

Assessing Uncertainty in Future Performance Predictions of Lumped Parameter Models Using the Randomized Maximum Likelihood Method

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ABSTRACT

In this work, we present a methodology within the context of stochastic simulation for assessing uncertainty in future pressure changes simulated by using history-matched lumped models for low-temperature geothermal systems. Specifically, we consider the randomized maximum likelihood method (RML) for the assessment of uncertainty. We show that this methodology allows us to incorporate into the performance predictions any uncertainties in both the model and the measured data. In this way, we are able to characterize or appraise the uncertainty in the predicted future pressure changes. Once the uncertainty in predicted performance is characterized or assessed, it is possible to make reservoir management decisions that account for an incomplete knowledge of the actual geothermal system. One synthetic example application is presented to show the use of the methodology proposed in this work.

1. INTRODUCTION

The behavior of low-temperature geothermal reservoirs under exploitation can be simulated using either analytical lumped-parameter models (Grant *et. al.*, 1982; Axelsson, 1989; Axelsson *et. al.*, 2005, Sarak *et. al.*, 2005) or distributed (numerical) models (Bodvarsson *et. al.*, 1986; O'Sullivan *et. al.*, 2001). Numerical models are, of course, more general than lumped-parameter models in that one can account for spatial variations in thermodynamic conditions and reservoir properties as well as for different well spacing and locations. However, they require a large amount of input data for modeling, simulation and prediction studies. This work specifically focuses on modeling of low-temperature geothermal reservoirs through the use of simple lumped-parameter models.

Over the last years, lumped-parameter models have been used for history matching and predicting pressure (or water level) changes in low-temperature geothermal systems in Iceland, Turkey, The Philippines, China, Mexico and other countries. Axelsson *et. al.* (2005) and Sarak *et. al.* (2005) have presented several field applications of various lumped-parameter models to low-temperature geothermal systems. When lumped-parameter models are used, model parameters can be obtained by applying nonlinear least-squares estimation techniques in which measured field pressure (or water level) data are history matched to the corresponding model response (Axelsson, 1989, and Sarak *et. al.*, 2005). Then, by using history-matched models, the future performance (in terms of pressure changes or water levels) of the reservoir can be predicted for different production/re-injection scenarios to optimize the management of a given low-temperature geothermal system.

The ultimate goal in any geothermal reservoir study is to predict future performance and even more important to predict the uncertainty in future predictions under different management options. This is necessary to determine the production/re-injection practices that will provide sustainable exploitation of the geothermal system in consideration. Uncertainty in all future predictions of pressure changes is inherent due to (i) measurement errors or noise in observed data, (ii) modeling errors, (iii) span of the available observed data (pressure change data and production history), and (iv) nonlinear relationship between model parameters and observed response.

The objective of this paper is to discuss the uncertainty in performance predictions and provide a methodology for the assessment of uncertainty in performance predictions. This is accomplished with a stochastic method of modeling that incorporates uncertainties both in the model and observed data to future performance predictions. Specifically, we consider the application of the randomized maximum likelihood method (RML) for the assessment of uncertainty to lumped-parameter modeling. This method has been shown to be quite efficient for the assessment of uncertainty in performance predictions for nonlinear problems (Kitanidis *et. al.*, 1995; Oliver *et. al.*, 1996; Liu and Oliver, 2003; Gao *et. al.*, 2005).

The paper begins with a brief review of lumped-parameter models considered in this study. Then, history matching and performance prediction problems within the context of maximum likelihood and randomized maximum likelihood methods are discussed. Finally, a synthetic example application is presented to demonstrate the methodology proposed in this study for the assessment of uncertainty in performance predictions by lumped-models for low-temperature geothermal systems.

2. LUMPED PARAMETER MODELING

The lumped-parameter modeling considered here is very similar in concept to the one presented originally by Axelsson (1989) and identical to the one presented later by Sarak *et. al.* (2003a, 2003b, and 2005). As in these works, our lumped-parameter models are based on the conservation of mass only and hence are valid for low-temperature liquid reservoirs under the assumption that variations in temperature within the system can be neglected (i.e. the simulated systems are assumed to be isothermal).

Lumped-parameter modeling can be regarded as a highly simplified form of numerical modeling. In numerical models, a geothermal system is represented by many (>100 to 10^6) gridblocks. On the other hand, in lumped-parameter modeling, a geothermal system is represented by only a few homogeneous tanks and is visualized as consisting of mainly three parts: (1) the central part of the reservoir; (2) outer parts of the reservoir, and (3) the recharge source. The

first two are treated as series of homogeneous tanks with average properties. The recharge (or constant pressure) source can be connected to the other parts of the reservoir or directly to the central part of the reservoir and is treated as a “point source” that recharges the system. If there is no connection to the recharge source, the model would be closed, otherwise would be open. Two different open lumped-parameter models are depicted in Fig. 1.

The model shown in Fig. 1(a) represents a two-tank open lumped model, where the first tank, in which production/injection occurs, represents the innermost (or central) part of the geothermal system. The changes in pressure in this part are monitored and production/injection rates are recorded. In the second tank, representing the outer part of the reservoir that is connected to the recharge source, there is neither production nor injection, and it recharges the central reservoir. Fluid production causes the pressure in the reservoir to decline, which results in water influx from the outer to the central part of the reservoir. The recharge source represents the outermost part of the geothermal system.

When using the lumped-parameter models considered in this work (Fig. 1), the simulated model (output) response represents pressure or water level changes for an observation well for a given net production history (input). The number of model parameters increases as the number of tanks or the complexity of the lumped model increases.

Here and throughout, α represents the recharge constant between the tanks in kg/(bar-s), κ represents the storage capacity (or coefficient) of a tank in kg/bar, and p_i represents the initial pressure of the recharge source in bar. The geothermal system is assumed to be in hydrodynamic equilibrium initially; i.e., the initial pressure, p_i , is uniform in the system. In cases for which the initial system pressure (or initial water level), p_i , is known, p_i can be eliminated from the unknown set of model parameter vector. Further details about the lumped-parameter models used in this study can be found in Sarak *et al.* (2003a, b, and 2005).

3. HISTORY MATCHING PROBLEM

After a period of production from a geothermal reservoir, and based on the production/injection rate history given, a lumped-parameter model can be matched to the observed

pressure (or water level) data to obtain the parameters of that particular model.

Here and throughout, \mathbf{y}_{obs} refers to the vector of measured or observed pressure change data, and contains all N_d pressure change measurements that will be used for estimating the model parameters by nonlinear regression. We let \mathbf{C}_D be the $N_d \times N_d$ symmetric positive-definite covariance matrix for pressure change measurement errors, and assume that measurement errors for pressure data are Gaussian with mean zero (vector) and covariance matrix \mathbf{C}_D [i.e., $N(\mathbf{0}, \mathbf{C}_D)$]. $N(\mathbf{0}, \mathbf{C}_D)$ represents a normal distribution with mean zero and covariance matrix \mathbf{C}_D . Throughout, a boldface capital letter denotes a matrix, while a boldface lower case letter denotes a column vector.

Letting \mathbf{e} denote the N_d -dimensional vector of errors for observed data and $\mathbf{m} = (m_1, m_2, \dots, m_M)^T$ denote the vector of unknown model parameters that are estimated, it follows that

$$\mathbf{y}_{obs} = \mathbf{f}(\mathbf{m}) + \mathbf{e} \quad (1)$$

Here \mathbf{f} refers to the N_d -dimensional vector of computed pressure-change data from a considered lumped model, for a given \mathbf{m} . M represents the total number of unknown model parameters.

As noted above, \mathbf{e} is $N(\mathbf{0}, \mathbf{C}_D)$. Thus, the likelihood function for the model conditional to observed data is given by (Bard, 1974)

$$L(\mathbf{m} | \mathbf{y}_{obs}) \propto \exp \left\{ -\frac{1}{2} [\mathbf{y}_{obs} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_D^{-1} [\mathbf{y}_{obs} - \mathbf{f}(\mathbf{m})] \right\} \quad (2)$$

where the superscripts “ T ” and “ -1 ” represent transpose of a vector and inverse of a matrix, respectively. The maximum likelihood estimate of \mathbf{m} , which honors measured pressure data, is obtained by maximizing Eq. 2, or equivalently, minimizing the objective function $O(\mathbf{m})$ given by

$$O(\mathbf{m}) = [\mathbf{y}_{obs} - \mathbf{f}(\mathbf{m})]^T \mathbf{C}_D^{-1} [\mathbf{y}_{obs} - \mathbf{f}(\mathbf{m})] \quad (3)$$

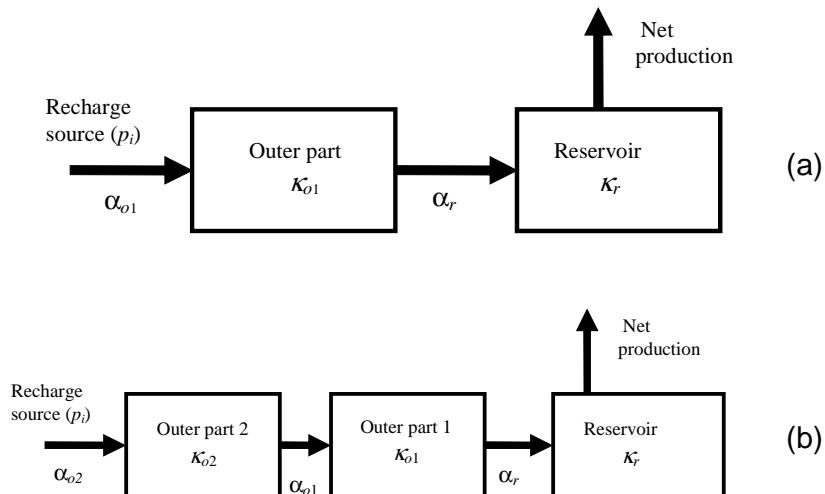


Figure 1: Two different lumped-parameter models. (a) two tank open lumped parameter model (b) three tank open lumped parameter model

Eq. 3 is the objective function for the well known general nonlinear least-squares method, and assumes that the data error covariance matrix \mathbf{C}_D is known. Often we do not have enough information to construct the data error covariance matrix \mathbf{C}_D that may account correlation between measurement errors. Hence, the commonly used assumption is that the $N_d \times N_d$ \mathbf{C}_D is a diagonal matrix in Eq. 3. If data measurement errors are independent random variables with mean zero and known variance $\sigma_{d,j}^2$ for each observed data $y_{obs,j}$, then \mathbf{C}_D is a diagonal matrix with diagonal entries equal to $\sigma_{d,j}^2$, $j=1,2,\dots,N_d$. In this case, Eq. 3 reduces to the well known weighted least-squares objective function:

$$O(\mathbf{m}) = \sum_{j=1}^{N_d} \left[\frac{y_{obs,j} - f_j(\mathbf{m})}{\sigma_{d,j}} \right]^2 \quad (4)$$

If we further assume that error variances $\sigma_{d,j}^2$ are identical, i.e., $\sigma_{d,j}^2 = \sigma_d^2$ for all j , then Eq. 4 with $1/(\sigma_d^2)$ deleted defines the objective function for the un-weighted (ordinary) least-squares procedure.

The lumped-parameter model responses are nonlinear with respect to the model parameters. Thus, Eq. 3 (or 4) calls for nonlinear minimization techniques. Over the past, we have found that the gradient based algorithms such as the Levenberg-Marquardt method based on a restricted procedure described by Fletcher (1987) is quite efficient to minimize Eq. 3 or 4.

It is important to note that within the context of maximum likelihood estimation, the observed data \mathbf{y}_{obs} would represent a single realization of the observed data from a normal distribution with mean zero and known covariance matrix, \mathbf{C}_D , and thus the model vector \mathbf{m} is considered as a random variable because different realizations of \mathbf{y}_{obs} would provide different estimates of \mathbf{m} . Thus, when history-matching problem is viewed within the context of the principle of maximum likelihood estimation, one can attach statistical measures to quantify the quality of a match as well as the uncertainty of the model parameters estimated. The standard statistical measures used for assessing the quality of a match and the reliability of estimated parameters are the root-mean-square error (RMS) and confidence (usually 95% percent) intervals.

The value of RMS defined by Eq. 5 shows the quality of fit quantitatively.

$$RMS = \sqrt{\frac{1}{N_d} \sum_{i=1}^{N_d} \left[y_{obs,i} - f_i(\mathbf{m}^*) \right]^2} \quad (5)$$

where \mathbf{m}^* represents the optimized parameter vector. The lower the RMS value, the better the fit between field and computed data. As we will discuss later, this does not necessarily mean that the lumped-model giving the smallest RMS value be the most appropriate model for the history-matched data and should give the most reliable predictions.

While it is important to improve the overall match of available data, it is equally or even more important that the history-matched model be able to predict reliably the uncertainty (from a statistical point of view) in predictions due to the fact that a certain amount of error (i.e., modeling and measurement errors, etc) will always be introduced into the estimated parameters from the history-matching

process. In history matching, increasing complexity of the lumped-model (or equivalently increasing the number of tank and hence the model parameters) may improve the overall fitting of the model to the current data at the expense of destroying and ignoring the underlying statistical basis for nonlinear least-squares parameter estimation based on the principle of maximum likelihood.

Statistical confidence intervals are known as a useful tool to give a quantitative evaluation of model discrimination and assessment of uncertainty in the estimated parameters (Dogru *et al.*, 1977; Anraku and Horne, 1995). From the least-squares theory under the assumption that a nonlinear model can be linearized around the optimal estimate \mathbf{m}^* , we know that confidence intervals contain information about both the statistical standard deviation (s , see Eq. 7) of the match (related to the RMS value) and the sensitivity of observed data to the parameters. In general, the larger the confidence interval, the higher the uncertainty in the estimated model parameters. The $\gamma \times 100\%$ approximate confidence intervals are computed from (Bard, 1974; Dogru *et al.* 1977)

$$m_i^* - t(1 - \gamma/2, N_d - M) s \sqrt{[(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*})^{-1}]_{ii}} \leq \hat{m}_i \leq m_i^* + t(1 - \gamma/2, N_d - M) s \sqrt{[(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*})^{-1}]_{ii}} \quad (6)$$

where \mathbf{m}^* denotes the estimate obtained by minimizing O (Eq. 3), m_i^* denotes the estimate of i^{th} model parameter at the minimum, \mathbf{G}_{m^*} denotes the $N_d \times M$ sensitivity matrix (containing derivatives of observed data with respect to model parameters) evaluated at the estimate \mathbf{m}^* , \hat{m}_i represents the true, but unknown value of the model parameter, m_i , and $t(1 - \gamma/2, N_d - M)$ is the value that cuts off $(1 - \gamma)/2 \times 100\%$ in the upper tail of t -distribution with $N_d - M$ degrees of freedom. (Taking $\gamma = 0.95$ in Eq. 6 gives 95 percent confidence intervals.) In Eq. 6, s is computed from

$$s = \sqrt{\frac{O(\mathbf{m}^*)}{N_d - M}} \quad (7)$$

It may be worth noting that s computed from Eq. 7 is a dimensionless quantity. If the quantity s is significantly greater than unity, this indicates either that the lumped-model is inappropriate to reproduce the pressure data, or that the magnitude of the pressure measurement errors reflected by the covariance matrix \mathbf{C}_D is underestimated. If the model is correct, and s computed from Eq. 7 is significantly larger than 1, this indicates that the covariance matrix \mathbf{C}_D used in Eq. 3 underestimates the true, but unknown covariance matrix \mathbf{C}_D . If the model used for observed data is correct, and we use the correct error covariance matrix, then we should expect s to be close to unity.

We should perhaps dedicate a few words to the philosophy underlying the use of RMS and confidence intervals to discriminate the best fitting model among the candidate lumped models selected for history matching. There is a relationship between the confidence intervals and the RMS. This relationship could be complicated in the models having large number of model parameters and when the parameters show correlation among them. One may expect

the uncertainty as reflected by the confidence intervals for some parameter estimates to increase with the increasing complexity of a model, while the value of RMS (or equivalently the quality of a fit as defined by Eq. 7) improves. However, in our view, as long as the lumped model selected is appropriate and there are sufficient observed data available to support the model, all parameters should have “acceptable” confidence interval ranges and the RMS value should be close to the standard deviation of measurement errors in observed pressure data. Then, one can accept the model. Otherwise, one rejects the model because confidence intervals do not support the model from a statistical point of view. In short, in our view, the best fitting lumped model is the one providing not only the smallest possible acceptable confidence intervals for all parameters but also the smallest possible RMS value among the lumped-models used for history matching. Here, our definition is that an estimate of a parameter is acceptable if its confidence interval range is less than 95% of the estimated value itself. As also shown later by a synthetic example application, it is important to compute the confidence intervals for the parameters in addition to the RMS (or the value of s given by Eq. 7). The application of this same methodology to a few field examples has also been demonstrated by Sarak *et al.* (2005).

4. PREDICTION OF FUTURE PERFORMANCE

The most important objective of lumped-parameter modeling is to predict future responses to given production scenario. The parameter estimation (or history-matching) procedure provides values of the model parameters to be inserted into the prediction model equations. Although forecasting future performance is similar to parameter estimation in that they both involve the history matching of measured data, in performance prediction we have also to tackle the element of uncertainty in the model parameters and in the data observations, caused by measurement errors. First we discuss the linear least-squares theory, and then the more general nonlinear case how to characterize the uncertainty in future predicted response.

Assuming that a nonlinear model can be linearized around the optimum parameter vector and that the measurement errors on the observed past and future data to be observed are independent, identically distributed random variables with mean zero and variance σ_d^2 (i.e., \mathbf{C}_D in Eq. 3 is a diagonal matrix with all diagonal entries equal to σ_d^2 or $\sigma_{d,i}^2 = \sigma_d^2$ in Eq. 4), we can show that the variance of the predicted value of $y_{p,i}$ at a given time t_i such that $t_i > t_{N_d}$ is given by (Bard, 1974; Dogru *et al.*, 1977; Sen and Srivastava, 1990)

$$\sigma_{p,i}^2 = \sigma_d^2 + s^2 \mathbf{g}_i^T \left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1} \mathbf{g}_i \quad (8)$$

Here, \mathbf{g}_i is the M -dimensional sensitivity vector of predicted response at any time t_i :

$$\mathbf{g}_i^T = \left[\frac{\partial y_{p,i}}{\partial m_1}, \frac{\partial y_{p,i}}{\partial m_2}, \dots, \frac{\partial y_{p,i}}{\partial m_M} \right] \quad (9)$$

where each sensitivity is evaluated at the optimized parameter vector, \mathbf{m}^* obtained from history matching period. $\left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1}$ is the $M \times M$ approximate Hessian

matrix evaluated at \mathbf{m}^* and also represents the covariance matrix of the estimated \mathbf{m}^* . The diagonal entries of this matrix represent the variances of model parameters, while its off diagonal entries represent the covariances (or correlations) between the two model parameters. This matrix does not vary with the prediction time t_i because it is determined from the history-matching period by minimizing Eq. 3. If σ_d^2 is not known, it can be estimated from the history-matching period:

$$\sigma_d^2 = \frac{\sum_{i=1}^{N_d} [y_{obs,i} - f_i(\mathbf{m}^*)]^2}{N_d - M} \quad (10)$$

As is clear from Eq. 8, the uncertainty (or $\sigma_{p,i}^2$) in the predicted response is controlled by the variance and covariance of the estimated parameters through the matrix $\left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1}$ and the quality of fit s^2 computed from history matching period as well as the sensitivity of the predicted response to the estimated model parameters through the vector \mathbf{g}_i computed in the prediction period. In general, the behavior of $\sigma_{p,i}^2$ can be quite complicated depending on the magnitudes of σ_d^2 (or the quality of match, s^2) and $\mathbf{g}_i^T \left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1} \mathbf{g}_i$. However, theoretically, we would expect that as the complexity of the model increases, the magnitude of s^2 (or the variance σ_d^2 in the case where it is unknown and estimated from Eq. 10) decreases, while that of $\mathbf{g}_i^T \left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1} \mathbf{g}_i$ increases.

We usually expect that the behavior of the predicted response is more controlled by the magnitude of $\mathbf{g}_i^T \left(\mathbf{G}_{m^*}^T \mathbf{C}_D^{-1} \mathbf{G}_{m^*} \right)^{-1} \mathbf{g}_i$ than that of s^2 for the lumped models. In fact, as shown later, our results show that when an over parameterized model instead of the correct lumped model is used for history matching, the uncertainty in predicted performance is overestimated, while using a less parameterized model provides an underestimated variance in the predicted response. Approximate confidence limits based on Eq. 8 for the predicted responses can be also constructed (Dogru *et al.*, 1977; Sen and Srivastava, 1990). These limits can characterize the uncertainty in future predicted responses.

As mentioned above, Eq. 8 is based on linearization of the predicted response around the optimal parameter vector. Hence, Eq. 8 may not provide a reliable estimate of the uncertainty on the predicted response if the linearization is not valid.

For nonlinear problems, it has been shown that although it is approximate, the randomized maximum likelihood method (RML) does a good job for assessing the uncertainty in the predicted response (Kitanidis *et al.*, 1995; Oliver *et al.*, 1996; Liu and Oliver, 2003; Gao *et al.*, 2005). By these authors, the RML has been considered within the Bayesian estimation framework for under-determined problems (i.e., the unknown model parameters far exceeds the number of observed data, $M > N_d$) with a prior model for the parameters. Here, we apply the RML method for the lumped-parameter modeling without a prior model, which usually constitutes an over-determined problem ($N_d \gg M$).

In this case, the RML would provide sampling of the likelihood probability density function for the model conditional to observed data, given by (Bard, 1974)

$$p(\mathbf{m} | \mathbf{y}_{obs}) = c \exp \left\{ -\frac{1}{2} O(\mathbf{m}) \right\} \quad (11)$$

where $O(\mathbf{m})$ is given by Eq. 1 and c is a normalizing constant.

In the RML sampling procedure of Eq. 11, a conditional realization of the model parameters to observed data can be generated as follows: **(i)** provide an initial guess of the model parameter vector \mathbf{m} , **(ii)** add noise to the observed data (i.e., a realization of observed data) by

$\mathbf{y}_{uc} = \mathbf{y}_{obs} + \mathbf{C}_D^{1/2} \mathbf{z}_u$, where \mathbf{z}_u is an N_d -dimensional vector of independent standard random normal deviates. In the applications considered in this paper, $\mathbf{C}_D^{1/2}$ is a diagonal matrix with entries equal to the square roots of the corresponding diagonal entries of \mathbf{C}_D ; **(iii)** generate a conditional realization of the model parameter vector \mathbf{m}_r^* by minimizing Eq. 1 with \mathbf{y}_{obs} replaced by \mathbf{y}_{uc} ; **(iv)** check to see if the estimated model parameter vector \mathbf{m}_r^* gives an acceptable match of the data. This is done based on the assumption that if \mathbf{m}_r^* is a legitimate realization of Eq. 11, then it should satisfy:

$$1 - 5 \sqrt{\frac{2}{N_d - M}} \leq O_N(\mathbf{m}_r^*) \leq 1 + 5 \sqrt{\frac{2}{N_d - M}} \quad (12)$$

where $O_N(\mathbf{m}_r^*) = O(\mathbf{m}_r^*) / 2(N_d - M)$ is the normalized objective function. Eq. 12 is obtained from the fact that $O(\mathbf{m}_r^*)$ is a chi-squared (χ^2) distribution with mean $N_d - M$ and variance $2(N_d - M)$ (Barlow, 1989; Gao et al. 2005). In Eq. 12, it is assumed that the realization should be within five standard deviations. If the realization \mathbf{m}_r^* does not satisfy Eq. 12, then it may mean that the nonlinear

minimization of Eq. 1 has resulted in a local minimum or that the noise level in observed data has been underestimated/overestimated or the model is not appropriate (Gao et al., 2005). To generate n conditional realizations, we repeat the procedure described by items **(i)** through **(iv)** n times. After n acceptable realizations of the model parameter vector, \mathbf{m}_r^* , for $r=1,2,\dots,n$, are generated, we can predict n realizations of the future response using these n \mathbf{m}_r^* realizations in the lumped-parameter model considered for a given future production scenario. Then, we can characterize the uncertainty in the predicted response by constructing the histogram and/or cumulative frequency based on these n realizations of the predicted responses at any given prediction time t_i such that $t_i > t_N$.

5. EXAMPLE APPLICATION

Here, we consider one synthetic example application to demonstrate the application of the methodology proposed in this work.

Suppose that our true model for the geothermal system is a two-tank open lumped-parameter model as shown in Fig. 1(a). The true pressure change and net production history data for a twenty-year period are shown in Fig. 2. (Note that pressure change is initial pressure minus the reservoir pressure, and thus as reservoir pressure decreases with production, pressure change increases.) The true input model parameters are shown in the second column of Table 1.

To simulate a “real case,” we corrupted the true pressure change data using normal random variables with mean zero and variance $\sigma_d^2 = 0.49$ bar². Note that we assume that the data error covariance matrix \mathbf{C}_D is a diagonal matrix with diagonal entries equal to $\sigma_d^2 = 0.49$. The corrupted pressure change data represent our observed data (\mathbf{y}_{obs}) and contain 193 data points. The observed pressure data are shown in Fig. 2 by solid circular data points, whereas the true pressure change data are shown by a solid curve. The net production rate history is shown as the red dashed curve in Fig. 2 and is assumed to contain no errors.

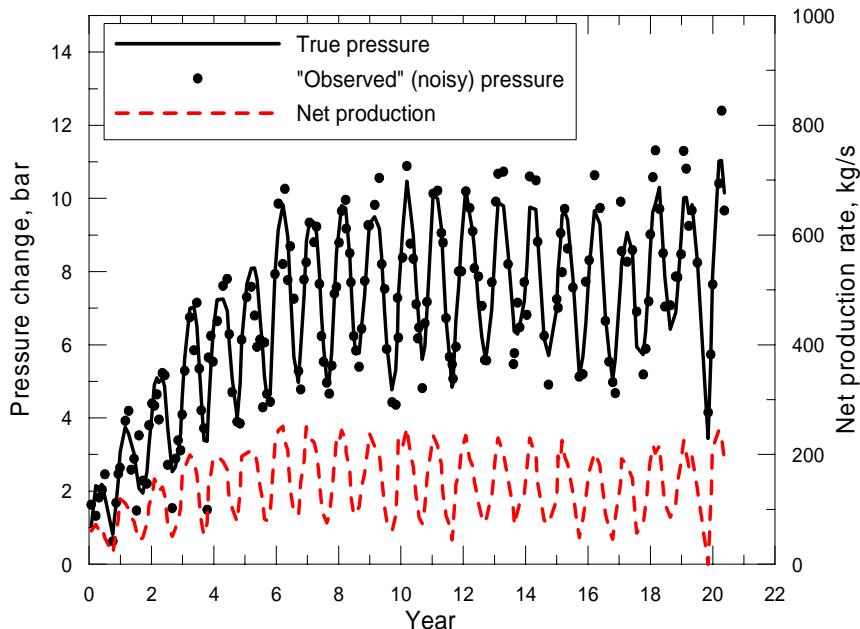


Figure 2: True and observed (noisy) pressure change data and net production rate history.

Table 1: Estimated model parameters by history matching of observed data with three different lumped parameter models.

Model Parameters	True parameters	Estimated parameters		
		2-tank closed	2-tank open	3-tank closed
κ_r (kg/bar)	8.9×10^7	9.5×10^7 ($\pm 1.2 \times 10^7$)	8.9×10^7 ($\pm 1.3 \times 10^7$)	8.9×10^7 ($\pm 1.3 \times 10^7$)
α_r (kg/bar-s)	30	27.9 (± 1.0)	30.6 (± 1.7)	30.6 (± 2.1)
κ_{o1} (kg/bar)	1.1×10^{10}	2.4×10^{10} ($\pm 2.1 \times 10^9$)	1.2×10^{10} ($\pm 3.1 \times 10^9$)	1.2×10^{10} ($\pm 6.5 \times 10^9$)
α_{o1} (kg/bar-s)	37	-	31.0 (± 6.1)	31.0 (± 43)
κ_{o2} (kg/bar)	-	-	-	2.9×10^{13} ($\pm 1.3 \times 10^{17}$)
RMS (bar)	0.7	0.73	0.69	0.69

As in the real life, we assume that we did not know the true lumped-parameter model, and that we can consider the two-tank closed, two-tank open, and three-tank closed models as three candidates of the lumped-parameter models to be used for history matching. The two-tank closed model is similar to the two-tank open model in Fig. 1(a), but without recharge source to the outer tank. This model contains three unknown parameters (κ_r , α_r , κ_r) to be estimated by history-matching. The two-tank open model contains four unknown parameters (α_{o1} , κ_{o1} , α_r , κ_r). The three-tank closed model is similar to the three-tank open model, but without recharge to the outer tank 2 and contains five unknown parameters (κ_{o2} , α_{o1} , κ_{o1} , α_r , κ_r). So, a two-tank closed model represents a less parameterized model, while the three-tank closed model represents an over parameterized model.

We performed history matching of the observed data (circular data points in Fig. 2) with each of these three different models. The estimated parameters and their associated 95% confidence intervals (given in parentheses) are summarized in Table 1.

An inspection of the results of Table 1 indicates that the two-tank open and three-tank closed models yield the same RMS values of 0.69 bar, which are very close to the actual noise (0.7 bar) in the observed data. On the other hand, the less parameterized two-tank closed model yields slightly higher RMS value (0.73 bar) than the other two lumped-parameter models. The common model parameters in all the three models are κ_r , α_r , and κ_{o1} . It is clear that the estimated values of these parameters by history matching are close to the true values for the two-tank open and three-tank closed models, but not very close to the true values for the two-tank closed model. Particularly, the estimated values of κ_r and κ_{o1} for the two-tank closed model are far from the corresponding true values.

In addition, it can be seen that the less parameterized two-tank closed model gives the narrowest (and “acceptable”) confidence intervals for all of the estimated parameters,

while the over parameterized three-tank closed model gives the widest confidence intervals for all parameters. The three-tank closed model gives very wide (unacceptable) confidence intervals for the parameters κ_{o2} and α_{o1} . The reason for obtaining quite wide confidence intervals for κ_{o2} and α_{o1} can be explained as follows: The observed data do not have any sensitivity at all to the parameter κ_{o2} in the three-tank closed model and thus we have very large uncertainty in its estimated value, as expected. An inspection of correlation coefficients (not given here) between the parameters indicates that there is a strong negative correlation between κ_{o2} and α_{o1} in the three-tank closed model. Because of this correlation, the uncertainty for the parameter κ_{o2} is reflected to the parameter α_{o1} , and hence we have large uncertainty for the estimated value of α_{o1} .

Based on our view that the best fitting model of all the possible models for a given low-temperature geothermal system is the one having both the smallest possible RMS for the fit, and acceptable confidence intervals for all parameters, one will be able to identify the most appropriate model as the two-tank open model, which is the true model used in this example application. If the confidence intervals were not computed for the estimated parameters and we used only the RMS as a criterion to identify the most appropriate model, we would identify both the two-tank open and three-tank closed models as two possible candidates to be used for performance predictions. On the other hand, if only the confidence intervals were used as a criterion for model identification, we would identify the two-tank closed model as the most appropriate model because it gives the narrowest (and acceptable) confidence intervals for all the parameters. As we show next, identifying the true (or the most appropriate) model from the history-matching period is essential to be able to correctly characterize the uncertainty in performance predictions.

Next, we used the RML method to generate future predictions of pressure changes for characterizing the uncertainty in predictions. First, we generated 1000 realizations of observed data by $\mathbf{y}_{uc} = \mathbf{y}_{obs} + \mathbf{C}_D^{1/2} \mathbf{z}_u$ with 1000 different seeds of \mathbf{z}_u . (Our results not shown here indicate that a thousand realizations are sufficient to characterize the uncertainty in predictions.) Then, we history matched each of the 1000 generated realizations of \mathbf{y}_{uc} to estimate the model parameters for each lumped-parameter model. All the realizations of model parameters for each lumped model considered were legitimate because they satisfied Eq. 12. Consequently, we obtained 1000 acceptable realizations of the estimated model parameters for each lumped-model considered. Then, each realization of the model parameter vector \mathbf{m}_r^* is input into the

corresponding lumped-parameter model to predict the future pressure performance for an additional twenty-five year period with a constant net production rate of 187 kg/s. Thus, for each conditional realization of the model parameters for a given lumped-parameter model, we predicted the future pressure changes for a total of 45 years.

Shown in Figures 3, 4, and 5 are predicted 1000 realizations of future pressure changes for the two-tank closed, two-tank open, and the three-tank closed models, respectively. The blue circular points in Figs. 3-5 are used to represent a prediction generated with the true lumped-model (two-tank open) parameters. Note that the true prediction (blue curve) will not be known in reality. Here, it is shown for comparison purposes. The green triangular data points represent the predictions based on the estimated parameters by history matching the observed data set \mathbf{y}_{obs} .

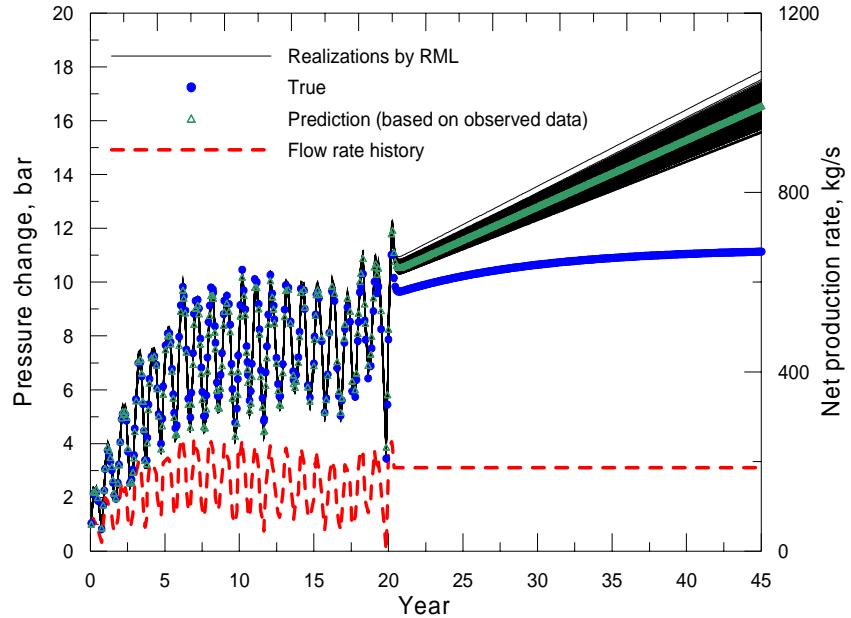


Figure 3: Realizations of predicted pressure change generated by RML, 2-tank closed model.

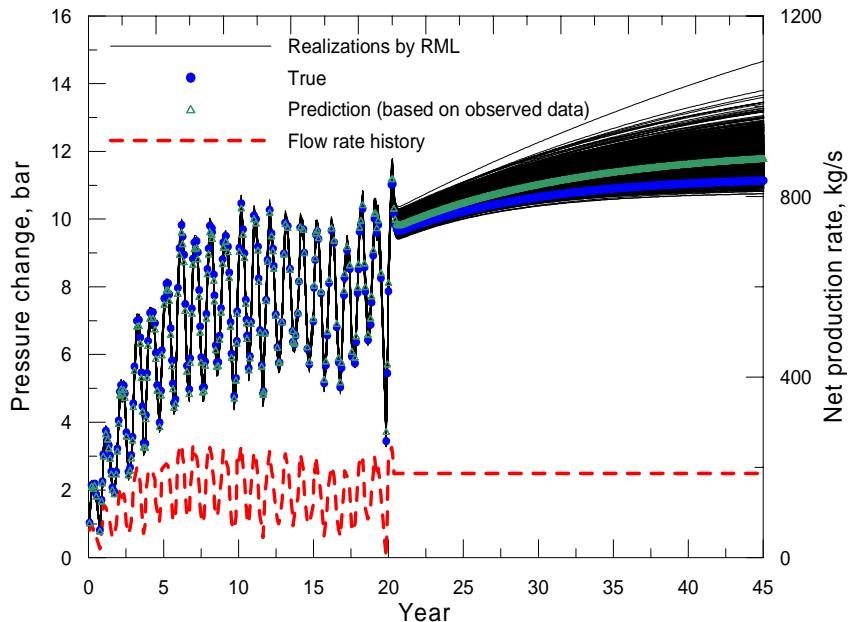


Figure 4: Realizations of predicted pressure change generated by RML, 2-tank open model.

The results of Fig. 3 indicate that the performance predicted with the two-tank closed model is biased; all realizations result in pressure change greater than the truth. Note that the band of the predictions (or the uncertainty in predictions) is quite narrow (or small). In fact, this is due to the fact that the uncertainty in the estimated model parameters by history-matching of 1000 realizations, as clearly reflected by the confidence intervals (see Table 1), are quite small, and thus the predictions made using these estimates have a narrow band. The main conclusion is that a less parameterized model, which is the two-tank closed model in this example, is unable to correctly characterize the uncertainty in predicted future performance, though it provides the least uncertainty in predictions; or in other words, it highly underestimates the uncertainty in predicted response.

The results of Fig. 4 for the two-tank open model, which is the correct model, indicate that the truth lies within the band of predictions (i.e., predictions are unbiased) and that the uncertainty in predictions can be correctly characterized by the RML method because the model chosen for predictions is correct. On the other hand, the three-tank closed model, an over parameterized model, gives the largest uncertainty in predictions (Fig. 5). The main reason is that some model parameters in this model do not show much sensitivity to the observed data and hence their estimated values by history matching have wide variations from one realization to another, as clearly identified by their wider confidence intervals (see last column of Table 1 for κ_{02} , α_{01} , κ_{01}) and this large uncertainty in these estimated parameters are reflected as large uncertainty in performance predictions. Interestingly, the band of the predictions by the three-tank closed model contains the truth, which indicates no bias in predictions. However, this does not mean that the uncertainty in performance predictions is characterized correctly, which cannot be the case in this example application.

Figure 6 presents a box-and-whisker plot of future pressure changes (predicted by RML) at the 45th year for all the lumped-parameter models considered. As expected, the bands of uncertainty for the two-tank open and three-tank

closed models include the truth, while that of the two-tank closed model fails to do so by giving biasedness in predictions. Assuming that the RML method provides the correct assessment of uncertainty for the future pressure changes for the two-tank open model, then it can be stated that the RML predictions based on the 3-tank closed model cannot provide the correct characterization of the uncertainty because they yield a larger band of uncertainty than those of the two-tank open model.

We should note that based on the results and methodology given in this study, it is unclear to us whether the methodology of Axelsson *et al.* (2005) who propose only two realizations of predictions; one with an open and the another with a closed lumped-parameter model, can provide the assessment of the inherent uncertainty in future pressure changes. In addition, it is unclear to us whether the future prediction changes can be expected to lie somewhere between the predictions of open and closed models, as they claim. To investigate this, we generated three predictions of future pressure changes based on the parameter estimates obtained by history-matching of the observed data set y_{obs} (shown as circular data points in Fig. 2) with the two-tank closed and open models and three-tank open model (see Table 1 for the estimated parameters). These future predictions, in comparison with the true prediction, are shown in Fig. 7. As is clear from Fig. 7, the true prediction is not contained within these predictions nor can the uncertainty in predictions be properly characterized because measurement errors in observed data are not accounted for in their methodology.

The results of this example clearly indicates that one can only characterize the uncertainty in performance predictions correctly by the most appropriate lumped-model representative of the geothermal system in question. Identifying the most appropriate model to be used for generating appropriate realizations of the future performance by the RML method may be achieved if one inspects both the RMS and confidence intervals during the history-matching period by considering several candidate lumped-models.

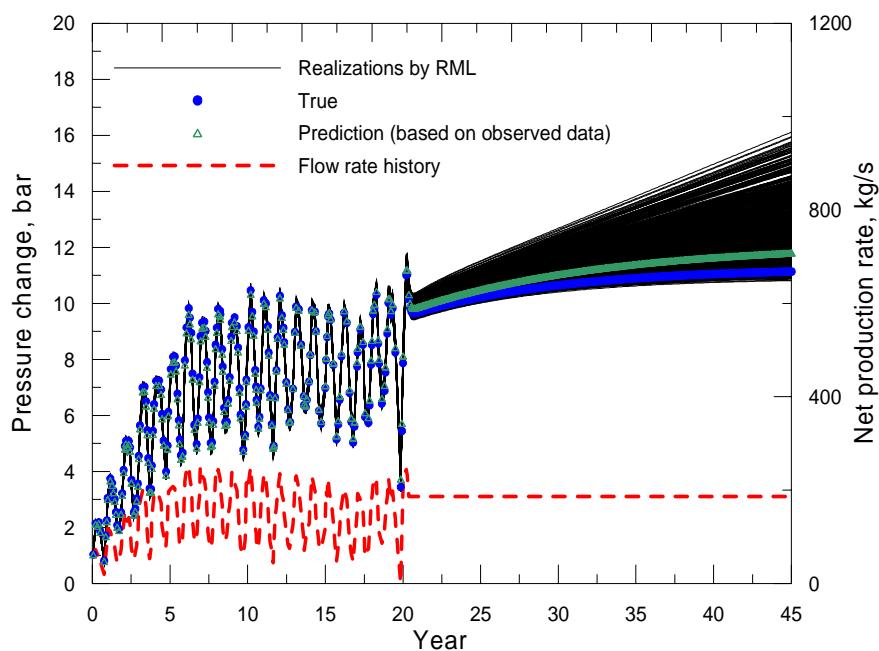


Figure 5: Realizations of predicted pressure change generated by RML, 3-tank closed model.

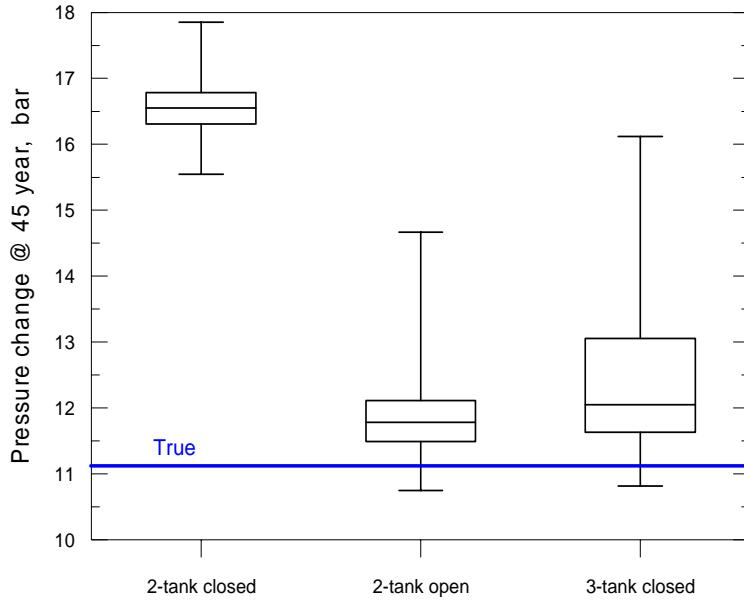


Figure 6: Box and whisker plot of predicted future pressure changes generated by RML for the three different lumped-parameter models, at the 45th year.

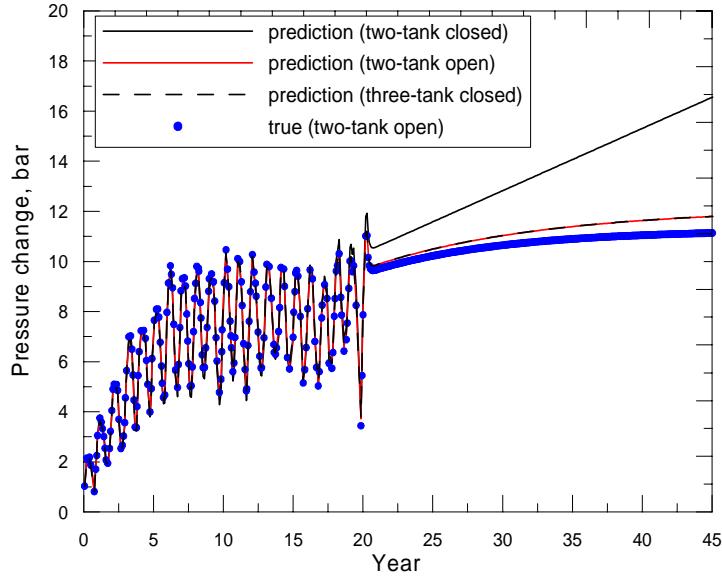


Figure 7: A comparison of predictions of future pressure changes based on estimated model parameters obtained from history matching of the observed data with the two-tank closed and open models and three-tank closed model.

6. FIELD APPLICATION

The application of the RML method on synthetic examples of lumped parameter models for geothermal reservoirs have been successfully performed in the previous section. Here, we extend the application of RML method to a real field. The field at study is the Balcova-Narlidere Geothermal Field. This field is known as the oldest geothermal system in Turkey and is situated 10km west of Izmir. The geothermal water with a temperature ranging from 80°C to 140°C is produced from the wells with depths ranging from 48.5m to 1100m.

The application of RML will be performed on the data collected from one of the wells in the field. The data consists of net rate (production rate – injection rate) information starting from 01/01/2000 and corresponding water level data starting from 17/06/2001. All data have been collected until 10/11/2005. Here it is important to note that the net rate history is obtained from the entire field

whereas the water level data is collected only from a single well. Fig. 8 illustrates the collected data to be used in the RML application.

The RML method has been performed on the above data for 5 different lumped parameter models (1 tank open model, 2 tank open/closed models and 3 tank open/closed models). For each model 100 realizations of matched responses are generated and the results are compared.

During the application the water levels have been converted to pressure changes. Hence all computations are performed on the pressure changes. The realizations of the observed data is obtained by adding random noise from a $N(0,0.8125)$ distribution. The pressure change data is treated stationary and hence the variance of the noise is obtained as the deviation from the overall mean.

Once the history matching is complete we make future predictions with the optimal model parameters. The rate

history for the future 10 years is obtained by taking the rates of the last year of the matching period and increasing them by %20 each year.

Figs 9-13 illustrate the history matching period and the future 10 year predictions for the 1 tank open model, 2 tank

closed/open models and 3 tank closed/open models respectively. The blue circled points in the figures represent the observed water level data, the solid red lines represent the results of 100 simulations and finally the solid black line represents the flow rate.

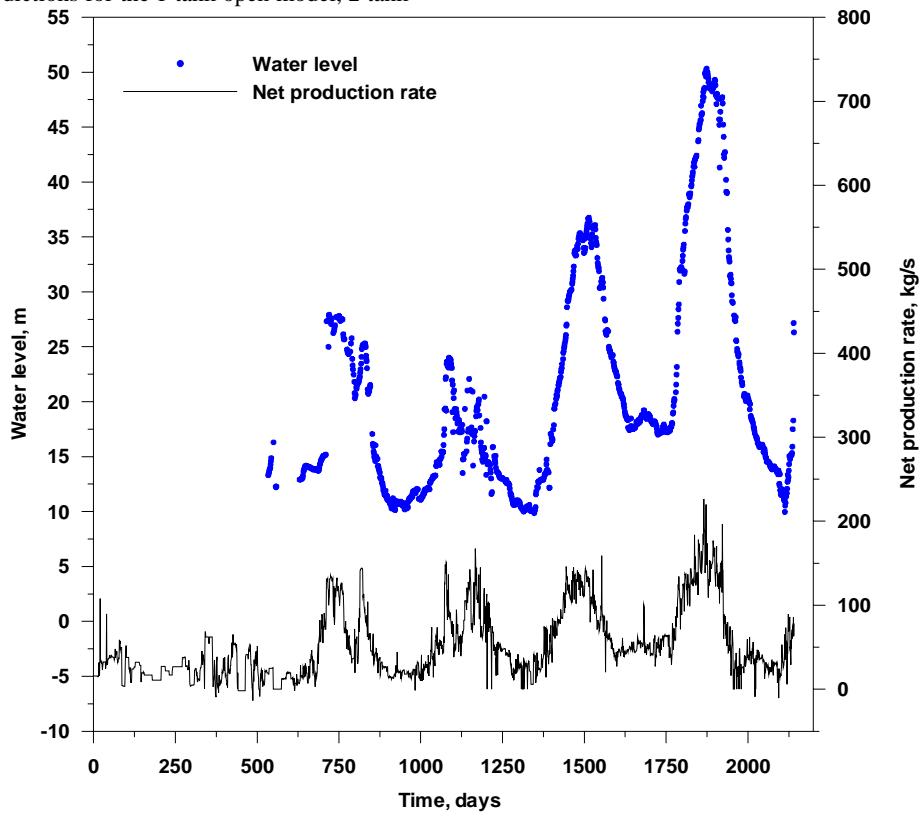


Figure 8: Observed rate and water level data.

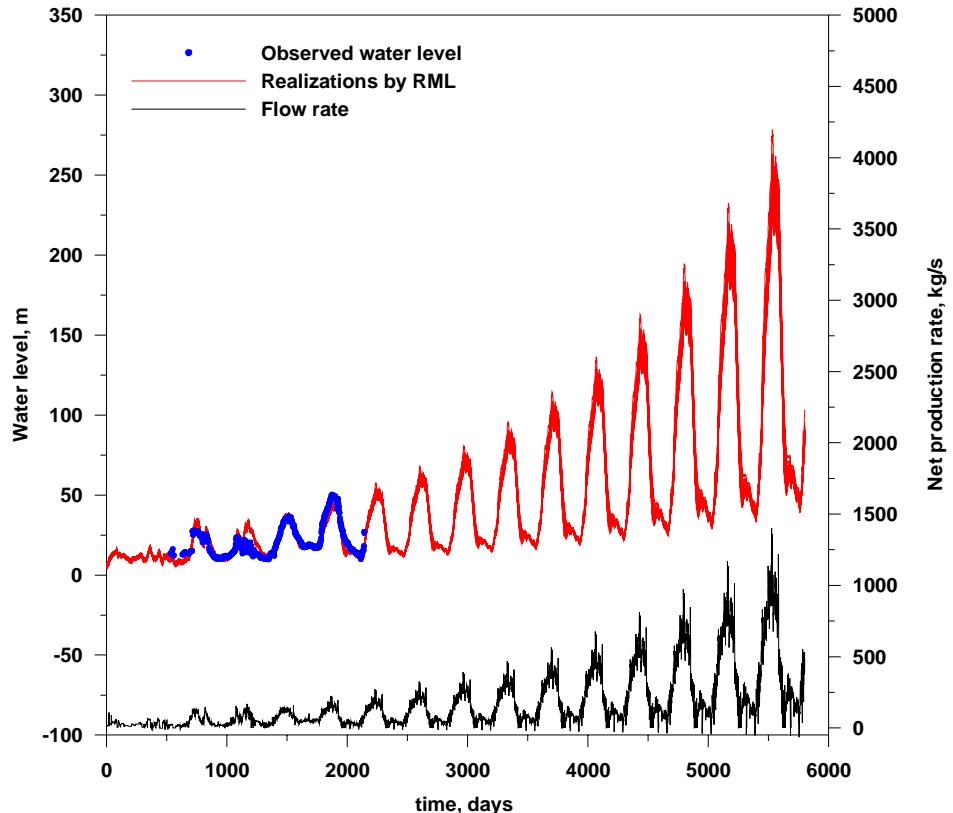


Figure 9: Realization of predicted pressure change generated by RML, 1-tank open model.

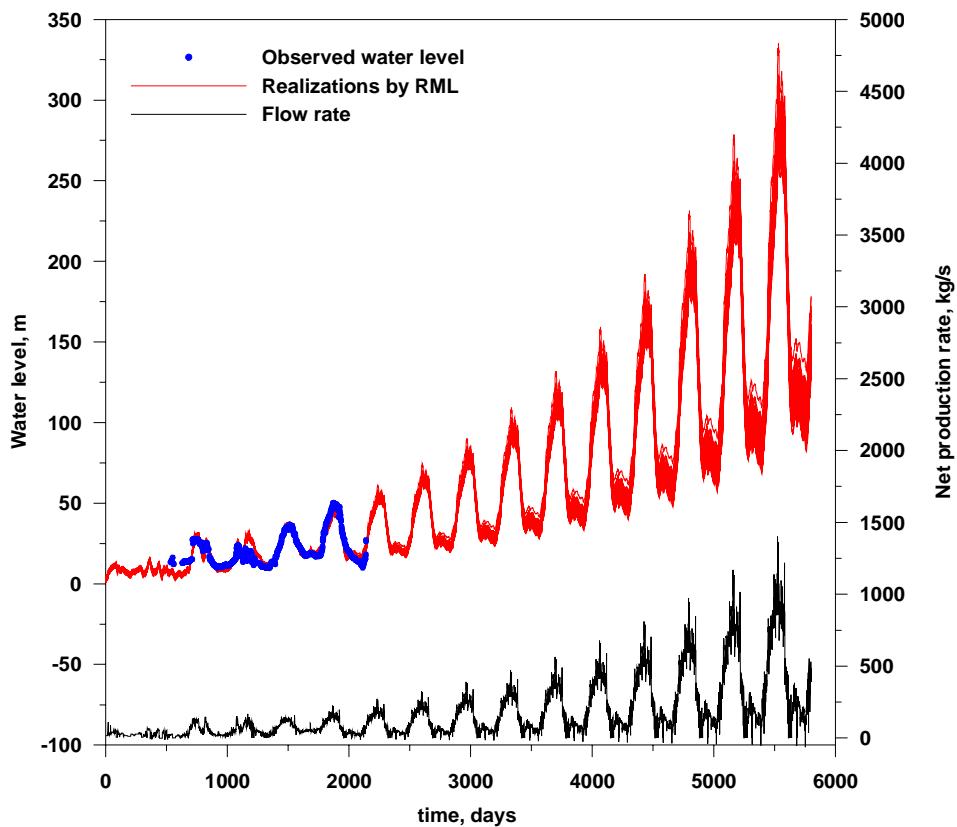


Figure 10: Realization of predicted pressure change generated by RML, 2-tank closed model.

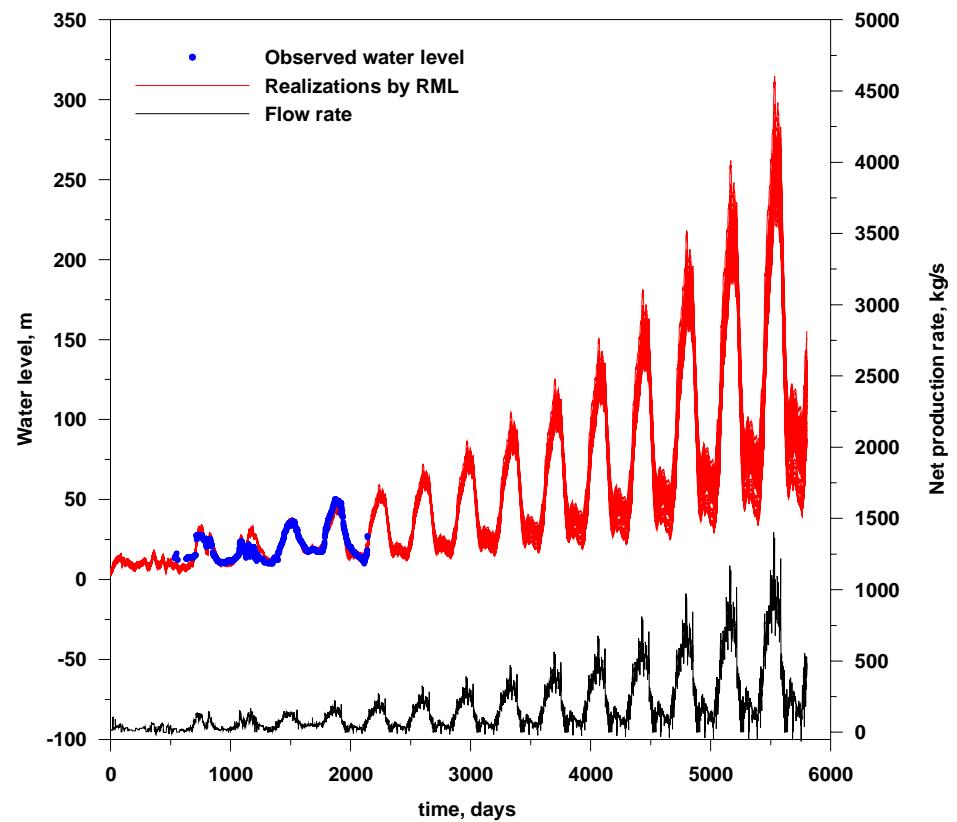


Figure 11: Realization of predicted pressure change generated by RML, 2-tank open model.

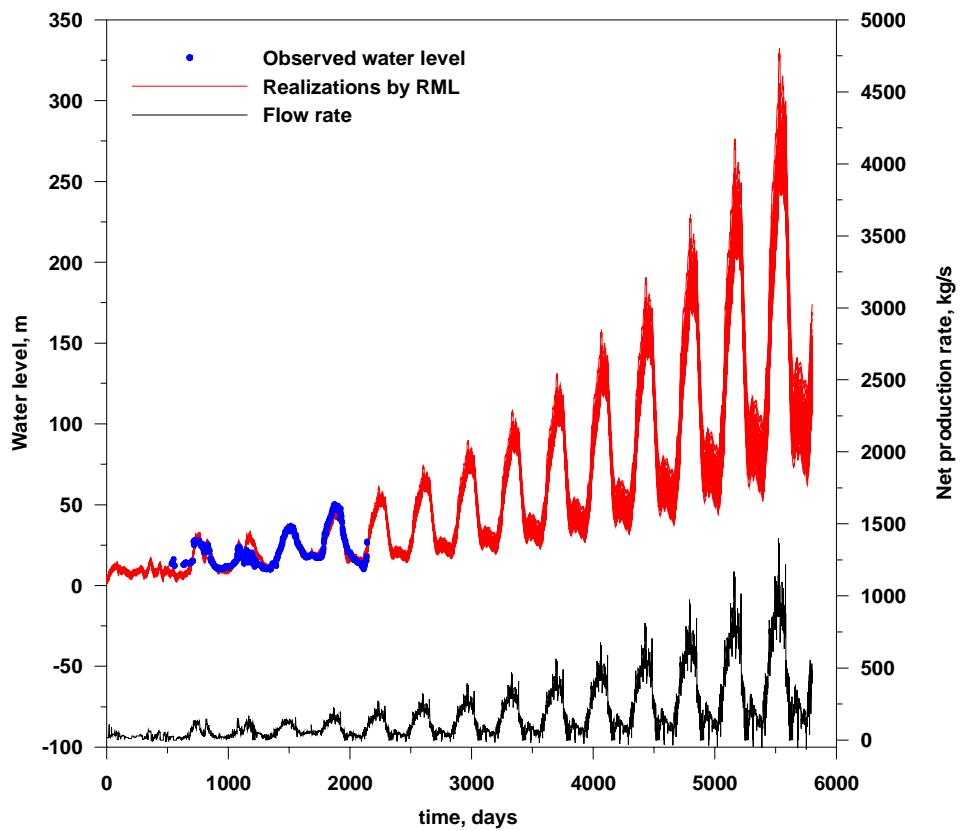


Figure 12: Realization of predicted pressure change generated by RML, 3-tank closed model.

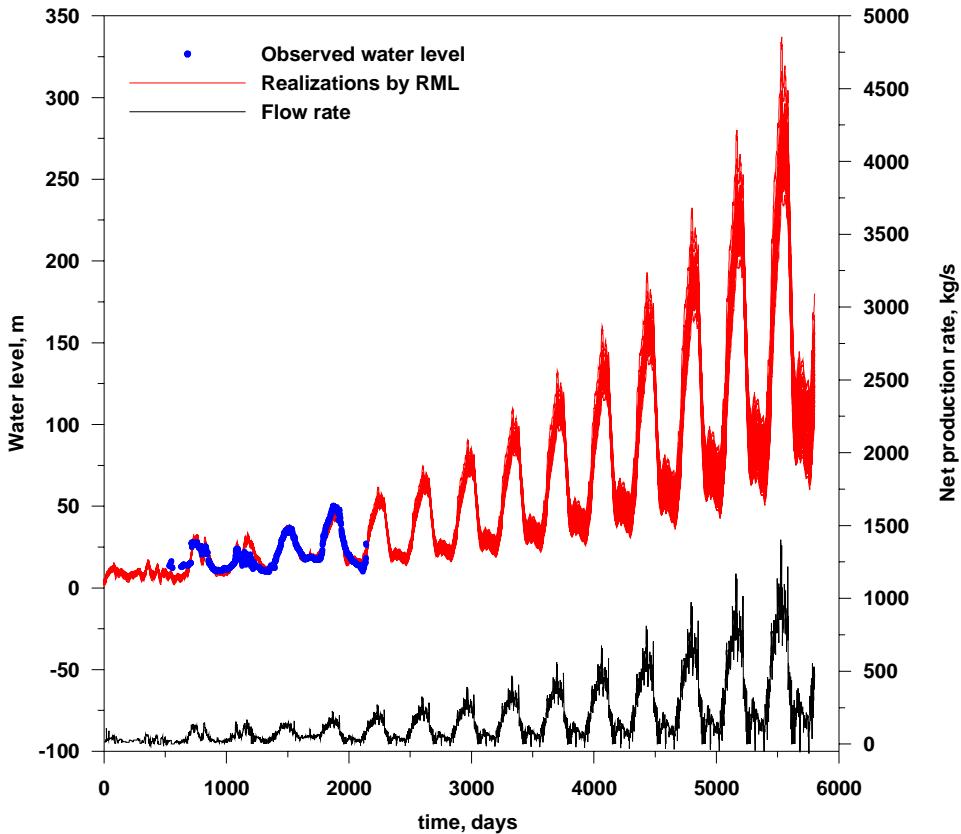


Figure 13: Realization of predicted pressure change generated by RML, 3-tank open model.

Based on the results provided by the Figs 9-13, we can conclude that the band of the predictions (or the uncertainty in predictions) is narrowest for the 1 tank-open model and widest for the 2 tank open model.

Table-2 summarizes the history matched parameters and various statistics (95% confidence intervals and RMS). All these parameters and their statistics have been obtained by averaging the results of 100 realizations. An inspection of the RMS values indicate that they are all almost the same and can be considered close to the actual noise that was added to the observed data for generating realizations of the data. The confidence intervals in this case will probably be the discriminating measure for the best model that represents the actual system. κ_{o1} and α_{o1} for the 2-tank open model, κ_{o1} , α_{o1} and κ_{o2} for the 3-tank closed model and α_{o1} , κ_{o2} and α_{o2} for the 3-tank open model are unacceptable due to their high confidence intervals (Here our definition is that an estimate of a parameter is acceptable if its confidence interval range is less than 95% of the estimated value itself). Hence they can immediately be ruled out as candidates for describing the Balcova-Narlidere geothermal field.

The reason for obtaining such wide confidence intervals for the above mentioned model parameters can be explained as follows: The observed data do not show any sensitivity to these parameters. Hence the uncertainty regarding these parameters are large.

Based on the above observations, since the RMS values are relatively close to each other, we compare the confidence intervals for the parameters of the 1-tank open model and the 2-tank closed model. The 1-tank open model in this case seems to best represent the real field due to the lower confidence intervals of the model parameters.

To better represent the response differences of the different models, Fig. 14 illustrates the box and whisker plot of the responses at 5750 days. The highest water level is reached with the 2-tank closed model. This is expected since, the system is closed. The largest band of uncertainty on the other hand is provided by the 2-tank open model. It is interesting to see that the model chosen to represent the real field has the narrowest band of uncertainty and provides the smallest water level at the same time.

7. CONCLUSIONS

The following conclusions are obtained from this study:

- i. A single realization of the predicted response to a given production scenario is not sufficient to make reservoir management decisions that account for an incomplete knowledge of the actual geothermal system.
- ii. One needs to generate a multiple of realizations of the predicted future pressure changes to a given production scenario for assessing the uncertainty inherent in performance predictions due to noise in observed data. The RML method can be used for this purpose.
- iii. One can correctly characterize the uncertainty in performance predictions if and only if the lumped-parameter model chosen is correct. The most appropriate lumped-model for given observed data should be identified by inspecting both the RMS and the confidence intervals for the estimated model parameters from history matching with several candidate lumped-parameter models.

Table 2: Estimated model parameters by history matching of observed data with five different lumped parameter models.

Model Parameters	Estimated parameters for lumped-models				
	1-tank open	2-tank closed	2-tank open	3-tank closed	3-tank closed
κ_r (kg/bar)	8.4×10^7 (%13.95)	7.4×10^7 (%16.36)	8.0×10^7 (%16.49)	7.7×10^7 (%18.65)	7.6×10^7 (%24.51)
α_r (kg/bar-s)	44.2 (%5.5)	47.2 (%5.8)	46.2 (%6.3)	46.2 (%28)	46.4 (%9.8)
κ_{o1} (kg/bar)	--	1.5×10^{10} (%25.08)	1.8×10^{12} (%1 $\times 10^4$)	2.1×10^{10} (%315)	2.2×10^{10} (%54.4)
α_{o1} (kg/bar-s)	--	--	10.6 (%1 $\times 10^7$)	2.9 (%1 $\times 10^7$)	0.823 (%1 $\times 10^5$)
κ_{o2} (kg/bar)	--	--	--	3.0×10^{13} (%3 $\times 10^8$)	8.5×10^{13} (%4 $\times 10^5$)
α_{o2} (kg/bar-s)	--	--	--	--	622.5 (%2 $\times 10^6$)
RMS (bar)	0.99	0.97	0.98	0.97	0.97

- iv. Using a less parameterized lumped-parameter model for predictions gives biased predictions and highly underestimates the uncertainty in future predictions, while using an over-parameterized lumped-parameter model gives unbiased predictions, but overestimates the uncertainty in future predictions.
- v. The RML method has successfully been implemented to real field data.
- vi. Based on the RML results, the authors believe that the Balcova-Narlidere geothermal system can best be represented by a 1-tank open model.
- vii. The uncertainty analysis show that the uncertainty band in the predictions is also lowest with the 1-tank open model.

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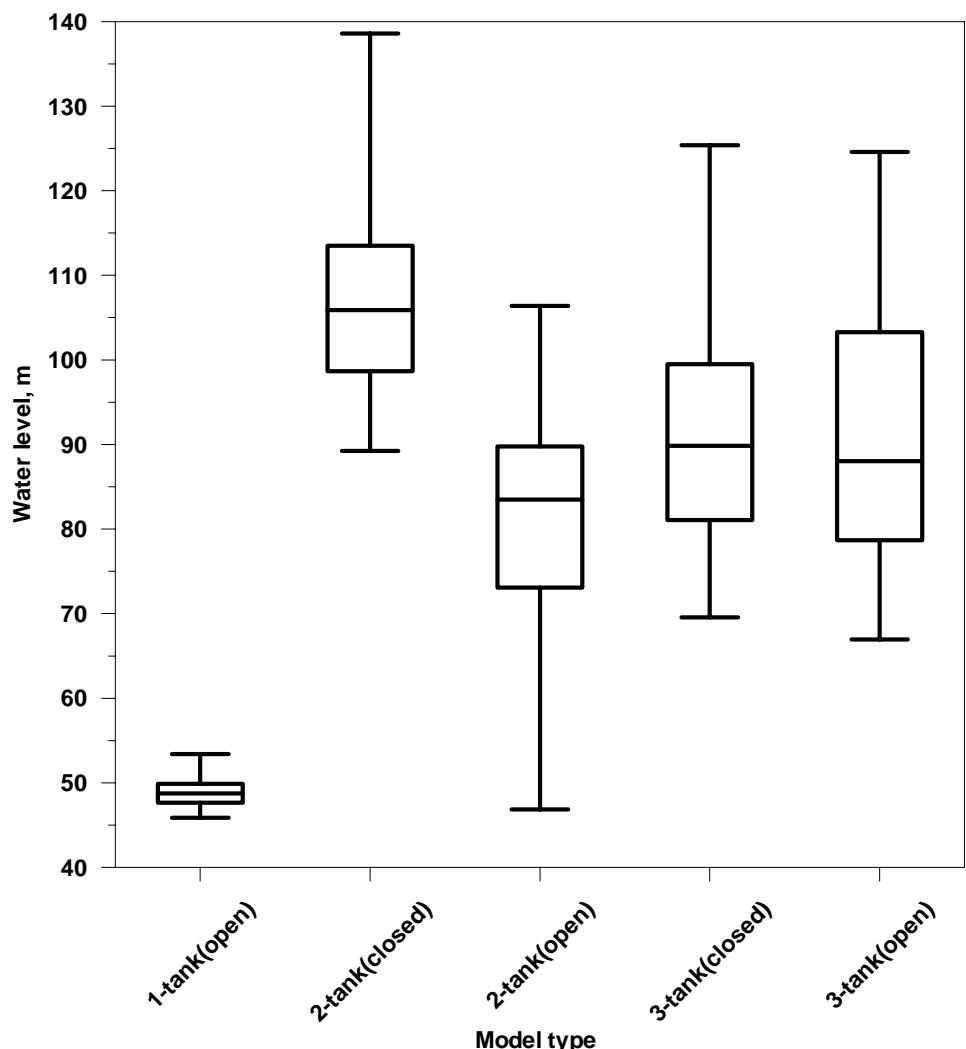


Figure 14: Box and whisker plot of water levels for five different models at 5750 days.

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