (1-\varphi)\,\rho_s$, the heat equation for the isotropic skeleton is deduced:

$$K_B\,\gamma_B\,T_0\frac{\partial \varepsilon_B}{\partial t}+c_p\,\rho\left(\frac{\partial T}{\partial t}+\vec{v}\cdot\vec{\nabla}T\right)=k_T\nabla^2 T+q_V \tag{37}$$

This formula is a thermoporoelastic equation coupling the rate of changes of the strains and the heat transported by conduction. To take into account the reservoir heat production and the effect of the advective transport of energy by the moving fluid in the porous rock, the fluid velocity term and the volumetric heat produced are included. $\boldsymbol{v}$ is the Darcy's velocity and q_V is the volumetric thermal energy production [W/m^3].

4.4 The 4D Tensorial Formulation of Geothermal Poroelasticity

As we did for the isothermal poroelasticity, we can write in a single four-dimensional tensorial equation the results of thermoporoelasticity relating stresses and strains. We solve first equation (32) in terms of the pore pressure:

$$p-p_0 = M\left(\zeta-\zeta_0\right)-C\varepsilon_B-M\varphi\left(\gamma_\varphi-\gamma_f\right)\left(T-T_0\right) \tag{38}$$

Expanding the terms of equations (30) and (36), and grouping them in their respective positions, we obtain the 4D thermoporoelastic matrix formulae, which include the thermal tensions in the total stress tensor:

$$\boldsymbol{\sigma}_T-\boldsymbol{\sigma}_T^0 = \begin{pmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_f \end{pmatrix} =$$

$$\varepsilon_B\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & -C \end{pmatrix}+2G\begin{pmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} & 0 \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} & 0 \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}+ \\ -\left(p-p_0\right)\begin{pmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & -M\dfrac{\zeta-\zeta_0}{p-p_0} \end{pmatrix}+ \\ -K_B\left(T-T_0\right)\begin{pmatrix} \gamma_B & 0 & 0 & 0 \\ 0 & \gamma_B & 0 & 0 \\ 0 & 0 & \gamma_B & 0 \\ 0 & 0 & 0 & \dfrac{M\varphi}{K_B}\left(\gamma_\varphi-\gamma_f\right) \end{pmatrix} \tag{39}$$

Finally, from equation (33), we deduce an important relationship to compute the fluid pressure variation when the temperature changes keeping constant the confining pressure and the fluid content:

$$\left(\frac{\partial p}{\partial T}\right)_\zeta = -M\varphi\left(\gamma_\varphi-\gamma_f\right) \tag{40}$$

As a general trend, thermal expansion increases when temperature rises. The volumetric thermal expansivity γ_B is about 10^{-5} K^{-1} for solids, 10^{-4} K^{-1} for liquids and 10^{-3} K^{-1} for gases. For water, at 80°C and 1 bar, $\gamma_B = 5.0 \quad 10^{-4}$ K^{-1}. For the expansivity of the pores, $\gamma_\varphi \approx 1.0 \quad 10^{-5}$ K^{-1}. For low porosity hard volcanic rocks such as granite, a measured value of $\gamma_B = 5.0 \quad 10^{-5}$ K^{-1} was obtained.

4.5 A Consolidation Diffusion Equation

Consolidation is a transient process occurring in porous rocks when pore pressure equilibrium is re-established after a change in the stress state. This process involves a flow of the pore fluid through the porous rock. This fluid flow obeys Darcy's law. From the definition of the variable fluid mass content ζ given in equation (17):

$$\zeta\left(\vec{u},t\right)=-\varphi\vec{\nabla}\cdot\vec{u} \;\Rightarrow\; \frac{\partial\zeta}{\partial t}=-\varphi\vec{\nabla}\cdot\frac{\partial\vec{u}}{\partial t} \tag{415}$$

Neglecting gravity, Darcy's velocity is given by:

$$\frac{\vec{v}}{\varphi}=\frac{\partial\vec{u}}{\partial t} \;\text{ and }\; \vec{v}=-\frac{k}{\mu}\vec{\nabla}p_f \tag{42}$$

Where k is the rock permeability and μ is its viscosity. Equating both equations (41) and (42) we obtain:

$$\vec{\nabla}\cdot\vec{v}=\varphi\vec{\nabla}\cdot\frac{\partial\vec{u}}{\partial t}=-\frac{k}{\mu}\vec{\nabla}\cdot\vec{\nabla}p_f \;\Rightarrow\; \frac{\partial\zeta}{\partial t}=\frac{k}{\mu}\nabla^2 p_f \tag{43}$$

This parabolic type partial differential equation is a consolidation diffusion equation, which characterizes the transient laminar fluid flow in a porous rock as function of the fluid pressure and of the variation of fluid content. We apply the diffusion equation (43) to a vertical stratum of homogeneous porous rock, of volume V_B. We suppose that the lateral strains are negligible, the rock has high permeability, especially at the top of the column, and it is impermeable at the bottom, at a depth equal to h_0. Assuming that the vertical stress does not change with time, the differential problem is:

$$p_f\left(h_0,t\right)=p_a;\; \frac{\partial p_f}{\partial z}\left(0,t\right)=0;\; p_f\left(z,0\right)=p_0 \\ \boxed{\frac{\partial p_f}{\partial t}=C_D\nabla^2 p_f} \tag{44}$$

Where the poroelastic diffusion coefficient is:

$$C_D=\frac{M\left(\lambda_U+2G\right)-C^2}{\lambda_U+2G}\frac{k}{\mu}\;\left[\frac{m^2}{s}\right] \\ z\in\left[0,h_0\right];\; t\geq 0 \tag{45}$$

5. DYNAMIC POROELASTICITY EQUATIONS

The preceding brief reconstruction of Biot's theory and the definition of the corresponding parameters, give the basis to establish a full tensorial model to represent linear, isotropic poroelasticity for Hookean rocks. This formulation is very convenient to be solved using the Finite Element Method. The fundamental poroelastic differential equation is the tensorial form of Newton's second law in continuum porous rock dynamics:

$$\overrightarrow{div}\,\vec{\sigma}_T + \vec{F} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}\ ;\ \overrightarrow{div}\,\vec{\sigma}_T = L^T \cdot \vec{\sigma}_T \qquad (46)$$

$$\text{where :}\quad \vec{\sigma}_T = C_B \cdot \vec{\varepsilon}_T\ ;\ \vec{\varepsilon}_T = L \cdot \vec{u}$$

The terms $\boldsymbol{\sigma}_T$ and $\boldsymbol{\varepsilon}_T$ are the equivalent vectorial form of tensorial equations (20) and $\boldsymbol{C}_B$ is the matrix of poroelastic constants derived from this expression. While **F** is the body force acting on the rock and the tensor differential operator L is given by:

$$L^T = \begin{pmatrix} \partial_x & 0 & 0 & \partial_y & \partial_z & 0 & \partial_x \\ 0 & \partial_y & 0 & \partial_x & 0 & \partial_z & \partial_y \\ 0 & 0 & \partial_z & 0 & \partial_x & \partial_y & \partial_z \end{pmatrix} \Rightarrow$$

$$L \cdot \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \vec{\varepsilon}_T = (\varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \varepsilon_{xy}\ \varepsilon_{xz}\ \varepsilon_{yz}\ e_r) \qquad (47)$$

Where $\boldsymbol{u} = (u_x, u_y, u_z)$ is the displacement vector of equation (16). Using the operator L in equation (46), the dynamic poroelastic equation becomes:

$$\left(L^T \cdot C_B \cdot L\right) \cdot \vec{u}\ +\ \vec{F} = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \qquad (48)$$

5.1 Solution of Thermoporoelastic Equations: The Finite Element Method

Equation (48) includes Biot's poroelastic theory. It can be formulated and numerically solved using the Finite Element Method (FEM). Let Ω be the bulk volume of the porous rock, and let ∂Ω be its boundary, $\boldsymbol{u}$ is the set of admissible displacements in equation (48); $\mathbf{f}_b$ is the volumetric force and $\mathbf{f}_s$ is the force acting on the surface ∂Ω. After doing some matrix algebra we arrive to a FEM fundamental equation for every element V^e in the discretization:

$$K^{\mathrm{e}} \cdot \vec{d}^{\,e} + M^{\mathrm{e}} \cdot \frac{\partial^2 \vec{d}^{\,e}}{\partial t^2} = \vec{F}^{\,e};\ e = 1, M \qquad (49)$$

$\mathbf{d}^e$ is a vector containing the displacements of the nodes in each V^e. Equation (49) approximates the displacement $\boldsymbol{u}$ of the poroelastic rock. $\mathbf{F}^e$ is the vector of total nodal forces. $\boldsymbol{K}^e$ and $\boldsymbol{M}^e$ are the stiffness and equivalent mass matrices for the finite element V^e. The mathematical definitions of both matrices are:

$$K^{\mathrm{e}} = \int_{\mathrm{V^e}} B^{\mathrm{T}} \cdot C_{\mathrm{B}} \cdot B\,\mathrm{dV}\ ;\quad e = 1, M$$

$$B = L \cdot N\ ;\ M^{\mathrm{e}} = \int_{\mathrm{V^e}} \rho N^{\mathrm{T}} \cdot N\,\mathrm{dV} \qquad (50)$$

Where N is the matrix of shape functions that interpolate the displacements. Matrix B is called the strain poroelastic matrix (Liu and Quek, 2003).

6. DEFORMATIONS OF A POROELASTIC AQUIFER

This section contains two applications of the theory herein presented. The first one reproduces the deformation of an isothermal aquifer already published (Leake & Hsieh, 1997). The second one illustrates the form in which a temperature change can affect the poroelastic deformation of the same aquifer. In the first example, we assume that the aquifer contains cold water at 20°C, with a density of 1000 kg/m^3. After, we consider a higher temperature of 250°C (50 bar, 800.4 kg/m^3). The program code of the model and the computations were done using COMSOL-Multiphysics (2006). The description of the reservoir is as follows.

6.1 Isothermal Deformation of a Poroelastic Aquifer

Three sedimentary layers overlay an impermeable bedrock in a homogeneous and isotropic basin where faulting created a bedrock step (BS) near the mountain front. Figure 1 illustrates the idealized geometry of the basin and the BS. Figure 2 shows the mesh constructed to model this problem with finite elements. The sediment stack totals 420 m at the deepest point of the basin (x = 0 m) but thins to 120 m above the BS (x > 4000 m). The top two layers of the sequence are each 20 m thick. The first and third layers are aquifers; the middle layer is relatively impermeable to flow.

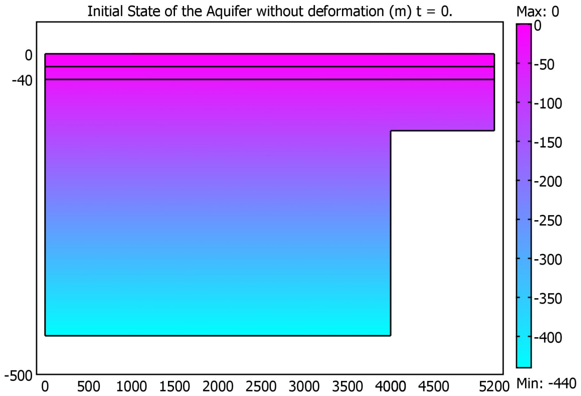

Figure 1: The simplified geometry of the aquifer and the impermeable bedrock in the basin. Initial state

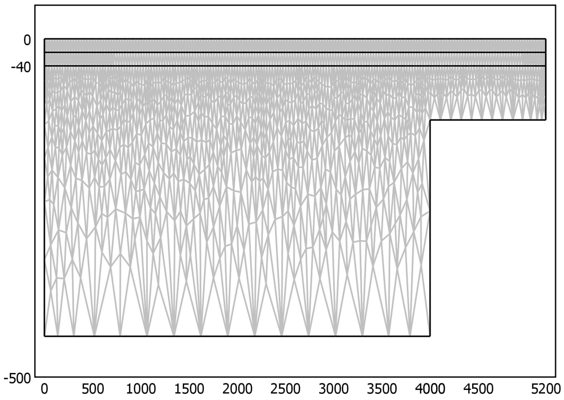

Figure 2: The mesh of the basin with 2967 finite elements

The water in the aquifer obeys Darcy's law for head h:

$$\frac{\partial}{\partial x}\left(K_X\,\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_Y\,\frac{\partial h}{\partial y}\right)+q_V=S_S\,\frac{\partial h}{\partial t} \qquad (51)$$

Where K_X and K_Y are the hydraulic conductivities and S_S is the specific storage. The flow field is initially at steady state, but pumping from the lower aquifer reduces hydraulic head by 6 m per year at the basin center. The head drop moves fluid away from the step. The fluid supply in the upper reservoir is limitless. The period of interest is 10 years. For the computations, data of Table 1 were used. To facilitate the comparison of the results with the non-isothermal case, both group of graphics are shown together in figures (3) to (8) in the same page. A discussion of numerical results follows.

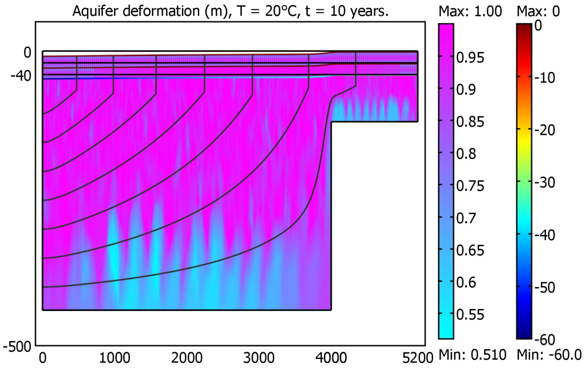

Figure 3:Poroelastic deformation of the basin with water at 20°C after ten years of fluid extraction. Streamlines represent the fluid - porous rock coupling

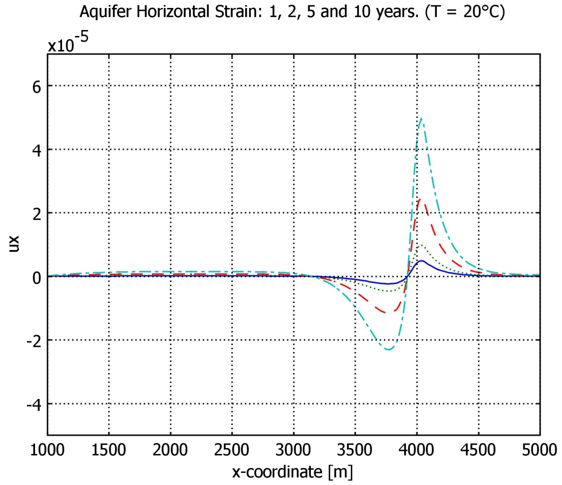

Figure 4: Horizontal strain at the basin with a BS. Case of cold water at 20°C

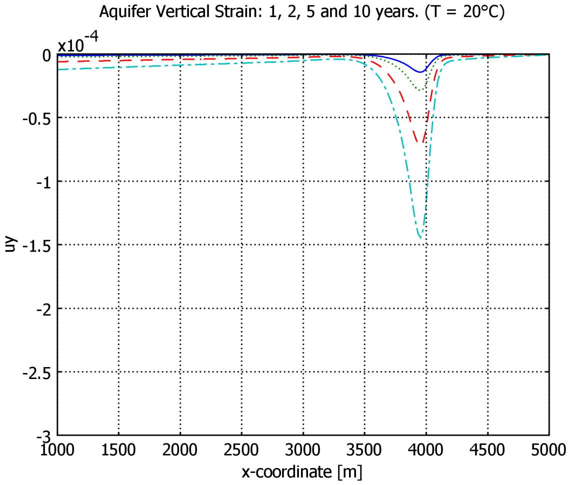

Figure 5: Vertical strain at the basin with a BS. Case of cold water (20°C)

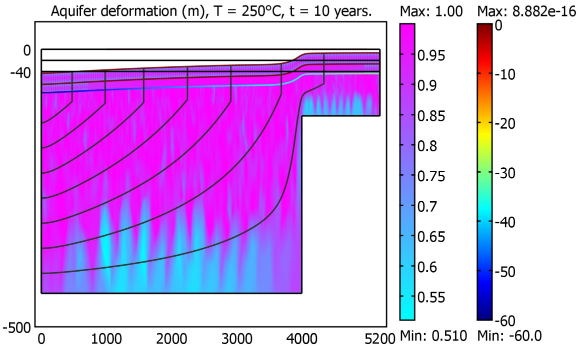

Figure 6: Thermoporoelastic deformation of the basin with hot water (250°C) after ten years of fluid extraction. Streamlines represent the fluid - porous rock coupling

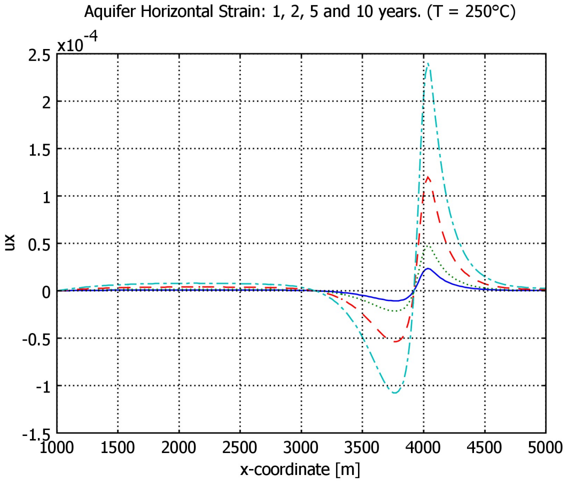

Figure 7: Horizontal strain at the basin with a BS for the case of geothermal water at 250°C

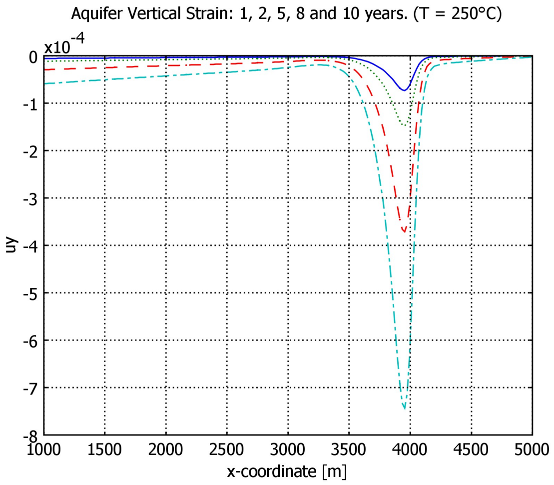

Figure 8: Vertical strain at the basin with a BS for the case of geothermal water at 250°C

6.2 Non-Isothermal Deformation of the Aquifer and Discussion of Results

The same three sedimentary layers overlay the impermeable bedrock, but in this non-isothermal case, the water in the aquifer is a geothermal fluid, at 250°C. The two examples presented herein were solved using the Finite Element Method (COMSOL - Multiphysics, 2006) for a known problem of linked fluid flow and solid deformation near a bedrock step in a sedimentary basin. The problem in both examples concerns the impact of pumping for a basin filled with sediments draping an impervious fault block.

The basin is composed of three layers having a total depth of 500 m and is 5000 m long in both cases. The Darcy's law (eq. 51) for simple water is coupled to the rock deformation via equation (22) through the porosity φ, which is implicit in the storage coefficient S_S:

$$S_S = \rho_f \, g \left(C_B + \varphi \, C_f \right) \qquad (52)$$

Where g (9.81 m/s^2) is gravity acceleration, ρ_f is the water density, C_B (0.22 10^{-9} 1/Pa) is the bulk rock compressibility and C_f (0.4 10^{-9} 1/Pa) is the compressibility of water. All units are in the SI. Figures (3) and (6) show simulation results of the basin for years 1, 2, 5, and 10, respectively, for cold and for hot water. The results of the first example are shown in figures (3, 4 and 5). They correspond to a coupled isothermal poroelastic deformation when the water in the aquifer is cold (fluid density of 1000 kg/m^3, temperature of 20 °C, and pressure of 50 bar).

The results of the second simulation (Figs. 6, 7 and 8) correspond to a coupled thermoporoelastic deformation when the water in the aquifer is under geothermal conditions (fluid density of 800.4 kg/m^3, temperature of 250°C, and pressure of 50 bar). Figures (4) and (7) compare the horizontal strains, illustrating the evolution of lateral deformations that compensate for the changing surface elevation above the bedrock step. Figures (5) and (8) compare the vertical strains in both cases, cold and geothermal respectively. Note that except in figures (3) and (6), vertical scales are different in both examples for clarity. Table 2 contains some data used for the computations.

Table 2. Numerical Values of the Parameters Used in the Simulations.

Hydraulic conductivity, upper, lower aquifers	K_X = 25 (m/day)	Poroelastic storage coefficient, upper aquifer	$S_S = 10^{-6}$
Hydraulic conductivity confining layer	K_Y = 0.01 (m/day)	Poroelastic storage coefficient, lower aquifer	$S_S = 10^{-5}$
Biot-Willis coefficient (cold water)	b = 0.3	Biot-Willis coefficient (hot water)	b = 1.0
Young's modulus	E = 8.0 10^8 (Pa)	Poisson's ratio	ν = 0.25

CONCLUSIONS

■ All crustal rocks forming aquifers and geothermal reservoirs are poroelastic and the fluid presence inside the pores affects their geomechanical properties. Several other factors affect the behavior of reservoir rocks: porosity, pressure, temperature, type of fluid, fissures and faults. The elasticity of aquifers and geothermal reservoirs is evidenced by the compression resulting from the decline of the fluid pressure, which can shorten the pore volume. This reduction of the pore volume can be the principal source of fluid released from storage.

■ A full 4D tensorial formulation of the classic Biot's poroelasticity theory was presented and its extension to non-isothermal phenomena was included. The equations developed are useful for the analysis of linear porous geothermal reservoirs with an elastic skeleton containing interconnected pores and non-isothermal fluids. With the inclusion of the fourth dimension, it is relatively simple and clear to extend the theory of solid linear elasticity to thermoporoelastic rocks, taking into account the effect of both, the fluid phase and the temperature.

■ The immediate physical experience shows that the supply or extraction of heat produces deformations in the rocks. Any variation of temperature induces a thermo-poroelastic behavior that influences their elastic response. Two numerical examples of a poroelastic aquifer were presented: one with cold water, and another with geothermal water. For cold water, the estimated value of the vertical strain ε_z is about -1.5×10^{-4}, while for hot water ε_z is -7.5×10^{-4}. Therefore, the poroelastic deformations are much higher in geothermal reservoirs than in cold isothermal aquifers. In the first case the bulk modulus of water K_w= 0.45 GPa, corresponding to T = 250°C. For cold aquifer K_w= 2.5 GPa.

■ The bulk modulus of water affects other poroelastic coefficients, including the expansivity of rocks The expansivity of geothermal rocks is relatively small, but its effects can produce severe structural damages in rocks subjected to strong temperature gradients, as happens during the injection of cold fluids. This is of great importance for enhanced geothermal systems, when the injected fluid circulates in the underground. This action can change both, the permeability and the rock thermal conductivity.

■ The final conclusion is that changes in applied stress produce changes in fluid pressure or in fluid mass. Any change in fluid pressure or in temperature or in fluid mass can produce a change in the volume of the porous rock.

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