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Numerical Modeling of Some Geothermal Sources in Gradient Media

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ABSTRACT

Geothermal heat flows are important in a wide range of areas. During the last decade new modeling approaches have been developed. The results of numerical modeling of some geothermal fields caused by local volumetric distributions of sources in the Earth's crust are presented in this paper. Inverse solutions defining the regions occupied by thermal sources when the heat flow on the Earth's surface is known are also provided. This inverse problem is unstable, so special methods of regularization of the solution are offered.

1. INTRODUCTION

Accurate knowledge of the temperature distributions in the shallow crust is of great practical importance for understanding many geological and drilling phenomena.

A common problem in the interpretation of surface heat flow data is the determination of the temperature distributions at depth from observations. This requires knowledge of the variation in thermal conductivity as well as determination of the distribution of heat sources. If the thermal conductivity and the distributions of the heat sources are specified, the temperature distributions in a section of the crust in conductive equilibrium can then be determined. Because of the lack of knowledge of the conductivity and the distribution of the heat sources, determination of the thermal regime of the crust from the heat flow data is an ill-posed problem.

Classical formulations of the downward continuation are generally restricted to cases of source-free media with known thermal conductivities, thereby circumventing the difficulty of no uniqueness. A priori information is introduced mainly to alleviate the problem of instability. For example, Brott et al. (1981) described a continuation technique utilizing the method of equivalent point sources, and Bodvarsson (1971, 1973) presented methods based on Fourier transforms. Mareschal et al. (1985) proposed a variation of Bodvarsson's (1973) formulation. More recent works on the downward continuations of heat flow have attempted to address the problem of variations of sources and conductivity.

These recent approaches may be classified into two groups. One group consists of the trial-and-error and Monte Carlo type formulations, in which the forward problem is solved a large number of times with the model parameters sampled according to their a priori distributions. The boundedness of the parameter space ensures the stability of the solutions. Examples are the formulations of Royer and Danis (1988) and Cermak and Bodri (1986). The second group consists of those formulations that solve the inverse problem directly. Huestis (1979, 1981) described a method for finding the extreme bounds of subsurface temperature distributions from the surface by applying the formulation of Backus and Gilbert (1968, 1970) to downward continuation. The methods in which a priori information is incorporated in the form of soft bounds (e.g. probability distributions), and which yield optimal solutions together with the appropriate error estimates, are of more practical use and have become increasingly more popular in recent years. An example of this is the constrained linear least squares formulations of Stromeyer (1984). A simultaneous inversion for all model parameters was given by Beck and Shen (1989), who formulated the downward continuations as a constrained least norm problem.

2. FORMULATION AND SOLUTION OF THE PROBLEM

2.1 .The Direct Problem

Let the thermal field caused by the local volumetric distribution V of thermal sources be described by q(M), where M=(x, y, z), in a gradient media (i.e. media in which thermal conductivity K(M) changes arbitrarily). A diagram of this model is given in Figure 1. The temperature T(M) on the Earth's surface is considered to be T(z=0)=0, and on the surface where z = H, $\frac{\partial T}{\partial z}=0$. This leads to the problem presented in Equation (1):

$$\begin{cases} div\left(K\left(M\right)gradT\left(M\right)\right)=-q\left(M\right) \\ T\left(M\right)\big|_{z=0}=0 \\ \left.\frac{\partial T\left(M\right)}{\partial z}\right|_{z=H}=0. \end{cases} \tag{1}$$

$$0<z<H\ ,$$

Equation (1) is a problem concerning the modelling of an anomalous field caused by sources q(M). Using the Green's function $G(M,M_0)$ (Kostyanev, 1982; Dmitriev and Kostyanev, 1994), the thermal field can be represented in the following way:

$$T\left(M\right)=\iiint_V q\left(M_0\right)G\left(M,M_0\right)dV_{M_0}. \tag{2}$$

If it is assumed that there are constant distribution of sources (i.e. q(M) = q = constant) and conductivity (i.e. K(M) = K = constant), Equation (3) is obtained.

$$T\left(M\right)=\frac{q}{4\pi K}\iiint_V G_0\left(M,M_0\right)dV_{M_0}. \tag{3}$$

where

$$G_0(M,M_0) = \\ = \int_0^\infty J_0(\lambda\rho)\left(e^{-\lambda(z-z_0)} - e^{-\lambda(z+z_0)} + 2e^{-\lambda H}\frac{(Sh\lambda z.Sh\lambda z_0)}{Ch\lambda H}\right)d\lambda. \qquad (4)$$

The measurement on the Earth's surface is the vertical heat flow − i.e. (Q(x, y)=K $\frac{\partial T}{\partial z}$) at z=0. According to (3) and (4), we have:

$$Q(x,y) = \frac{q}{2\pi}\iiint_V L(\rho,z_0)dV_{M_0}. \qquad (5)$$

where

$$L(\rho,z_0) = \int_0^\infty J_0(\lambda\rho)\left(e^{-\lambda z_0} + \frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\right)\lambda d\lambda \quad (6)$$

$$= \frac{z_0}{\left(\rho^2+z^2\right)^{\frac{3}{2}}} + \int_0^\infty J_0(\lambda\rho)\frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\lambda d\lambda.$$

The expression of $L(\rho,z_0)$ can be simplified using Equation 7:

$$\frac{1}{Ch\lambda H} = \frac{2e^{-\lambda H}}{1+e^{-2\lambda H}} = 2e^{-\lambda H}\sum_{n=0}^{\infty}(-1)^n e^{-2\lambda nH}. \quad (7)$$

Substituting (7) in (6), we obtain:

$$L(\rho,z_0) = \frac{z_0}{\left(\rho^2+z_0^2\right)^{\frac{3}{2}}} + \\ + \sum_{n=0}^{\infty}(-1)^n\int_0^\infty J_0(\lambda\rho)\left(e^{\lambda z_0} - e^{-\lambda z_0}\right)e^{-2\lambda(n+1)H}\lambda d\lambda.$$

However, knowing that

$$\int_0^\infty J_0(\lambda p)e^{-\lambda h}\lambda d\lambda = \frac{h}{\left(\rho^2+h^2\right)^{\frac{3}{2}}}.$$

we obtain:

$$L(\rho,z_0) = \\ \sum_{n=0}^{\infty}(-1)^n\left(\frac{2nH+z_0}{\left(\rho^2+(2nH+z_0)^2\right)^{\frac{3}{2}}} + \frac{2(n+1)H-z_0}{\left(\rho^2+(2(n+1)H-z_0)^2\right)^{\frac{3}{2}}}\right). \qquad (8)$$

If $H >> z_0$ and the dimension of the volume V occupied by geothermal sources is also much smaller than H, Equation (8) can be rewritten as Equation (9):

$$L(\rho,z_0) \cong \frac{z_0}{\left((x-x_0)^2+(y-y_0)^2+{z_0}^2\right)^{\frac{3}{2}}}. \qquad (9)$$

Substituting (9) into (5), we obtain:

$$Q(x,y) = \frac{q}{2\pi}\iiint_V \frac{z_0dx_0dy_0dz_0}{\left(\left((x-x_0)^2+(y-y_0)^2+{z_0}^2\right)^{\frac{3}{2}}\right)} \quad (10)$$

Equation (10) enables easy modelling of the heat flow on the Earth's surface due to a given volume V, which is occupied by geothermal sources.

2.2 The Inverse Problem

When the heat flow on the Earth's surface is known, the inverse problem of defining the region V occupied by geothermal sources can be solved. This problem is unstable, and special methods of regularization of the solution are necessary (Tikhonov and Arsenin, 1979; Kostyanev, 1982, 1991). Usually, additional information about the shape of the region V is used.

Suppose that the shape of V can be described by the existence of a mean plane at a certain depth $\bar{H}$. This means that the plane $z = \bar{H}$ intersects the region V in such a way that when $z < \bar{H}$ the shape of V is described by the function $z = z_1(x,y)$ and when $z > \bar{H}$ - by the function $z = z_2(x,y)$, as illustrated in Figure 1. If the intersection of the region V with the mean plane is denoted by D, then Equation (10) can be written in the following way:

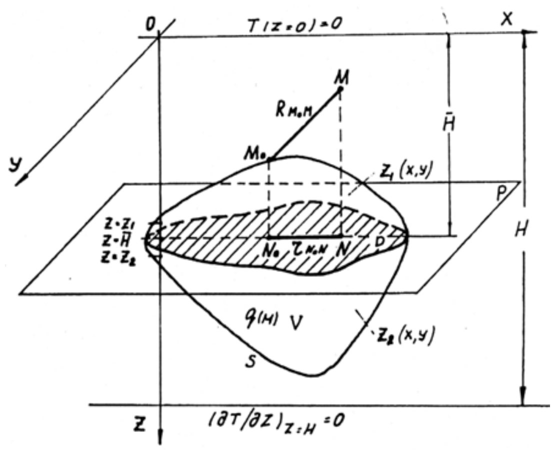

Figure 1

$$Q(x,y) = \frac{q}{2\pi}\iint_D dx_0dy_0\int_{z_1(x_0,y_0)}^{z_2(x_0,y_0)}\frac{\partial}{\partial z_0}\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}dz_0 \qquad (11)$$

Equation (11) can be used to determine the heat flow on the Earth's surface if $\bar{H}$, $z_1(x,y)$, and $z_2(x,y)$ are known.

Equation (11) can be rewritten in operational form as follows,

$$Q(x,y) = A\left[\bar{H}, z_1, z_2\right]. \quad (12)$$

where A is some nonlinear operator. The boundary of domain D is determined by $z_1(x,y) = \bar{H}$ or $z_2(x,y) = \bar{H}$.

If the heat flow measurements have errors, the approximate $\bar{Q}$ can be used, where $\left\|Q - \bar{Q}\right\| \le \delta$, and δ is the error of the measurement. An additional condition for the smoothness of the function must be given – for example, in the form of minimum of the special function:

$$\Omega\left(\bar{H}, z_1, z_2\right) = \beta\left(\bar{H} - H_0\right)^2 +$$

$$+\sum_{i=1}^{2}\iint_{D}\left(P_i\left(\frac{\partial z_i}{\partial x}\right)^2 + R_i\left(\frac{\partial z_i}{\partial y}\right)^2 + S_i {z_i}^2(x,y)\right) d_x d_y \quad (13)$$

where β is a weight coefficient and P_i (x, y) > 0, R_i (x, y) > 0, and S_i (x, y) > 0 are weight functions. H_0 is some hypothetical a priori value adjacent to the depth of the mean plane.

Thus, the inverse problem is reduced either to a problem of conditional extrema:

$$\begin{cases} \min_{\bar{H}, z_1, z_2} \Omega\left(\bar{H}, z_1, z_2\right), \\ \left\|A\left[\bar{H}, z_1, z_2\right] - \bar{Q}\right\| \le \delta, \end{cases} \quad (14)$$

or to a problem of unconditional extrema:

$$\min_{\bar{H}, z_1, z_2}\left[\left\|A\left(\bar{H}, z_1, z_2\right) - \bar{Q}\right\| + \alpha\Omega\left(\bar{H}, z_1, z_2\right)\right], \quad (15)$$

where α , the coefficient of regularization, is determined by the condition

$$\left\|A\left(\bar{H}_\alpha, z_1^\alpha, z_2^\alpha\right) - \bar{Q}\right\| = \delta, \quad (16)$$

and $\bar{H}_\alpha, z_1^\alpha, z_2^\alpha$ is the solution of Equation (15) at a given value of α .

Thus, both the shape and the depth of the region V are determined. It should be noted that the main supposition, or the existence of the mean plane, is not applicable to all types of V regions. If the region does not have a mean plane, it is proposed that an equivalent region processing mean plane be found. A similar problem arises if the region is not smooth or has a complicated configuration. An equivalent region with the “smooth” shape of the surface, from the class of regions processing a mean plane, has to be found. The described method has been applied to the investigation of some geothermal sources in Southwest Bulgaria.

3. CONCLUSIONS

The technique described consists of inverting geothermal field data using a regularization method. It is useful to obtain a numerical estimation of the temperature in the Earth’s crust directly from subsurface heat flow data. Numerical modeling of the geothermal field based on application of regularizing operators of any kind enables one to obtain the shape and depth of an anomalous area. More accurate measurements and more complete a priori information lead to more precise results.

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