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Correlation Between Resistivity Index, Capillary Pressure and Relative Permeability

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ABSTRACT

It is known that the three important parameters: resistivity, capillary pressure, and relative permeability, are all a function of fluid saturation in a porous medium. This implies that there may be a correlation among the three parameters. The models representing such relationships show that capillary pressure and relative permeability could be inferred from resistivity data using the analytical mathematical models derived theoretically. In fact the other two could be inferred using these models if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known. Using this approach, it would be possible to quickly obtain a distribution of capillary pressure and relative permeability characteristics as a function of depth and location across an entire reservoir.

1. INTRODUCTION

Capillary pressure and relative permeability are the key parameters that govern fluid flow in geothermal reservoirs. Determination of capillary pressure and relative permeability are traditionally conducted in the laboratory. However it is expensive, difficult, and time-consuming to measure capillary pressure and relative permeability in many cases, especially in steam-water flow. It is difficult to maintain exact reservoir conditions in taking a core from the reservoir and bringing it to the surface, and it is not possible to conduct the measurements in real time. On the other hand, it is easier to measure resistivity in the reservoir; a large number of resistivity measurements are available from well logging, even in real time. Routine well testing can only provide the effective permeability of the rock. Most of the approaches to evaluating permeability from resistivity well logging are based on empirical relationships between porosity and permeability.

Literature on the relationship between capillary pressure and resistivity index has been scarce. Szabo (1974) proposed a linear model to correlate capillary pressure with resistivity by assuming the exponent of the relationship between capillary pressure and water saturation is equal to that of the relationship between resistivity and water saturation. This assumption may not be reasonable in many cases. The linear model proposed by Szabo (1974) can be expressed as follows:

$$\frac{R_t}{R_0} = I = a + bP_c \tag{1}$$

where R_o is the resistivity of rock at a water saturation of 100%, R_t is the resistivity at a specific water saturation of S_w, I is the resistivity index, P_c is the capillary pressure, a and b are two constants.

The results from Szabo (1974) demonstrated that a single straight line, as predicted by the model (Equation 1), could not be obtained for the relationship between capillary pressure and resistivity index. Longeron et al. (1989) measured the resistivity index and capillary pressure under reservoir conditions simultaneously. Longeron et al. (1989) didn’t attempt to correlate the two parameters.

Li and Williams (2006) developed a correlation between resistivity and capillary pressure theoretically. The model was derived according to the fractal modeling of porous media.

As mentioned previously, it is difficulty to measure both capillary pressure and relative permeability. But it is relatively easier to measure capillary pressure, especially when mercury-intrusion approach is applied. It may be because of this that several mathematical models have been proposed to infer relative permeability from capillary pressure data. In 1949, Purcell (1949) developed a method to calculate the permeability using capillary pressure curves measured by mercury-injection. Later, Burdine (1953) introduced a tortuosity factor in the model. Corey (1954) and Brooks and Corey (1966) summarized the previous work and modified the method by representing capillary pressure curve as a power law function of the wetting-phase saturation. The modified model was known as the Brooks and Corey relative permeability model. Li and Horne (2006) reported that steam-water relative permeability could be calculated from capillary pressure data.

It would be helpful to establish the relationship between relative permeability and resistivity index. However, literature on the relationship between relative permeability and resistivity index has been scarce as well. Pirson et al. (1964) proposed a model to calculate relative permeability from resistivity data; the model reported by Pirson et al. (1964) was empirical. Li (2007) derived a model to infer relative permeability from resistivity index and verified the model using experimental data.

In this study, analytical mathematical models correlating resistivity index, capillary pressure, and relative permeability were reviewed. It is shown that capillary pressure and relative permeability can be inferred from resistivity data. Actually the other two could be inferred using these models if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known.

2. THEORY

Resistivity, capillary pressure, and relative permeability have similar features. For example, all are a function of fluid saturation in a porous medium. This implies that there should be a correlation among the three parameters. The models representing such relationships are discussed in this section.

Calculation of Wetting-phase Relative Permeability from resistivity index

Li (2007) derived the relationship between relative permeability and resistivity index:

$$k_{rw} = S_w^* \frac{1}{I} \tag{2}$$

k_{rw} is the relative permeability of the wetting phase. S_w^* is the normalized saturation of the wetting-phase and is expressed as follows:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \tag{3}$$

where S_{wr} is the residual saturation of the wetting phase.

The resistivity index, as a function of the wetting-phase saturation, can be represented using the Archie's equation (1942):

$$I = \frac{R_t}{R_0} = S_w^{-n} \tag{4}$$

where n is the Archie's saturation exponent.

Relative permeability of the wetting-phase can be calculated using Eq. 2 from resistivity index data once the residual saturation of the wetting-phase is available. Note that the residual saturation of the wetting-phase can be obtained from the experimental measurement of resistivity in the porous medium.

Calculation of Nonwetting-phase Relative Permeability

The wetting-phase relative permeability can be inferred from the resistivity data based on Eq. 2. However the relationship between nonwetting-phase relative permeability and resistivity has not been established. The computation of nonwetting-phase relative permeability will be described as follows.

According to Li and Horne (2006), the wetting-phase relative permeability can be calculated using the Purcell approach (1949):

$$k_{rw} = (S_w^*)^{\frac{2+\lambda}{\lambda}} \tag{5}$$

where λ is the pore size distribution index and can be calculated from capillary pressure data. After the relative permeability curve of the wetting-phase is obtained using Eq.2, the value of λ can be inferred using Eq. 5.

According to the Brooks-Corey model (1966) and the study by Li and Horne (2006), the relative permeability of the nonwetting-phase can be calculated once the value of λ is available. The equation is expressed as follows:

$$k_{rnw} = (1 - S_w^*)^2 [1 - (S_w^*)^{\frac{2+\lambda}{\lambda}}] \tag{6}$$

One can see that the entire relative permeability set (both wetting and nonwetting phases) can be inferred from resistivity index data using Eqs. 2 and 6.

Calculation of Capillary Pressure

There are two approaches to determining capillary pressure once resistivity index data are available. The first approach is to calculate capillary pressure using the Brooks and Corey capillary pressure model (Brooks and Corey, 1966):

$$P_{cD} = (S_w^*)^{-1/\lambda} \tag{7}$$

where P_{cD} is the dimensionless capillary pressure (P_c/p_e); p_e is the entry capillary pressure and λ is the pore size distribution index.

As pointed out previously, the value of λ could be inferred once resistivity index data are available. Therefore the dimensionless capillary pressure can be determined using Eq. 7 with the value of λ.

The second approach to determining capillary pressure is the application of the model developed by Li and Williams (2006):

$$P_{cD} = (I)^{\beta} \tag{8}$$

where β is the exponent in the relation between disjoining pressures and film thickness. One can see from Equation 8 that the dimensionless capillary pressure can be calculated from the resistivity index once the value of β is known.

According to the above description, Eqs. 2, 5, 6, 7, and 8 constitute the interrelationship among resistivity index, capillary pressure, and relative permeability. This implies that if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known, the other two could be inferred using these models (also see Figure 1). As shown in Figure 1, assuming that the resistivity index data are available, the wetting-phase relative permeability can be calculated using Eq. 2. Then the value of λ can be estimated using Eq. 5. Finally the nonwetting-phase relative permeability can be determined using Eq. 6. Capillary pressure can be estimated using Eq. 7 or 8 when resistivity index data are known.

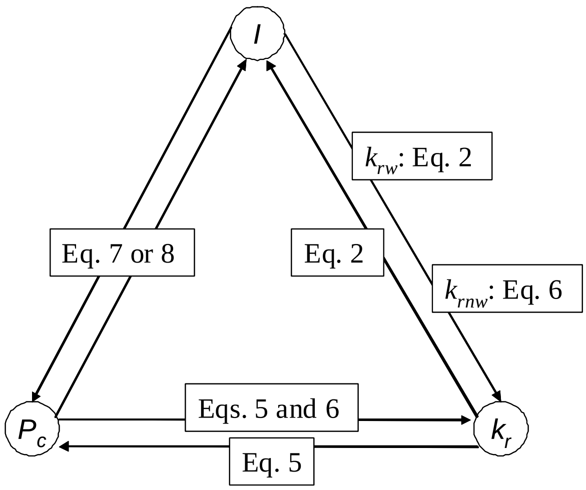

Figure 1: Procedure of inferring saturation function from each other

Note that the end points of relative permeability at initial water saturation or at residual fluid saturation and the entry capillary pressure can not be inferred using the above models. Those values need to be determined using different approaches. For example, the relative permeability of oil phase at the initial water saturation may be estimated from well testing data.

3. RESULTS

In this section, the results of relative permeability calculated using resistivity index and capillary pressure data are analyzed and compared with experimental data. Also discussed are the relationships between resistivity index and capillary pressure.

Calculation of Relative Permeability with Resistivity Index and Capillary Pressure Data

The relationship between relative permeability and resistivity index (Eq. 2) was verified using the experimental data of resistivity and capillary pressure measured by Sanyal (1972) in rocks (Berea, Boise sandstone and limestone) with different permeability (Li, 2007). All of the experimental data used in this study were obtained in drainage process. Firstly, the values of oil/water relative permeability were calculated with the experimental data of resistivity index using Eqs. 2 and 6. Secondly, the oil/water relative permeability data were calculated using the capillary pressure data (Li and Horne, 2006). According to the study by Li and Horne (2006), relative permeability could be calculated accurately using the capillary pressure technique. Finally the results of relative permeability inferred from resistivity index and capillary pressure data respectively were compared.

Figure 2 shows the oil/water relative permeability data obtained from resistivity index and capillary pressure in Berea sandstone sample with a porosity of 20.4% and a permeability of 300 md. As shown in Figure 2, the relative permeability data inferred from the resistivity index data are close to those calculated using the capillary pressure data. The oil relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the experimental capillary pressure data.

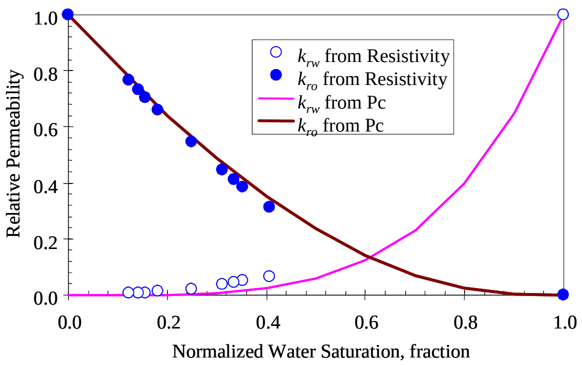

Figure 2: Relative permeability calculated from resistivity and capillary pressure data in Berea sandstone at a temperature of 175°F

Sanyal (1972) also conducted the experimental measurements of resistivity index and capillary pressure at different temperatures in the same core sample. Figure 3 demonstrates the relative permeability data calculated from both the resistivity index data and the capillary pressure data measured at a temperature of 300°F. One can see in Figure 3 that both the oil and water relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the capillary pressure data.

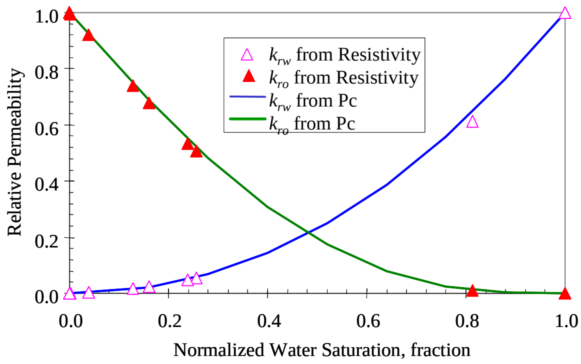

Figure 3: Relative permeability calculated from resistivity and capillary pressure data in Berea sandstone at a temperature of 300°F

It is useful to test the approach in different rocks. Figure 4 shows the relative permeability data calculated from both the resistivity index and the capillary pressure data measured in the Boise sandstone core sample with a porosity of 32% and a permeability of 960 md. The oil and water relative permeabilities calculated from the resistivity index data are also close to those calculated from the capillary pressure data.

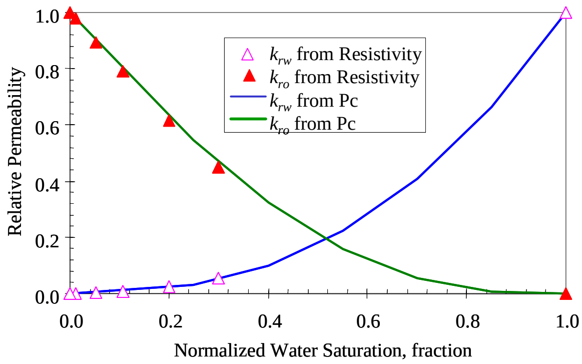

Figure 4: Relative permeability calculated from resistivity and capillary pressure data in Boise sandstone at a temperature of 175°F

The results in the Boise sandstone at a higher temperature of 300°F are demonstrated in Figure 5.

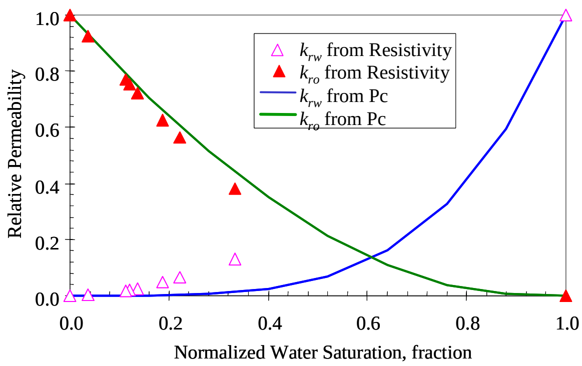

Figure 5: Relative permeability calculated from resistivity and capillary pressure data in Boise sandstone at a temperature of 300°F

One can see from Figure 5 that the values of the oil relative permeability calculated from resistivity index are close to those inferred from the capillary pressure data. However the water relative permeability calculated from resistivity index data is smaller than those inferred from capillary pressure.

For the limestone core sample with a porosity of 19% and a permeability of 410 md, the results of relative permeability calculated from the resistivity index and the capillary pressure data (measured at a temperature of 300°F) are plotted in Figure 6. The oil relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the capillary pressure data in the limestone core sample.

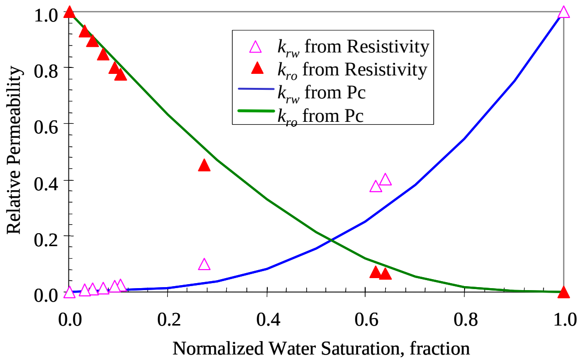

Figure 6: Relative permeability calculated from resistivity and capillary pressure data in limestone at a temperature of 300°F

According to the above results, the difference between the relative permeability inferred from the resistivity index and those calculated from the experimental data of capillary pressure is acceptable in terms of reservoir engineering applications.

Comparison of Relative Permeability Inferred From Resistivity Index with Experimental Data

In the last section, the relative permeability data calculated from resistivity index are compared with those computed from capillary pressure, instead of experimental data of relative permeability. In this section, the relative permeability data will be inferred from resistivity index and compared with experimental data directly (Li, 2006).

The experimental data of resistivity and gas/water relative permeability measured by Pirson et al. (1964) in eight core samples (sandstone) with different permeability were used to test the models (Eqs. 2 and 6). The permeability of the core samples ranged from 10 to 280 md. The values of porosity, permeability, and initial water saturation (S_{wi}) are listed in Table 1.

Table 1. Properties of Core Samples.

Core #	Porosity, f	Permeability, md	S_{wi}, f
1	0.25	280	0.37
2	0.26	250	0.39
3	0.19	70	0.35
4	0.20	15	0.48
5	0.21	10	0.46
6	0.27	100	0.50
7	0.17	40	0.28
8	0.26	75	0.40

Nitrogen was the nonwetting-phase and brine with a concentration of 5% NaCl was the wetting phase. The resistivity and relative permeability were measured simultaneously at an ambient temperature. The results of relative permeability inferred from resistivity index data using Eqs. 2 and 6 were compared with the experimental data.

Figure 7 shows the comparison of gas and water relative permeability calculated from resistivity index with experimental data in core sample No.1. Both the gas and water relative permeability data calculated from resistivity index data using the mathematical models (Eqs. 2 and 6) were almost equal to the experimental data at the same water saturation.

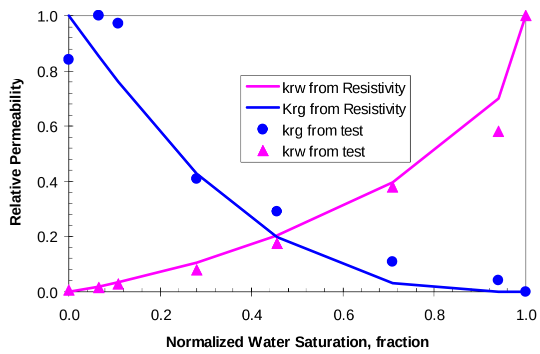

Figure 7: Comparison of relative permeability calculated from resistivity with experimental data (core No.1, ϕ=0.25, k=280 md, S_{wi}=0.37)

For core sample No. 2, the results are plotted in Figure 8. The water relative permeability data calculated using Eq. 2. are approximately equal to the experimental data. But for the gas phase, the calculated relative permeability is smaller than the experimental data.

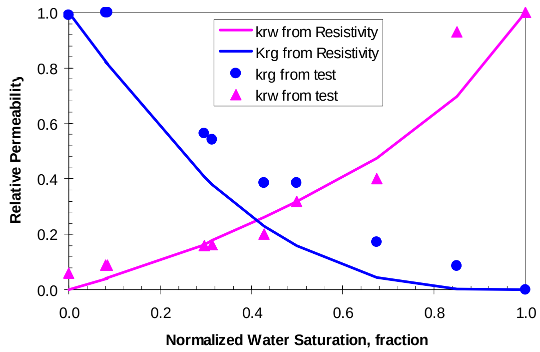

Figure 8: Comparison of relative permeability calculated from resistivity with experimental data (core No.2, ϕ=0.26, k=250 md, S_{wi}=0.39)

For all of the rest six core samples, the results are shown in Figs. 9 to 14. One can see that the models (Eqs. 2 and 6) work better in core samples with greater permeabilities than in those with lower permeabilities. The calculated gas phase relative permeability is smaller than experimental data in core samples with low permeabilities. One of the possible reasons may be due to gas slippage in two phase flow (Li and Horne, 2004). The gas slip effect in two phase flow was not considered in the experimental data of relative permeability. Note that the gas slippage is greater in core samples with low permeabilities than in those with high permeabilities.

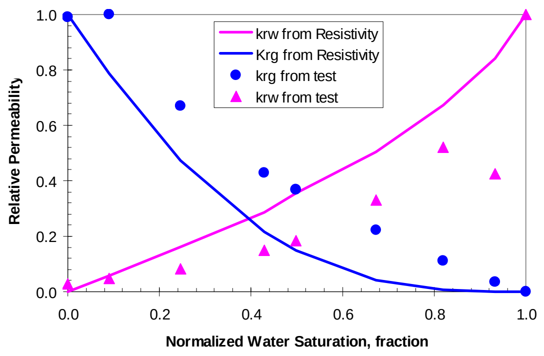

Figure 9: Comparison of relative permeability calculated from resistivity with experimental data (core No.3, ϕ=0.19, k=70 md, S_{wi}=0.35)

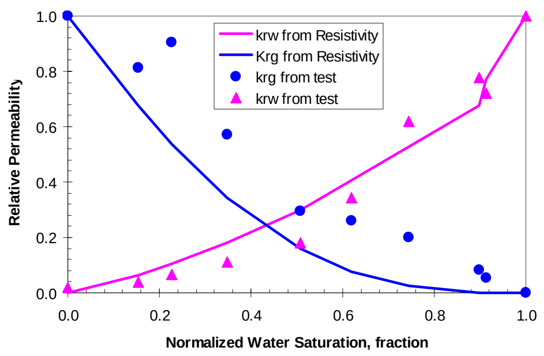

Figure 10: Comparison of relative permeability calculated from resistivity with experimental data (core No.4, ϕ=0.20, k=15 md, S_{wi}=0.48)

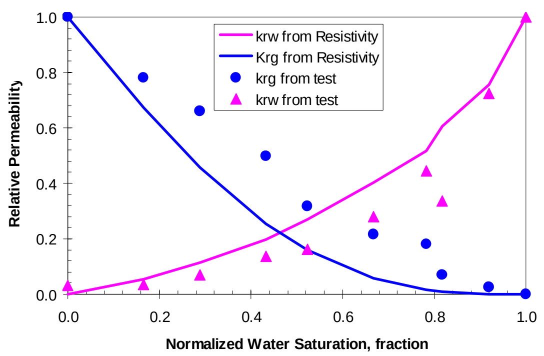

Figure 11: Comparison of relative permeability calculated from resistivity with experimental data (core No.5, ϕ=0.21, k=10 md, S_{wi}=0.46)

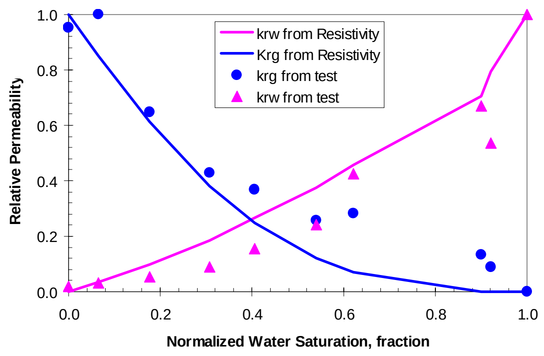

Figure 12: Comparison of relative permeability calculated from resistivity with experimental data (core No.6, ϕ=0.27, k=100 md, S_{wi}=0.50)

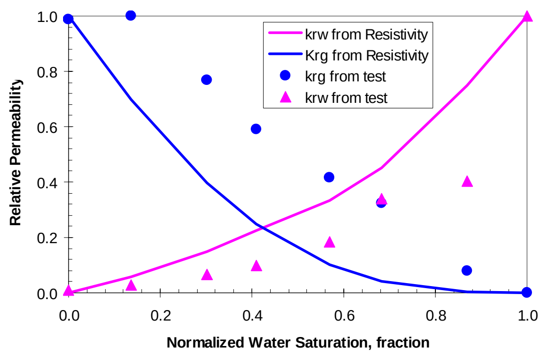

Figure 13: Comparison of relative permeability calculated from resistivity with experimental data (core No.7, ϕ=0.17, k=40 md, S_{wi}=0.28)

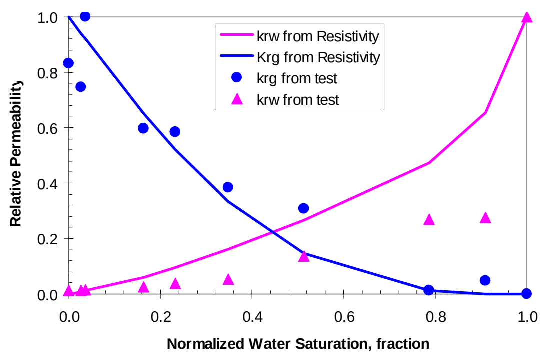

Figure 14: Comparison of relative permeability calculated from resistivity with experimental data (core No.8, ϕ=0.26, k=75 md, S_{wi}=0.40)

Verification of Relationship Between Capillary Pressure and Resistivity Index With Experimental Data

The experimental data of gas-water capillary pressure and resistivity measured simultaneously by Li and Williams (2006) were used to test the relationship between capillary pressure and resistivity index (Eq. 8). The experiments were conducted at a room temperature.

All of the core samples were sandstones and were obtained from one oil reservoir but different formations. Group 1 core samples were from one formation with a high permeability and Group 2 were from another formation with a low permeability. The permeability in Group 1 ranged from 437 to 3680 md; the permeability in Group 2 ranged from 0.028 to 387 md (see Table 2).

Table 2. Properties of Core Samples (Li and Williams, 2006)

	Core	ϕ (f)	k (md)	S_{wr} (f)	m	n
Group 1	1	0.272	941	0.112	1.80	1.87
	3	0.281	1192	0.116	1.68	1.86
	6	0.191	999	0.134	1.65	1.82
	8	0.227	3680	0.067	1.67	2.00
	10	0.321	437	0.167	1.83	2.11
	16	0.262	1916	0.078	1.66	1.97
Group 2	152	0.114	1.49	0.519	2.21	2.49
	153	0.077	0.028	0.796	2.32	2.39
	204	0.179	0.560	0.617	2.20	1.82
	299	0.185	4.63	0.446	2.19	2.13
	334	0.234	387.	0.222	2.00	2.02
	336	0.163	35.3	0.388	2.03	2.23
	418	0.211	74.0	0.454	2.09	2.26
	479	0.210	28.3	0.560	2.18	1.91

The relationships between capillary pressure and resistivity index of Group 1 are shown in Figure 15. On the log-log plot, a straight line exists in the range with great values of capillary pressure and resistivity index (corresponding to small water saturations), as predicted by the model (Eq. 8).

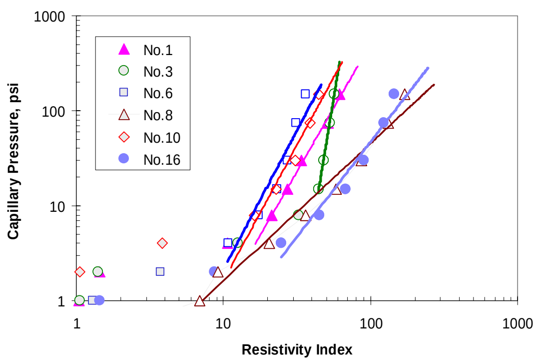

Figure 15: Relationship between capillary pressure and resistivity index in the core samples (Group 1, high permeability)

Figure 16 shows the relationships between capillary pressure and resistivity index of Group 2 core samples with low permeability. The results shown in Figure 16 demonstrate the validity of Eq. 8 in low permeability core samples. Comparing Figure 16 with Figure 15, one can see that the model (Eq. 8) works better in core samples with low permeability than those with high permeability.

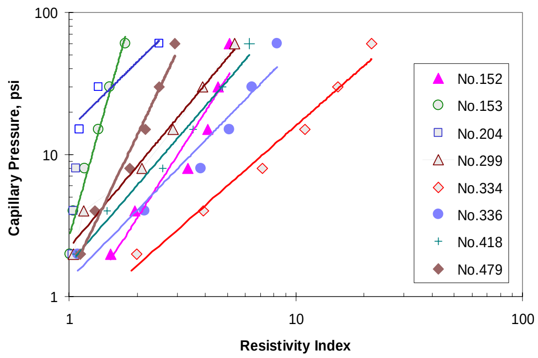

Figure 16: Relationship between capillary pressure and resistivity index in the core samples (Group 2, low permeability)

As demonstrated in Figure 15, Eq. 8 works properly for high values of capillary pressure and resistivity (corresponding to low values of water saturations) in core samples with a high permeability. At high water saturations, the experimental data deviate the power law model. A possible reason may be that the distribution of water saturation may not be a fractal at high water saturations. In this case, water (wetting phase) remains in both small and big pores. It has been demonstrated that the part of the rock with big pores may not be a fractal (Katz and Thompson, 1985; Li, 2004). This was also pointed out by Toledo *et al*. (1994). Note that Eq. 8 is only suitable for a specific range of water saturation with low values. In the case of low permeability core samples, the number of data points that deviate the power law model is significantly less. This may be due to the unique fractal property of low permeability core samples. In low permeability rock, most of the pores are small and the pore system may be a fractal. When water (wetting phase) saturation is below a specific value (for example, the percolation threshold), water exists as thin films and follows the surface shape of pores in the rock, which is a fractal and the power law applies. However this is yet to study in more detail.

CONCLUSIONS

The following conclusions may be drawn according to the present study:

The three saturation functions, resistivity index, capillary pressure and relative permeability, are coupled and can be inferred from each other using the mathematical models proposed in this paper if one of the three parameters is known;

Relative permeability can be calculated from the experimental data of both resistivity index and capillary pressure;

A power law model applies to the relationship between capillary pressure and resistivity index. The goodness of fitting to the experimental data is greater in low permeability rocks than in high permeability rocks.

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