Proceedings World Geothermal Congress 2010
Bali, Indonesia, 25-29 April 2010

Sustainability of a Geothermal Reservoir

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Keywords: Sustainability, geothermal field, analytical approach.

ABSTRACT

Most geothermal fields are exploited at a rate faster than the energy is replaced by the pre-production flow. Thus fields cannot be produced at a rate corresponding to the installed capacity of their heating facilities or power plants on a continuous basis, forever. In this sense they are not sustainable. However if after a time the field is shut-in the natural energy flow will slowly replenish the geothermal system and it will again be available for production. Therefore when operated on a periodic basis, with production followed by recovery, geothermal systems are renewable and sustainable. The difference between renewability and sustainability is a matter of time scale.

This paper addresses renewability and sustainability concept by considering a lumped parameter model with simple analytical solutions. An analytical approach is presented to analyze the parameters involved to estimate the time that a geothermal field takes to fully recover to its original state after shut-down at some production time.

A simplified lumped-parameter type of an approach to model the temperature behavior of a relatively low-temperature single liquid-phase geothermal reservoir is discussed here. New analytical equations and correlations are presented to investigate the pressure and temperature behavior of geothermal systems. Emphasis is given to understand the characteristics of temperature recovery following some production period. The time to reach a recovery depends on many factors as discussed in the paper. Primarily it depends on the production period. However, the natural recharge and reinjection conditions considerably affect the recovery.

1. INTRODUCTION

Sustainable use of all geothermal resources has become an issue of crucial importance. Sustainability can be defined as the ability to economically maintain the commercial capacity, over the amortized life of a geothermal heat and/or power project, by taking practical steps (such as, reinjection, make-up well drilling) to compensate for resource degradation (pressure drawdown and/or cooling) (Sanyal, 2005). Renewability is defined as the ability to maintain the project capacity indefinitely without encountering any resource degradation. Renewable capacity is therefore is smaller than the sustainable capacity. Renewable capacity of a field corresponds to the power capacity equivalent of the natural heat recharge, both conductive and convective, into the system. Sustainable capacity is supported by recovering of the stored heat in addition to natural heat recharge. Sanyal (2005) reviewed the results of some liquid-dominated geothermal fields supplying commercial projects and his study showed that the sustainable capacity of a field is about 5 to 45 times the renewable capacity, with ten times being the most likely. The renewability concerns the nature of a resource while the sustainability applies to how a resource is utilized.

Most geothermal fields are exploited at a rate faster than the energy is replaced by the pre-production flow. Thus fields cannot be produced at a rate corresponding to the installed capacity of their heating facilities or power plants on a continuous basis, forever. In this sense they are not sustainable. However if after a time the field is shut-in the natural energy flow will slowly replenish the geothermal system and it will again be available for production. Therefore when operated on a periodic basis, with production followed by recovery, geothermal systems are renewable and sustainable. The difference between renewability and sustainability is a matter of time scale.

Wright (1995), Pritchett (1998), Rybach et al. (1999), Rybach et al. (2000), Megel and Rybach (2000), Stefansson (2000), Axelsson et al. (2001), Rybach (2003), Axelsson et al. (2004 and 2005), Sanyal (2005) and Rybach and Mongillo (2006) studied the renewability and sustainability of geothermal resources. These studies indicated that geothermal resources can be considered renewable on time-scales of technological/societal systems and they will recover after abandonment on a larger time-scale than the production period.

This paper will address renewability and sustainability concept by considering a lumped parameter model with simple analytical solutions. An analytical approach is presented to analyze the parameters involved to estimate the time that a geothermal field takes to fully recover to its original state after shut-down at some production time.

2. MATERIAL AND ENERGY BALANCES

The basic considerations involved in geothermal reservoir engineering are: thermodynamics, physical and thermal properties of water, material and energy balances, fluid influx, and performance matching and predicting.

Two of the most important and basic approaches to geothermal reservoir engineering are a material balance and an energy balance. The schematic diagram of reservoir model is shown in Fig.1.

The reservoir system has a bulk volume *V* and contains fluid (water, steam) and rock. The fluid may also contain some non-condensable (NC) gases such as CO_2. It is assumed that there is a cumulative mass production of fluid with a corresponding cumulative heat production. The term representing the heat loss would normally contain terms for heat conduction to and from the reservoir, and for convective heat losses caused by natural discharges such as springs, fumaroles, etc. The recharge is considered separately. It is likely that it would be liquid.

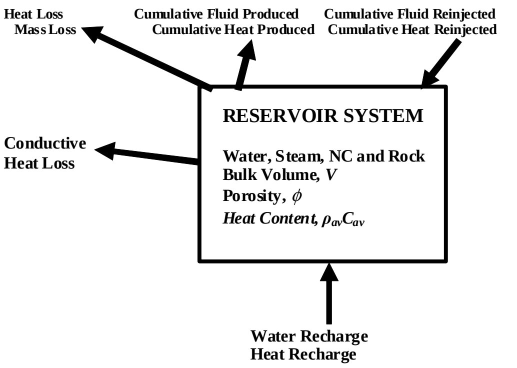


Fig. 1 Schematic diagram of reservoir model

A simple material-energy balance for a geothermal reservoir may be given in the following form:

$$\begin{bmatrix}Heat\\Produced\end{bmatrix}+\begin{bmatrix}Heat\\Loss\end{bmatrix}+\begin{bmatrix}Conductive\\Heat\ Out\end{bmatrix}-\begin{bmatrix}Heat\\Recharged\end{bmatrix}-\begin{bmatrix}Heat\\Reinjected\end{bmatrix}-\begin{bmatrix}Conductive\\Heat\ In\end{bmatrix}=\begin{bmatrix}Initial\\Heat\end{bmatrix}-\begin{bmatrix}Current\\Heat\end{bmatrix}$$

Although the balance given above seems like an energy balance, however, the mass balance is incorporated into the energy balance equation when the cumulative heat terms are defined in terms of cumulative mass terms. Thus the final balance obtained from such a formulation becomes a material-energy balance.

One important assumption in such a balance is complete thermodynamic equilibrium. Although this is a reasonable assumption within local pore space, it may not be reasonable for the entire reservoir. Some local temperature variations may occur in the reservoir. These kind of local variations are not considered in the formulation and the reservoir system is assumed to be a lumped system.

Another important observation regarding the balance is that the heat in and heat out terms due to conduction should almost always be negligible for the production case. In general, the heat conduction to the bottom of the reservoir should essentially equal the heat loss by conduction from the top of the reservoir. However, the effect of the heat in due to conduction could be considerable for the shut-in period after production resulted in significant reservoir temperature change.

Neglecting the conductive heat in and out terms as well as the heat loss term, the simple energy balance can be written as:

$$\begin{bmatrix}Heat\\Produced\end{bmatrix}-\begin{bmatrix}Heat\\Recharged\end{bmatrix}-\begin{bmatrix}Heat\\Reinjected\end{bmatrix}=\begin{bmatrix}Initial\\Heat\end{bmatrix}-\begin{bmatrix}Current\\Heat\end{bmatrix} \quad (1)$$

or in terms of cumulative heat (*Q'*):

$$Q'_p - Q'_r - Q'_{ri} = Q'_i - Q'_c \quad (2)$$

whereas the mass balance becomes:

$$\begin{bmatrix}Mass\\Produced\end{bmatrix}-\begin{bmatrix}Mass\\Recharged\end{bmatrix}-\begin{bmatrix}Mass\\Reinjected\end{bmatrix}=\begin{bmatrix}Initial\\Mass\end{bmatrix}-\begin{bmatrix}Current\\Mass\end{bmatrix} \quad (3)$$

or in terms of cumulative mass (*W*):

$$W_p - W_r - W_{ri} = W_i - W_c \quad (4)$$

Two further assumptions are made to simplify the problem. Expressing the reservoir performance for a model containing fluid existing as a compressed liquid is the first assumption. The second assumption is regarding the type of the lumped model. To further simplify we also assume the reservoir is represented by one tank lumped parameter model (Sarak et al., 2005a; Satman et al., 2005).

The heat and mass terms in Eqs. 2 and 4 are defined as follows:

$$Q'_p = w_p C_{pw} \int_0^t T dt$$

$$Q'_r = C_{rw} \int_0^t w_r T_r dt \quad \text{where} \quad w_r = \alpha(p_i - p)$$

$$Q'_{ri} = w_{ri} C_{pwri} T_{ri} t$$

$$Q'_i = V \rho_{av} C_{av} T_i \quad \text{where}$$
$$\rho_{av} C_{av} = \phi \rho_w C_{pw} T + (1-\phi) \rho_m C_{pm} T$$

$$Q'_c = V \rho_{av} C_{av} T$$

$$W_p = \int_0^t w_p dt\ , \quad W_r = \int_0^t w_r dt\ , \quad W_{ri} = \int_0^t w_{ri} dt$$

$$W_c = V \phi \rho_w$$

where w represents the flow rate, C_p the specific heat whereas *Q'* and W the cumulative heat and mass, respectively. Assuming that the recharge and reinjection temperatures are constant, taking the derivative of Eq. 2 with respect to time yields:

$$V \rho_{av} C_{av} \frac{dT}{dt} = -w_p C_{pw} T + C_{rw} T_r \alpha(p_i - p) + w_{ri} C_{pwri} T_{ri} \quad (5)$$

$$\Delta p = p_i - p = \frac{w_p - w_{ri}}{\alpha}(1 - e^{-Dt}) \text{ where } D = \frac{\alpha}{\kappa} \quad (6)$$

Here α (kg/bar-s) is the recharge constant whereas the storage coefficient κ (kg/bar) is defined as $\kappa = V \phi \rho_w c_t$

where V is the reservoir volume (m^3), ϕ is the porosity (fraction), ρ_w is the water density (kg/m^3), and c_t is the total compressibility (1/bar).

The details of the energy-material balance and its solutions are presented in the following sections. Recently Onur et al. (2008) presented the results of a new non-isothermal lumped-parameter model. Our modeling approach is similar to Onur et al.'s work. Their model is valid for the production period only. However our model covers both the production and recovery periods and thus has the capability for the renewability-sustainability studies.

3. FORMULATION OF THE PROBLEM

3.1 Closed Tank Model

We will concentrate on a case where the reservoir is represented by a tank which is a closed one. In other words, there is no recharge into the system. Two cases are discussed here.

Only Production Case: In the first case, only production is considered, recharge and reinjection are neglected. The schematic diagram of the model is shown in Fig. 2. The corresponding energy-material balance becomes:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_p C_{pw} T \qquad (7)$$

The solution of Eq. 7 is

$$T_i - T = \Delta T = T_i(1-e^{-at}) \text{ or } T = T_i e^{-at} \qquad (8)$$

where $$a = \frac{w_p C_{pw}}{V\rho_{av}C_{av}} \qquad (9)$$

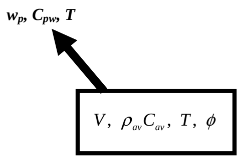


Fig. 2 Schematic of production case for a closed system

A particular extension of this case is the one in which the natural (pre-production) heat flow to the system, Q_n, is considered. It represents the net energy flow into the system (O'Sullivan and Mannington, 2005). Sanyal (2005) calls this term Renewable Capacity. It is defined as heat recharge rate into the reservoir (primarily convective with a small conductive component) and is equivalent to the total heat discharge from the surface over the thermal anomaly. Ideally, this term should be estimated at a pre-production state from a "heat budget" survey of the anomaly including conductive heat loss at the surface, convective heat discharge at surface manifestations, and subsurface convective heat loss regional aquifers. The schematic diagram of the model is shown in Fig. 3. For this case the balance yields:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_p C_{pw} T + Q_n \qquad (10)$$

The solution is given by:

$$T_i - T = \Delta T = \frac{aT_i - x}{a}(1-e^{-at}) \quad \text{or}$$

$$T = T_i - \frac{aT_i - x}{a}(1-e^{-at}) \qquad (11)$$

where $$x = \frac{Q_n}{V\rho_{av}C_{av}} \qquad (12)$$

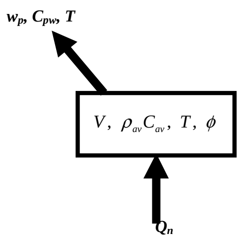


Fig. 3 Schematic of production case with natural heat flow for a closed system

Production and Reinjection Case: In the second case, production and reinjection are considered. The schematic diagram of the model is shown in Fig. 4. Then the corresponding energy-material balance becomes:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_p C_{pw} T + w_{ri} C_{pwri} T_{ri} \qquad (13)$$

The solution of Eq. 13 is

$$T_i - T = \Delta T = (T_i + \frac{g}{a})(1-e^{-at}) \quad \text{or}$$

$$T = T_i e^{-at} - \frac{g}{a}(1-e^{-at}) \qquad (14)$$

where $$g = \frac{w_{ri} C_{pwri} T_{ri}}{V\rho_{av}C_{av}} \text{ and } \frac{g}{a} = \frac{w_{ri} C_{pwri} T_{ri}}{w_p C_{pw}} \qquad (15)$$

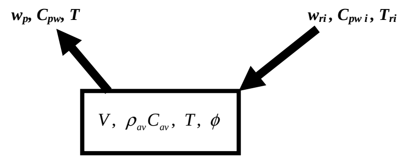


Fig. 4 Schematic of production and reinjection case for a closed system

If the natural heat flow to the system is considered then the respective schematic diagram of the model is shown in Fig. 5. For this case the balance yields:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_p C_{pw} T + w_{ri} C_{pwri} T_{ri} + Q_n \qquad (16)$$

The solution is given by:

$$T_i - T = \Delta T = T_i(1-e^{-at}) - \frac{g'}{a}(1-e^{-at})$$

or $$T = T_i e^{-at} - \frac{g'}{a}(1-e^{-at}) \qquad (17)$$

where $$g' = \frac{w_{ri} C_{pwri} T_{ri}}{V\rho_{av}C_{av}} + \frac{Q_n}{V\rho_{av}C_{av}} \qquad (18)$$

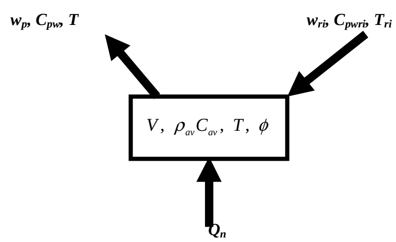


Fig. 5 Schematic of production and reinjection case with natural heat flow for a closed system

3.2 Open Tank Model

Two different cases are considered here: (1) production and recharge case, and (2) production, recharge and reinjection case.

Production and Recharge Case: In this case, the reservoir is modeled by an open system with production and natural recharge occurring simultaneously. The schematic diagram of the model is shown in Fig. 6.

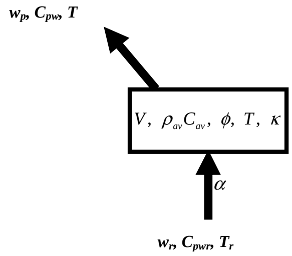


Fig. 6 Schematic of production and natural recharge case for an open system

The corresponding energy-material balance is:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_pC_{pw}T + C_{rw}T_r\alpha(p_i - p) \qquad (19)$$

Since

$$p_i - p = \frac{w_p}{\alpha}(1-e^{-Dt}) \quad \text{where } D = \frac{\alpha}{\kappa} \qquad (20)$$

then substituting Eq. 20 into Eq. 19 yields:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_pC_{pw}T + w_pC_{pwr}T_r(1-e^{-Dt}) \qquad (21)$$

The solution of Eq. 21 is given by

$$T = T_ie^{-at} + \frac{b}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) \qquad (22)$$

or

$$\Delta T = T_i(1-e^{-at}) - \frac{b}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) \qquad (23)$$

$$\text{where } b = \frac{w_pC_{pwr}T_r}{V\rho_{av}C_{av}} \text{ and } \frac{b}{a} = \frac{w_pC_{pwr}T_r}{w_pC_{pw}} \qquad (24)$$

If we assume $C_{pw} = C_{pwr}$, then $b/a = T_r$ and Eq. 22 becomes

$$T = T_ie^{-at} + T_r(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) \qquad (25)$$

or $\Delta T = T_i(1-e^{-at}) - T_r(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt})$ (26)

If $T_r = T_i$; $\Delta T = -T_i(\frac{a}{a-D})(e^{-at} - e^{-Dt})$ (27)

If this open tank model incorporating production and recharge also considers the natural heat flow (see Fig. 7), then the heat-material balance equation becomes:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_pC_{pw}T + w_pC_{pwr}T_r(1-e^{-Dt}) + Q_n \qquad (28)$$

The solution is

$$T = T_ie^{-at} + \frac{b}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) + \frac{x}{a}(1-e^{-at}) \qquad (29)$$

or

$$\Delta T = T_i(1-e^{-at}) - \frac{b}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) - \frac{x}{a}(1-e^{-at}) \qquad (30)$$

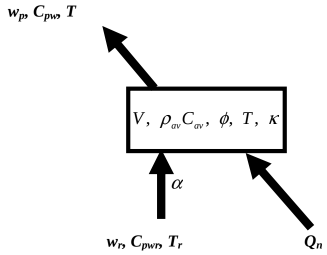


Fig. 7 Schematic of production and natural recharge case including the natural heat flow for an open system

Production, Reinjection and Recharge Case: In this case reservoir is modeled by an open system with production, natural recharge and reinjection occurring simultaneously. The schematic diagram of the model is shown in Fig. 8.

The energy-material balance is given by:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_pC_{pw}T + w_rC_{pwr}T_r + w_{ri}C_{pwri}T_{ri} \qquad (31)$$

where $w_r = w_{pn}(1-e^{-Dt})$ and $w_{pn} = w_p - w_{ri}$ (32)

w_{pn} is the net production rate which is defined to be the difference between the production rate and the reinjection rate. For the one tank open system, the natural recharge rate is proportional with the net production rate as given by Eq. 32. Substituting Eq. 32 into Eq. 31 yields:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_pC_{pw}T + w_{pn}C_{pwr}T_r(1-e^{-Dt}) + w_{ri}C_{pwri}T_{ri} \qquad (33)$$

The solution is:

$$T = T_ie^{-at} + \frac{b'}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) - \frac{g}{a}(e^{-at}-1) \qquad (34)$$

or

$$\Delta T = (T_i + \frac{g}{a})(1-e^{-at}) - \frac{b'}{a}(1+\frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) \qquad (35)$$

where

$$a = \frac{w_pC_{pw}}{V\rho_{av}C_{av}}, b' = \frac{w_{pn}C_{pwr}T_r}{V\rho_{av}C_{av}}, g = \frac{w_{ri}C_{pwri}T_{ri}}{V\rho_{av}C_{av}}.$$

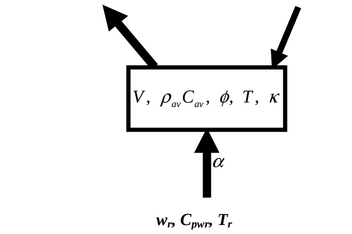


Fig. 8 Schematic of production, reinjection and natural recharge case for an open system

If this open tank model incorporating production, reinjection and recharge also considers the natural heat flow, then the energy-material balance equation becomes:

$$V\rho_{av}C_{av}\frac{dT}{dt} = -w_p C_{pw}T + w_{pn}C_{pwr}T_r(1-e^{-Dt}) + w_{ri}C_{pwri}T_{ri} + Q_n \quad (36)$$

The solution is

$$T = T_i e^{-at} + \frac{b'}{a}(1 + \frac{D}{a-D}e^{-at} - \frac{a}{a-D}e^{-Dt}) + (\frac{g}{a} + \frac{x}{a})(1-e^{-at}) \quad (37)$$

3.3 About the Cumulative Recharge

During Production Time: The natural recharge flow rate is given by:

$$w_r = \alpha(p_i - p) \quad (38)$$

For one tank model, the natural recharge flow rate is described in terms of the production rate as following:

$$w_r = w_p\left[1-\exp\left(-\frac{\alpha t}{\kappa}\right)\right] = w_p(1-e^{-Dt}) \quad (39)$$

The cumulative recharge by definition is written as:

$$W_r = \int_0^t w_r dt = \int_0^t w_p(1-e^{-Dt})dt = w_p\left[t - \frac{1}{D}(1-e^{-Dt})\right] \quad (40)$$

If reinjection is considered:

$$W_r = w_{pn}\left[t - \frac{1}{D}(1-e^{-Dt})\right] \text{ where } \quad w_{pn}=w_p-w_{ri} \quad (41)$$

During Shut-in Time: The natural recharge flow rate is given for production and shut-in times as follows:

$$w_p(t) = \begin{cases} w_p & for \quad 0 < t < t_p \\ 0 & for \quad t > t_p \end{cases} \quad (42)$$

The Duhamel's principle will be employed to calculate the cumulative natural recharge during shut-in time after a production period of t_p. The Duhamel's principle is defined as:

$$W_r(t) = w_p \int_0^{t_p} W'_{su}(t-\tau)dt = w_p \;{}_0^{t_p}\left|W_{su}(t-\tau)\right|$$

$$= -w_p\left[W_{su}(t-t_p) - W_{su}(t)\right] = w_p\left[W_{su}(t) - W_{su}(t-t_p)\right] \quad (43)$$

where t = *total time* = $t_p + \Delta t$ or $\Delta t = t - t_p$.

Hence,

$$W_r(t) = w_p\left[W_{su}(t) - W_{su}(\Delta t)\right] \quad (44)$$

Eq. 44 gives the cumulative recharge to the system until time t. If we are interested in calculating the cumulative recharge to the system throughout the shut-in period, then the following equation is employed:

$$\Delta W_r(\Delta t) = W_r(t) - W_r(t_p) \quad (45)$$

After using Eq. 40 to find $W_r(t)$ and $W_r(t_p)$ and substituting the resultant expressions in Eq. 44 gives:

$$\Delta W_r(\Delta t) = w_p\left[t - \frac{1}{D}(1-e^{-Dt}) - \Delta t + \frac{1}{D}(1-e^{-D\Delta t}) - t_p + \frac{1}{D}(1-e^{-Dt_p}\right] \quad (46)$$

Since $\Delta t + t_p = t$, then

$$\Delta W_r(\Delta t) = \frac{w_p}{D}\left[1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p}\right] \quad (47)$$

Equation 47 can be used to determine the amount of cumulative recharge to the system during shut-in time of Δt.

4. RETURNING to the ORIGINAL STATE WHEN the FIELD is SHUT-IN

4.1 Pressure Recovery During Shut-in

Equations 6 and 20 give the pressure drop during production period. Pressure recovery during shut-in period is given by:

$$p_i - p(\Delta t) = \Delta p = \frac{w_p}{\alpha}e^{-D\Delta t}(1-e^{-Dt_p}), \text{ for } \quad t(=t_p+\Delta t) > t_p \;.$$

w_{pn} replaces w_p in case of reinjection applications.

4.2 Heating of the Reservoir During Shut-in Due to Recharge Only

If the effect of the recharge induced by the pressure decline during the production period is negligible, then the natural heat flow, Q_n, is the only mechanism to heat the field after shut-in. If it is considered, then conduction and convection are both effective heat recharge mechanisms to heat the field after shut-in. Convection has two components; the first one is the convective contribution in the natural heat flow term (Q_n) and the other one is the convective flow as a result of recharge induced by the pressure decline.

Let us assume that the reservoir is produced until a production time of t_p and the reservoir temperature declined to T_p. During shut-in period, although production is ceased, the natural recharge mass and heat flow into the reservoir still continues since the reservoir pressure is below the recharge system pressure. Such a case is described by the following energy-material balance:

$$w_r C_{pwr} T_r = V\rho_{av}C_{av}\frac{d\Delta T}{d\Delta t}, \; \Delta T = T - T_p, \; \Delta t = t - t_p \quad (48)$$

Since $w_r = w_p e^{-D\Delta t}(1-e^{-Dt_p})$ (49)

then Eq. 48 can be written as:

$$w_p C_{pwr} T_r (1-e^{-Dt_p})e^{-D\Delta t} = V\rho_{av}C_{av}\frac{d\Delta T}{d\Delta t} \quad (50)$$

The solution of Eq. 50 is:

$$\Delta T = \Delta T_{conv} = \frac{C_{pwr}T_r}{V\rho_{av}C_{av}}\frac{w_p}{D}(1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p}) \quad (51)$$

Eq. 51 gives the temperature rise due to convective heat flow caused by natural recharge during the shut-in time.

If reinjection exists during the production period, then the following equation

$$\Delta T = \Delta T_{conv} = \frac{C_{pwr} T_r}{V \rho_{av} C_{av}} \frac{w_{pn}}{D} (1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p}) \quad (52)$$

describes the temperature rise.

Equations 51 and 52 are valid whenever the recharge temperature is greater than the reservoir temperature at the end of the production period, $T_r > T_p$.

However, if $T_r < T_p$ which is usually the case, then some natural heat entering into the system is used to heat the recharge water whose temperature is less than the reservoir temperature so that its temperature T_r reaches to T_p.

$$\Delta T = \Delta T_{conv} = \frac{C_{pwr} (T_p - T_r)}{V \rho_{av} C_{av}} \frac{w_p}{D} (1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p})$$

For $\Delta t_1 < \Delta t < \Delta t_2$:

$$\Delta T_{conv_{21}} = \frac{C_{pwr} (T_{p1} - T_r)}{V \rho_{av} C_{av}} \frac{w_p}{D} (e^{-Dt_2} - e^{-D\Delta t_2} - e^{-Dt_1} + e^{-D\Delta t_1})$$

For reinjection applications w_{pn} replaces w_p in the last two expressions.

4.3 Heating of the Reservoir During Shut-in Due to Natural Heat Flow Only

If there is only natural heat flow mechanism prevalent during shut-in time, the energy-material balance for this simple system is given by:

$$Q_n = V \rho_{av} C_{av} \frac{d\Delta T}{d\Delta t} \quad (53)$$

In that case, the natural heat flow to the system affects the reservoir temperature and heats the reservoir at a constant rate proportional to $x = Q_n/(V\rho_{av}C_{av})$ and the change of the reservoir temperature during shut-in time is described by the following expression:

$$\Delta T = \Delta T_{nat} = x\Delta t \text{ or } T_{nat} = T_p + x\Delta t \quad (54)$$

Eq. 54 gives the temperature rise due to natural heat flow during the shut-in time.

4.4 Heating of the Reservoir During Shut-in

The contributions of convective heat flow induced by pressure decline and natural heat flow are considered to describe the total change in reservoir temperature:

$$\Delta T = \Delta T_{tot} = \Delta T_{nat} + \Delta T_{conv} \quad (55)$$

If $T_r > T_p$, substituting Eqs. 51 and 54 into Eq. 55 gives:

$$\Delta T = \Delta T_{tot} = T(\Delta t) - T_p =$$
$$x\Delta t + \frac{C_{pwr} T_r}{V \rho_{av} C_{av}} \frac{w_p}{D} (1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p}) \quad (56)$$

w_p in Eq. 56 should be replaced by w_{pn} if reinjection is to be considered.

For the general case when $T_r < T_p$:

$$\Delta T = \Delta T_{tot} = T(\Delta t) - T_p =$$
$$x\Delta t - \frac{C_{pwr} (T_p - T_r)}{V \rho_{av} C_{av}} \frac{w_p}{D} (1 + e^{-Dt} - e^{-D\Delta t} - e^{-Dt_p})$$

or in a finite difference form

$$\Delta T_{tot_{21}} = x(\Delta t_2 - \Delta t_1) -$$
$$\frac{C_{pwr} (T_{p1} - T_r)}{V \rho_{av} C_{av}} \frac{w_p}{D} (e^{-Dt_2} - e^{-D\Delta t_2} - e^{-Dt_1} + e^{-D\Delta t_1})$$

4.5 About Q_n

O'Sullivan and Mannington (2005) presented a simple lumped parameter model to describe the renewability. In particular they investigated how long will it take for a geothermal reservoir to fully recover to its original state and what changes will occur during the recovery process.

They first discussed the natural or pre-production state of a geothermal system. In the natural state, the surface outflow rate of heat Q_{surf} is equal to the deep inflow rate Q_n, i.e.:

$$Q_{surf} = Q_n \quad (57)$$

The schematic diagram of the model is shown in Fig. 9.

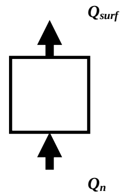


Fig. 9 Schematic of the natural state of a geothermal system

During production, energy is extracted at a rate Q_p by the production wells. The natural recharge Q_r occurs by the production induced pressure decline. Thus the rate Q_{extr} at which heat is being extracted from the system is given by:

$$Q_{extr} = Q_p + Q'_{surf} - Q_n - Q_r \quad (58)$$

The energy flow from the surface Q'_{surf} is not the same as in the natural state. In general, the surface heat flows in the compressed liquid reservoirs decrease as the production proceeds. They ignored the surface energy flow and the induced recharge (or assumed canceling each other) and thus Eq. 58 becomes:

$$Q_{extr} \approx Q_p - Q_n \approx (PR - 1)Q_n \quad (59)$$

where

$$\text{Production Ratio} = PR = \frac{\text{Produced Energy Flow}}{\text{Natural Energy Flow}} = \frac{Q_p}{Q_n} \quad (60)$$

The schematic diagram of the model is shown in Fig. 10.

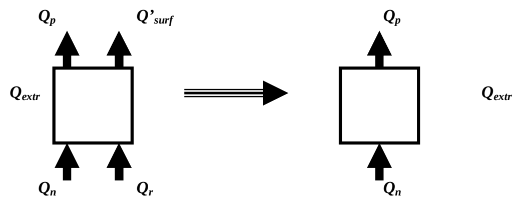


Fig. 10 Schematic of the production state of a geothermal system

They studied the Wairakei-Tauhara geothermal system to analyze the extraction behavior. Using the energy flow rate of 400 MW_{th} for the natural (or pre-production) state and the energy flow rate of 1900 MW_{th} for the production state, Eq. 60 gives PR of 4.75 and Eq. 59 indicates that the production regime at Wairakei is extracting energy from the system at a rate of approximately 3.75 times the natural energy supply. They commented saying that as hot water and steam have been produced cold water has flowed laterally and vertically downwards to replace it. This has resulted in the cooling off of some of the production wells. Thus the field is exploited at a rate faster than the energy is replaced by the pre-production flow.

If after some time the field is shut-in, their lumped-parameter energy balance above the rate of energy recovery Q_{reco} is given by:

$$Q_{reco} = -Q''_{surf} + Q_n + Q_r \qquad (61)$$

They assumed that the surface flow Q''_{surf} and the induced recharge Q_{reco} are small and can be ignored (see Fig. 11) then:

$$Q_{reco} \approx Q_n \qquad (62)$$

By equating the total heat extracted to the total heat recovered it follows that:

$$Q_{reco} t_{reco} \approx Q_{extr} t_{extr} \qquad (63)$$

Then Eqs. 59, 62 and 63 together give

$$t_{reco} \approx (PR-1) t_{extr} \qquad (64)$$

Thus the ratio of the duration of the recovery to the duration of the production should be approximately one less than the ratio of produced energy flow to natural energy flow. They conclude that the recovery process will depend on the state of the reservoir, however Eq. 64 should give a reasonable estimate.

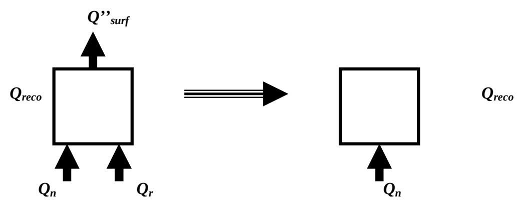


Fig. 11 Schematic of the recovery state of a geothermal system

However, their approach is a simple one, generally valid for steady-state conditions and it neglects effects of parameters such as recharge temperature, reinjection and production duration. As discussed in the following sections particularly the extraction (production) period is affected by these parameters and should be incorporated into modeling and prediction study.

4.6 Effects of Parameters on Pressure and Temperature Declines in Geothermal Systems

The energy production potential of geothermal systems is primarily determined by pressure decline due to production, but also by the available energy content. Pressure declines continuously with time, particularly in systems that are closed or with small recharge. Production potential is, therefore, often limited by lack of water rather than lack of thermal energy. The pressure decline is controlled by the size of the system, permeability of the rock and water recharge. Reinjection is considered an integral part of any sustainable geothermal utilization. It serves to dispose the waste-water, to counteract pressure draw-down, and to extract more thermal energy from the reservoir rock. The purpose of a successful management approach is to maintain and limit the pressure drawdown for a long production time.

Effect of Recharge Temperature (T_r): Fig. 12 shows the pressure and temperature behavior of an open geothermal system. The relevant system data are given in Table 1. An open system considering production and recharge but no reinjection is modeled. Eq. 29 is employed to describe the temperature behavior. It is assumed that $C_{pw}=C_{pwr}$.

Table 1. The relevant system data of the open geothermal system.

w_p = 270 kg/s, $\rho_{av}C_{av}$ = 2.5x10^6 J/(m^3.^{o}C), V = 12x10^9 m^3

C_{pw} = 4100 J/(kg.^{o}C), T_i = 210 ^{o}C, α = 41.22 kg/(bar.s)

κ = 4.3x10^9 kg/bar, D = 9.59x10^{-9} 1/s, a = 3.69x10^{-11} 1/s

x = 1x10^{-9} ^{o}C/s, Q_n = 30x10^6 J/s

The pressure drop in an open system is given by:

$$\Delta p = p_i - p = \frac{w_p}{\alpha}(1 - e^{-Dt}) \qquad (65)$$

which indicates a sharp decline initially and reaches to steady-state constant Δp finally. The steady-state Δp_{ss} is determined using $\Delta p_{ss} = w_p / \alpha$. The results in Fig. 12 show that production causes a pressure drop of 6.55 bar up to a production time of 100 year.

The temperature decline is smoother and depends on the recharge temperature. However, the recharge temperature does not affect the pressure behavior. The principal results of the modeling study for several T_r values are presented in Fig. 12.

Reservoir cooling is expected because of colder recharge and production. The magnitude of cooling depends on the recharge temperature. Lower recharge temperature yields higher reservoir temperature drop as expectedly.

Not only the pressure response but also the temperature response of the system exhibits a characteristic steady-state behavior. As can be seen in Fig. 13, the temperature response finally reaches to a steady-state temperature of 167.1 ^{o}C. This study indicates that the steady-state temperature is represented by the following equation:

$$T_{ss} = T_r + x/a = T_r + \frac{Q_n}{w_p C_{pw}}$$

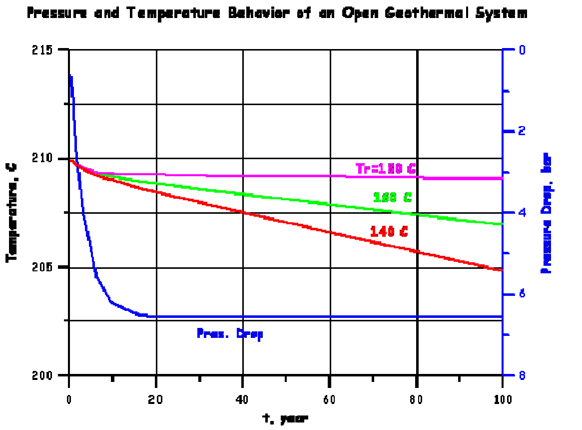


Fig. 12 Effect of recharge temperature on pressure and temperature behavior of an open geothermal system.

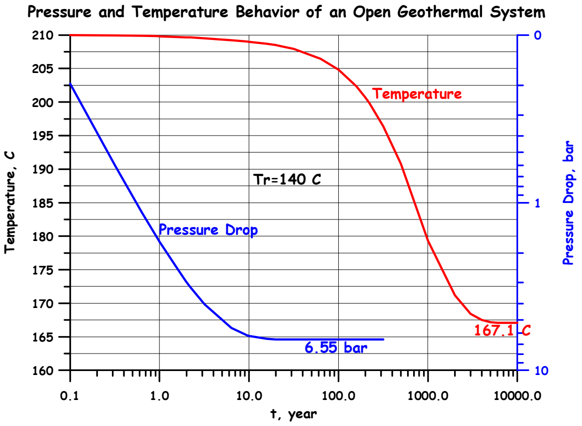


Figure 13. Steady-state pressure and temperature characteristics of an open geothermal system

Effect of Natural Heat Flow (Q_n): Fig. 14 shows the effect of natural heat flow Q_n on the pressure and temperature behavior of an open geothermal system. The system data given in Table 1 except that Q_n=53 MW$_t$ (53x10^6 J/s) rather than 30 MW$_t$ are used in modeling. Recharge temperature is assumed to be 160 ^{o}C. An open system considering production and recharge but no reinjection is modeled. Eq. 29 is employed to describe the temperature behavior. It is assumed that C_{pw}=C_{pwr} . Pressure drop is calculated by Eq. 65 and identical to the one given in Fig. 12 since the data required to find the pressure drop are the same.

Results in Fig. 14 show that the magnitude of reservoir cooling depends on the amount of natural heat flow. Higher Q_n yields lower reservoir temperature drop.

Effect of Reservoir Volume (V): Fig. 15 shows the effect of reservoir volume V on the pressure and temperature behavior of an open geothermal system. The system data given in Table 1 are used in modeling. Recharge temperature is assumed to be 160 ^{o}C. Results for two different values of reservoir volume are given, V=12x10^9 and 6x10^9 m^3. The corresponding a values are 3.69x10^{-11} and 7.38x10^{-11} 1/s, respectively. An open system considering production and recharge but no reinjection is modeled. Eq. 29 is employed to describe the temperature behavior. It is assumed that C_{pw}=C_{pwr}. Pressure drop is calculated by Eq. 65 and identical to the one given in Fig. 12 and 14 since the data required to find the pressure drop are the same.

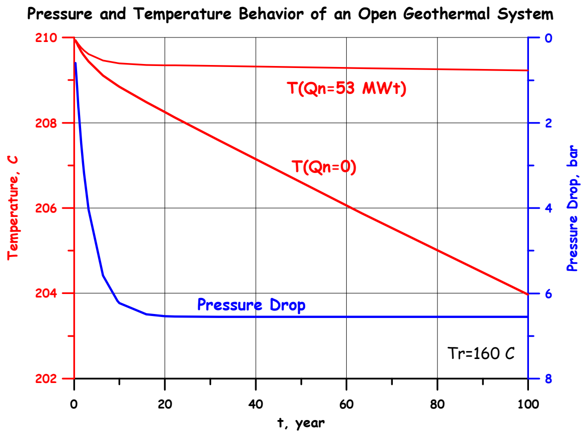


Fig. 14 Effect of natural heat flow on pressure and temperature behavior of an open geothermal system

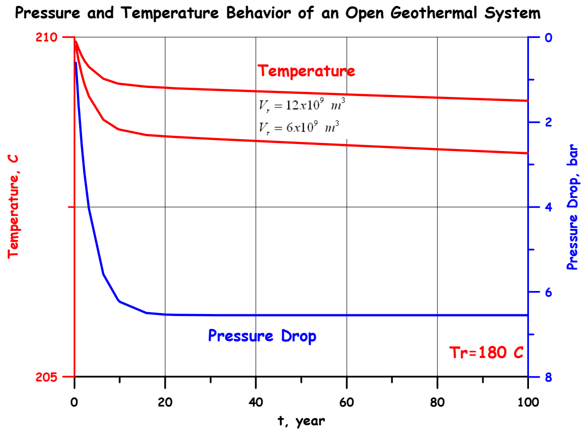


Fig. 15 Effect of reservoir volume on pressure and temperature behavior of an open geothermal system

Results in Fig. 15 show that the magnitude of reservoir cooling depends on the reservoir volume. Higher reservoir volume V yields lower reservoir temperature drop.

Effect of Outer Boundary Condition (Open, Closed): Fig. 16 shows the effect of outer boundary condition on the pressure and temperature behavior of a geothermal system. Open system corresponds to the case which recharge exists whereas the closed one is without recharge. The system data given in Table 1 except that Q_n=0 (no natural heat in) rather than 30 MW$_t$ are used in modeling. Recharge temperature is assumed to be 210 ^{o}C. An open system considering production and recharge but no reinjection is modeled. Eq. 29 is employed for describing the temperature behavior of the open system and Eq. 8 for the closed one. It is assumed that C_{pw}=C_{pwr} . Pressure drop is calculated by Eq. 65 for the open system. The pressure drop for the closed system is calculated using the following equation:

$$\Delta p = p_i - p = \frac{w_p}{\kappa} t \qquad (66)$$

Results in Fig. 16 show that production causes a pressure drop of 6.55 bar for the open system and 198 bar for the closed system up to a production time of 100 year. Cooling depends on the outer boundary condition. The open system containing heat recharge in compensates for some of the heat produced and thus a lower temperature drop occurs as compared to closed system.

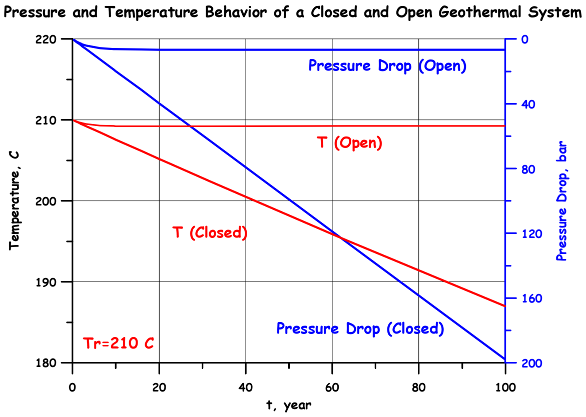


Fig. 16 Effect of outer boundary condition on pressure and temperature behavior of a geothermal system

Effect of Reinjection (w_{ri}): Fig. 17 shows the effect of reinjection on the pressure and temperature behavior of a geothermal system. The system data given in Table 1 are used in modeling. Recharge and reinjection temperatures are assumed to be 180 and 90 °C, respectively. Therefore, b' and g values are 3.32×10^{-9} and 1.66×10^{-9} °C/s. Eq. 37 is employed for describing the temperature behavior of the open system. It is assumed that $C_{pw}=C_{pwr}$. Pressure drop is calculated by Eq. 65 for the open system.

In Fig. 17 modeling results of two different cases are presented for comparison purposes. First case represents an open system with production and recharge (w_p=270 kg/s) whereas the second case again an open system, however, with reinjection as well as production and recharge (w_p=270 kg/s, w_{ri}=135 kg/s and w_{pn}=135 kg/s). In the second case, 50% of the production is assumed to be reinjected.

Results in Fig. 17 show the positive effect of reinjection on counteracting pressure drop and thus on maintaining the reservoir pressure. The pressure drop in reinjection application is less than the pressure drop without it. However, reinjection causes some cooling of the reservoir. Clearly the magnitude of cooling depends on the reinjection temperature. Lower reinjection temperature yields higher reservoir temperature drop as expectedly.

4.7 Pressure and Temperature Recovery After Shut-in in Geothermal Systems

Reservoir conditions (pressure and temperature) recover if production is stopped. Reversibility of the effects of production is a crucial phenomena to understand the renewability and sustainability issues of the geothermal systems. In general, pressure recovers on a time-scale comparable to the time-scale of production. However, temperature recovers on a much longer time scale. The mass change effects happen on a shorter time scale than the energy changes.

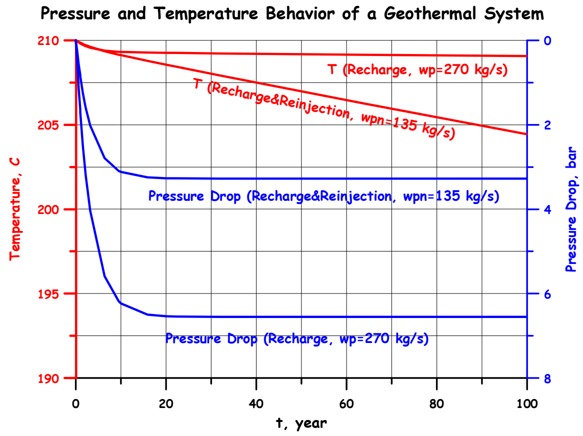


Fig. 17 Effect of reinjection on pressure and temperature behavior of a geothermal system

The purpose of this chapter is to investigate the behavior of a geothermal system if it is shut-in after some production period.

4.7.1 Analysis of Case 1

For the present study a scenario is considered where an open geothermal system with properties given in Table 1 is continued in production for a period of 100 years and then shut-in. In Fig. 18 the pressure and temperature are plotted. The rapid decline in pressure after production began is clear and there is also a rapid recovery after field shut-in. The corresponding plot of temperature versus time is also given in Fig. 18. It shows the slower decline followed by the expected slower recovery.

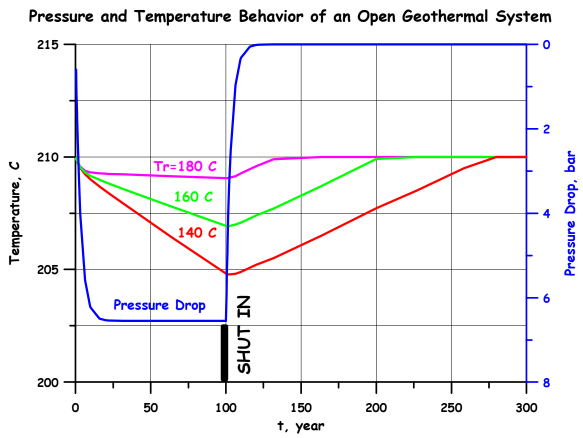


Figure 18 Pressure and temperature recovery

As shown in Fig. 18, temperature recovery is a function of the recharge temperature. Higher recharge temperature yields shorter recovery time. In other words recharge at a lower temperature causes more cooling and higher temperature drop in the reservoir. For a constant natural heat flow (Q_n), the recovery time is expected to be longer for lower recharge temperature.

Figure 19 presents the production and recharge rate data for the system during production and recovery periods. The results given in Fig. 19 are pressure dependent and the same results are valid for all the cases, no matter what the recharge temperature is.

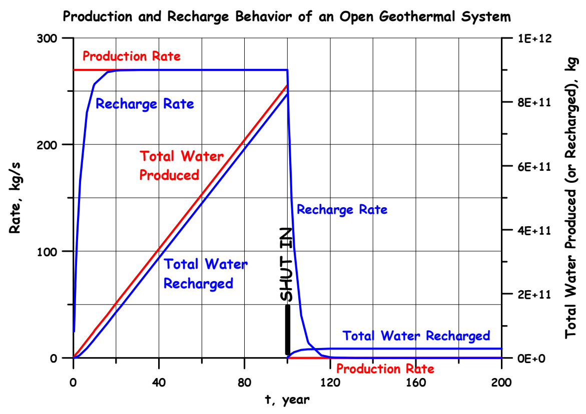


Figure 19 Production and recharge rates

In case of reinjection application, pressure and temperature behavior are given in Fig. 20. Data used for Fig. 17 applies. Recharge and reinjection temperatures are assumed to be 180 and 90 ^{o}C, respectively. Therefore, *b'* and *g* values are 3.32x10^{-9} and 1.66x10^{-9} ^{o}C/s. Eq. 37 is employed for describing the temperature behavior of the open system. It is assumed that $C_{pw}=C_{pwr}$. Pressure drop is calculated by Eq. 65 for the open system.

In Fig. 20 modeling results of two different cases are presented for comparison purposes. First case represents an open system with production and recharge (w_p=270 kg/s) whereas the second case again an open system, however, with reinjection as well as production and recharge (w_p=270 kg/s, w_{ri}=135 kg/s and w_{pn}=135 kg/s). In the second case, 50% of the production is assumed to be reinjected.

Results in Fig. 20 show the positive effect of reinjection on counteracting pressure drop and thus on maintaining the reservoir pressure. The pressure drop in reinjection application is less than the pressure drop without it. However, reinjection causes some cooling of the reservoir. Clearly the magnitude of cooling depends on the reinjection temperature. Lower reinjection temperature yields higher reservoir temperature drop during production period and thus it takes longer time to recover as expectedly.

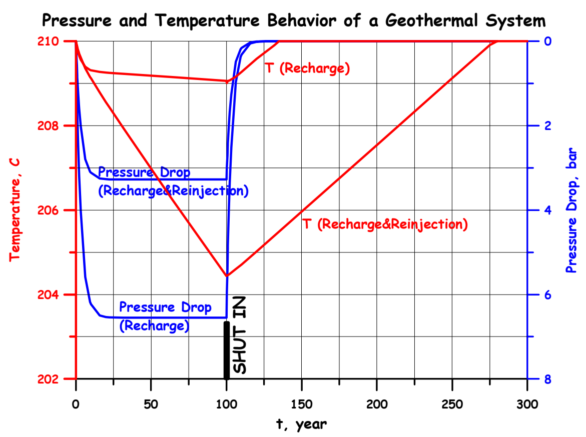


Figure 20 Effect of reinjection on pressure and temperature behavior

As a last case, the production rate is assumed to be 540 kg/s, two times of the previous case. And again half of the production is reinjected into the reservoir, w_{pn}=270 kg/s. Results for this case and for the case of w_p=270 kg/s and w_{pn}=135 kg/s are shown in Fig. 21 for comparison purposes.

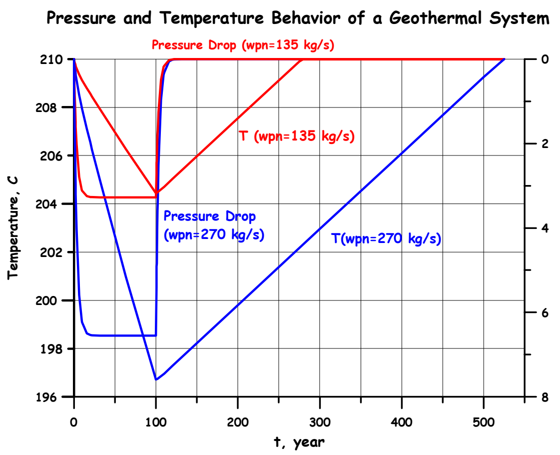


Figure 21 Effect of magnitude of reinjection on pressure and temperature behavior

4.7.2 Analysis of Case 2

The following data were incorporated to produce temperature and pressure response of an open geothermal reservoir.

Table 2. The relevant system data of the open geothermal system.

w_p = 1563 kg/s, $\rho_{av}C_{av}$ = 2.6x10^{6} J/(m^{3}.^{o}C), V = 2x10^{9} m^{3}

C_{pw} = 4100 J/(kg.^{o}C), T_i = 263 ^{o}C, α = 57.35 kg/(bar.s)

κ = 1.54x10^{10} kg/bar, D = 3.7x10^{-9} 1/s, a = 1.23x10^{-9} 1/s

x = 1.92x10^{-8} ^{o}C/s, Q_n = 100x10^{6} J/s , T_r = 180 ^{o}C

The reservoir is produced for 170 years and then shut-down. The pressure and temperature responses during the production and recovery periods are shown in Fig 22.

A steady-state pressure drop of 27.25 bar being equivalent to $\Delta p_{ss}=w_p/\alpha$ is reached at fairly short production time.

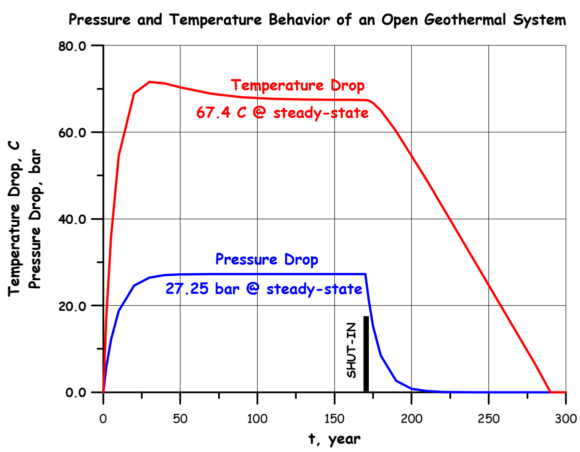


Figure 22 Pressure and temperature responses of the geothermal system

As shown in Fig. 22 the temperature drop is a sharp one initially, however it reaches to a stabilized steady-state value of 67.4 ^{o}C. This value can be determined from

$$\Delta T_{ss} = T_i - T_r - x/a = T_i - T_r - \frac{Q_n}{w_p C_{pw}}$$

or $$T_{ss} = T_r + \frac{Q_n}{w_p C_{pw}} \tag{67}$$

Notice that if such a steady-state temperature (T_{ss}) or temperature drop (ΔT_{ss}) is observed in temperature behavior of the system, then it is possible to estimate T_r providing that Q_n is known or vice versa.

Similarly the recovery time is related to reservoir volume:

$$V = \frac{\Delta t_{rec} Q_n}{\Delta T_{ss} \rho_{av} C_{av}} \tag{68}$$

Thus the reservoir volume can be estimated if parameters on the right hand side of the above equation are known.

5. RESULTS AND DISCUSSION

The main characteristics of the pressure and temperature responses occurring during production and shut-in periods are illustrated in Fig. 23.

Significant points regarding the pressure response:

1) The pressure behavior during the production period follows:

$$\Delta p = \frac{w_p}{\alpha}(1 - e^{-\alpha t/\kappa}) = \frac{w_p}{\alpha}(1 - e^{-Dt}) \tag{69}$$

and the steady-state pressure drop is reached much earlier than the steady-state temperature drop and is given by

$$\Delta p_{ss} = \frac{w_p}{\alpha} \tag{70}$$

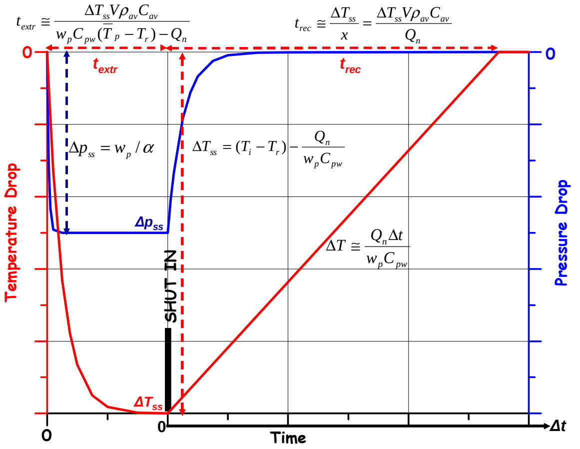


Fig. 23 The main characteristics of the pressure and temperature responses occurring during production and shut-in periods

2) The pressure behavior during the shut-in period is given by

$$\Delta p = \frac{w_p}{\alpha} e^{-D\Delta t}(1 - e^{-Dt_p}) \tag{71}$$

If κ is obtained from matches with the historical data by using an appropriate simulation model then the reservoir volume-porosity-total compressibility product, $V\phi c_t$, can be estimated.

Important points regarding the temperature response:

1) Depending on the mechanisms (production, recharge, reinjection, closed reservoir, open reservoir or any combination of these) involved in the system, the expressions presented in this paper are to be chosen to represent the temperature response.

2) If the production time is long enough, the temperature response reaches to a steady-state temperature drop given by:

$$\Delta T_{ss} = (T_i - T_r) - \frac{Q_n}{w_p C_{pw}} \tag{72}$$

and the producing (or extraction) time to reach the steady-state temperature drop is approximated by:

$$t_{extr} \cong \frac{\Delta T_{ss}}{a(\overline{T}_p - T_r) - x} = \frac{\Delta T_{ss} V \rho_{av} C_{av}}{w_p C_{pw}(\overline{T}_p - T_r) - Q_n} \tag{73}$$

where $\overline{T}_p$ is the average temperature of the water produced. An arithmetic average of initial reservoir temperature (T_i) and T_{ss} can be used as an initial estimate for $\overline{T}_p$. Notice that this approach is different from the O'Sullivan and Mannington approach. They ignore the effect of recharge and suggest using

$$t_{extr} \cong \frac{\Delta T_{ss}}{a\overline{T}_p - x} = \frac{\Delta T_{ss} V \rho_{av} C_{av}}{w_p C_{pw} \overline{T}_p - Q_n} \tag{74}$$

3) During the recovery period the change in the temperature drop is represented by

$$\Delta T \cong \frac{Q_n \Delta t}{w_p C_{pw}} \tag{75}$$

and the time to recover the temperature is given by

$$t_{rec} \cong \frac{\Delta T_{ss}}{x} = \frac{\Delta T_{ss} V \rho_{av} C_{av}}{Q_n} \tag{76}$$

One of the most important findings of this study is the ratio of the recovery duration to the duration of production:

$$\frac{t_{rec}}{t_{extr}} \approx PR\left(1 - \frac{w_{pn} T_r + w_{ri} T_{ri}}{w_p \overline{T}_p}\right) - 1 \tag{77}$$

where

$$PR = \frac{Produced\ Energy\ Flow}{Natural\ Energy\ Flow} = \frac{w_p C_{pw} \overline{T}_p}{Q_n}$$

$w_{pn}=w_p-w_{ri}$, w_{ri} is the reinjection mass rate, $\overline{T}_p$ is the average production temperature, t_{rec} is the recovery time and t_{extr} is the duration of past production.

O'Sullivan-Mannington (2005) neglects the effects of the recharge and reinjection and hence gives

$$\frac{t_{rec}}{t_{extr}} = PR - 1 \qquad (78)$$

One should prefer to use Eq. 77 to estimate the recovery-production period ratio. Eq. 78 works provided that recharge and reinjection effects are negligible.

Example Application 1: The Wairakei system has been under production since 1953. O'Sullivan-Mannington (2005) presented results of a modeling study considering a scenario where the Wairakei geothermal field in New Zealand is continued in production from 2003 for a further 50 years and then shut down for 500 years. Thus a history of 100 years of production followed by 500 years of recovery is considered. The purpose of their study was to investigate how long will it take for the reservoir to fully recover to its original state.

In Fig. 24 the pressures and temperatures in Western and Eastern Borefields, the main production areas of the geothermal system, are plotted for 1953-2353. The data for 1953-2003 are the actual measured field data whereas the rest obtained from their modeling study. There is a very rapid decline of pressure after production began in 1953 and also a rapid recovery after the field shut-down in 2053. The temperature versus time plot shows slower decline followed by slower recovery. The temperature recovery is slower in the Eastern Borefield, which is further away from the deep recharge.

The initial temperature (T_i) is 263 ^oC, the average mass flow rate (w_p) is 1563 kg/s (=135 ktonnes/day) and the corresponding temperature drop is 70 ^oC for the Western Borefield and 63 ^oC for the Eastern Borefield. Assuming that the Western Borefield represents the Wairakei system, let us use the characteristics of the temperature versus time plot given in Fig. 24 and employ the correlations and results developed in this study to estimate some important properties of the geothermal system. For this analysis we assume C_{pw}=4.1 kJ/(kg.^{o}C) and $\rho_{av}C_{av} = 2.6x10^6$ J/(m^3.^{o}C).

1) At the end of the production period of 100 years the resulting temperature drop is 70 ^{o}C. Thus we can use Eq. 72,

$$\Delta T_{ss} = (T_i - T_r) - \frac{Q_n}{w_p C_{pw}} \qquad \text{or}$$

$$263 - 193 = 70 = (263 - T_r) - \frac{x}{a} \qquad (79)$$

2) A recovery time of 100 years is observed so that using Eq. 76,

$$t_{rec} \cong \frac{\Delta T_{ss}}{x} = \frac{\Delta T_{ss} V \rho_{av} C_{av}}{Q_n} \qquad \text{or}$$

$100 \ years = 3.15x10^9 s = \frac{70}{x}$, and so x=2.2x10^{-8} ^{o}C/s (80)

3) Using Eq. 73,

$$t_{extr} \cong \frac{\Delta T_{ss}}{a(\overline{T}_p - T_r) - x} \qquad \text{or}$$

$$100 \ years = 3.15x10^9 s = \frac{70/(2.2x10^{-8})}{(a/x)(\overline{T}_p - T_r) - 1}$$

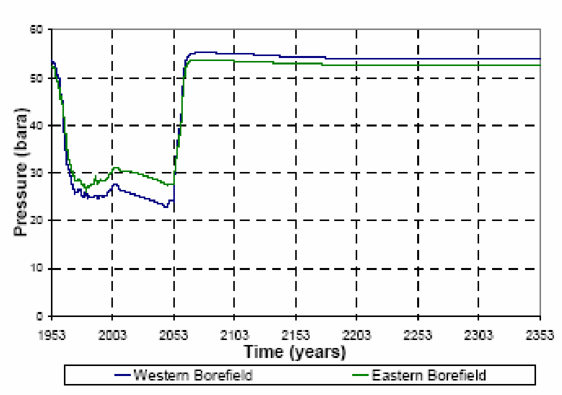
which yields $\frac{x}{a} = \frac{\overline{T}_p - T_r}{2}$ (81)

4) For practical purposes we can assume an arithmetic average value for the average production temperature ($\overline{T}_p$) so that $\overline{T}_p = \frac{T_i + T_{ss}}{2} = \frac{263+193}{2} = 228 \ ^oC$. Thus Eq. 81 yields:

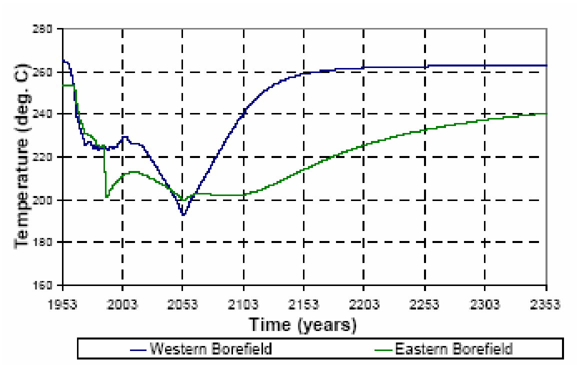
$$x/a = (228 - T_r)/2. \qquad (82)$$

Substituting this expression for *x/a* into Eq. 79 results

$70 = (263 - T_r) - \frac{228 - T_r}{2}$ which gives T_r = 158 ^{o}C. Since x and T_r are known then a=6.3x10^{-10} 1/s are obtained. From the definition of a, an estimate of the reservoir volume (V) is calculated to be 3.9x10^9 m^3. Also from the definition of x, the natural heat flow into the system (Q_n) is determined to be 224x10^6 J/s.



(a) Pressure history



(b) Temperature history

Figure 24. The pressures and temperatures in the Western and Eastern Borefields in the Wairakei geothermal system.

Moreover, analysis of the pressure history when combined with the results obtained from the analysis of the temperature history can yield an estimate of the ϕc_t product. Sarak et al. (2005b) analyzed the Wairakei pressure data given in Fig. 24 and determined κ = 1.54x10^{10} kg/bar. Since $\kappa = V\rho_w \phi c_t$ and we have an estimate of V=3.9x10^9 m^3, then ϕc_t for the geothermal system investigated is calculated to be ~0.005 1/bar.

Notice that analysis of this example application demonstrates the power of the correlations and expressions presented in this study. By simple analysis of the temperature versus time plot one can estimate the recharge temperature (T_r), the reservoir volume (V) and the natural heat flow into the system (Q_n). It is also possible to have an estimate of ϕc_t if κ is obtained from pressure analysis. Of course those parameters obtained from the analysis presented here should represent some apparent and average characteristics.

Example Application 2: Let us analyze the w_{pn}=270 kg/s case discussed earlier and its temperature-time plot is given in Fig. 21. For this case; w_p=540 kg/s, C_{pw}=4100 J/(kg.^{o}C), V=12x10^9 m^3, $\rho_{av}C_{av}$ =2.5x10^6 J/(m^3.^{o}C), Q_n=30x10^6 J/s, T_i=210 ^{o}C, T_r=180 ^{o}C, T_{ri}=90 ^{o}C, w_{pn}=270 kg/s, w_{ri}=270 kg/s. Fig. 21 indicates a temperature drop of about 13 ^{o}C after 100 years of production. For practical purposes we can assume 203 ^{o}C for the average production temperature.

The production ratio is $PR = \dfrac{540x4100x203}{30x10^6} \approx 15$

O'Sullivan-Mannington equation gives t_{rec}/t_{extr} = PR-1 = 14. However if we use Eq. 77:

$$\frac{t_{rec}}{t_{extr}} \approx PR\left(1 - \frac{w_{pn}T_r + w_{ri}T_{ri}}{w_p \overline{T}_p}\right) - 1 = 15\left(1 - \frac{270x180 + 270x90}{540x203}\right) - 1 = 4$$

Fig. 21 shows about 4.2 for t_{rec}/t_{extr}. This checks the validity of Eq. 77.

6. CONCLUSIONS

A simplified lumped-parameter type of an approach to model the temperature behavior of a relatively low-temperature single liquid-phase geothermal reservoir is discussed here.

New analytical equations and correlations are presented to investigate the pressure and temperature behavior of geothermal systems. Emphasis is given to understand the characteristics of temperature recovery following some production period.

The time to reach a recovery depends on many factors as discussed in the paper. Primarily it depends on the production period. However, the natural recharge and reinjection conditions considerably affect the recovery.

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