Mathematical Models of the Fluid Flowing for Geothermal and Hydrocarbon Wells

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ABSTRACT

Estimation of production response of the geothermal or hydrocarbon reservoir includes modeling i) the processes in reservoir during exploitation and ii) fluid flowing in the production and re-injection wells. Therefore, separate mathematical models for simulation of the conditions in reservoir and flowing in the wells are to be jointed as one concept.

The paper presents the comparative analysis of models for hydrocarbon and geothermal fluid flowing in the well. Analogy between two types of two-phase fluid (gas–condensate and steam–water) is drawn and differences are discussed. Some calculation results for wells of Karachaganak gas-condensate field and Mutnovsky geothermal field are presented.

1. INTRODUCTION

The model for calculating the main parameters of flowing in the wells (production flow rate, enthalpy, composition of extracted fluid) is necessary in development and exploitation of both geothermal and hydrocarbon fields. It is possible to draw analogy between flowing in the well for hydrocarbon (gas–condensate) and geothermal (steam–water) two-phase systems (Vereina, 2002).

In both cases two-phase fluid of multi-component composition is under consideration and mathematical model includes the same basic equations followed from conservation laws for mass and energy. In the same time, the following differences should be taken into account:

1. Geothermal system is non-isothermal while gas-condensate flowing may be assumed isothermal to simplify calculations.

2. In the case of gas-condensate well, it is possible to solve a problem assuming almost constant gas content, but for geothermal well water-steam phase transitions and steam content variations along the well are significant. The share of steam in geothermal fluid may vary in the range from 0 to 1 (e.g., when water-saturated seam is uncovered by a well, pressure drop can cause boiling along the well that will produce pure steam).

3. Closure equations are different because they describe the properties of certain fluid.

The present work considers several models. These are HOLA-simulator designed for modeling multi-feedzone geothermal well, and two models, non-isothermal two-rate and simplified (isothermal homogeneous), designed for estimating parameters of critical flowing for hydrocarbon wells. Some illustrating results for Mutnovsky geothermal field and Karachaganak gas-condensate field are also presented.

2. WELLBORE SIMULATOR HOLA AND ITS USE FOR PLOTTING PT-PROFILES

2.1. Brief description and governing equations

The multi-feedzone geothermal wellbore simulator HOLA developed by Grimur Bjornsson and Thordur Arason (Bjornsson et al., 1993) allows one to reproduce the temperature and pressure profiles in flowing wells and determine the contribution of each feedzone for given discharge conditions. The code is written in Fortran programming language and executable

The simulator HOLA uses the following basic assumptions. The flow within the well is assumed steady-state, since changing reservoir pressures are allowed. The simulator can be used for single and two-phase flows in vertical pipes, and calculates the flowing temperature and pressure profiles in a well. The properties of steam and water are calculated according to formulae presented by the International Formulation Committee in 1967.

Two sets of equations are used to represent the flow of fluid in a geothermal well. The flow in the well, between the feedzones, is represented by one-dimensional steady-state momentum, energy and mass balance equations. When a feedzone is encountered, the equations of mass and energy balance between the fluid in the well and the feedzone are used. To solve these equations fully defined boundary conditions (wellbore geometry, lateral mass and heat flow) and flow conditions at one end of the system (inlet conditions) are required. The governing equations are solved by numeric methods in small, finite steps along the well. If a feedzone is encountered, the known parameters of inflow, or outflow (mass and energy), are used to continue calculations.

The equations of mass, momentum and energy flux in a vertical well are written as:

$$\frac{dM}{dz} = 0 \tag{1}$$

$$\frac{dp}{dz} - \left[\left(\frac{dp}{dz} \right)_{fri} + \left(\frac{dp}{dz} \right)_{acc} + \left(\frac{dp}{dz} \right)_{pot} \right] = 0 \tag{2}$$

$$\frac{dE_t}{dz} \pm Q = 0 \tag{3}$$

The plus and minus signs correspond to downflow and upflow, respectively. The pressure gradient includes three terms: wall friction, acceleration of fluid and change in gravitational load over dz.

The equation of interaction between the well and the reservoir is:

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$$M_{feed} = PI_{feed}\left[\frac{k_{r1}\rho_1}{\mu_1} + \frac{k_{r2}\rho_2}{\mu_2}\right](p_r - p_w),$$

(4)

where relative permeabilities are calculated by linear relationships:

$$k_{rg} = s; \qquad k_{rl} = 1\text{-}s \qquad (5)$$

2.2. Plotting pressure and temperature profiles for Mutnovsky geothermal field

The wellbore simulator HOLA offers six modes of calculating downhole conditions in geothermal wells.
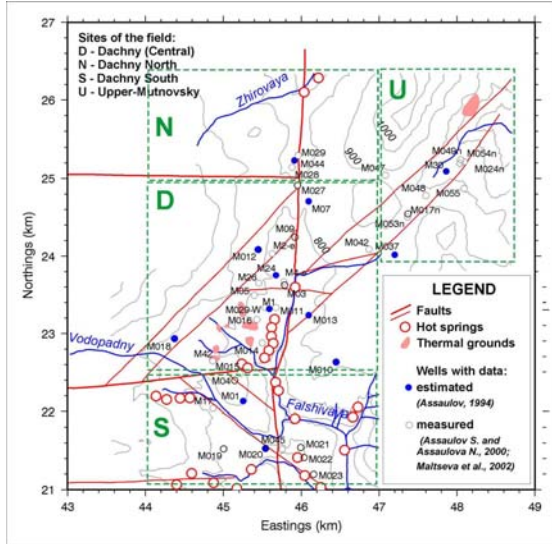

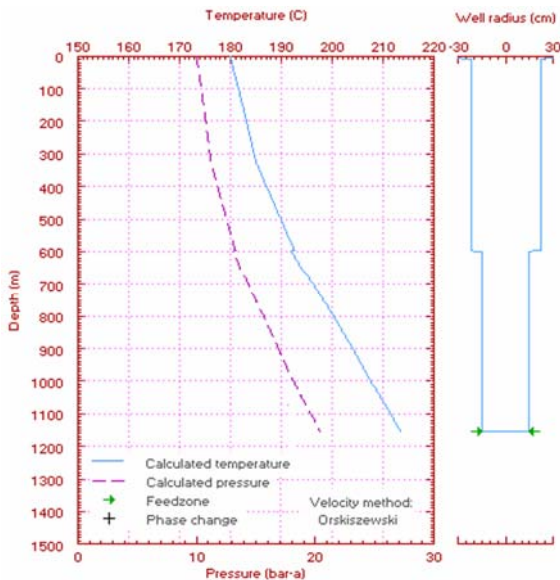Figure 1: Location of wells at the Mutnovsky geothermal field (Maltseva et al., 2002).

Figure 2: The pressure and temperature profiles in the well M01 located at the Dachny site of Mutnovsky geothermal field, plotted by HOLA-simulator.

The current paper considers the first mode of HOLA-simulator, the case of *Outlet conditions known at the wellhead* (Bjornsson et al., 1993). In this case simulator reproduces pressure and temperature profiles from given wellhead conditions and given flow rates and enthalpies at each feedzone (except the bottom one).
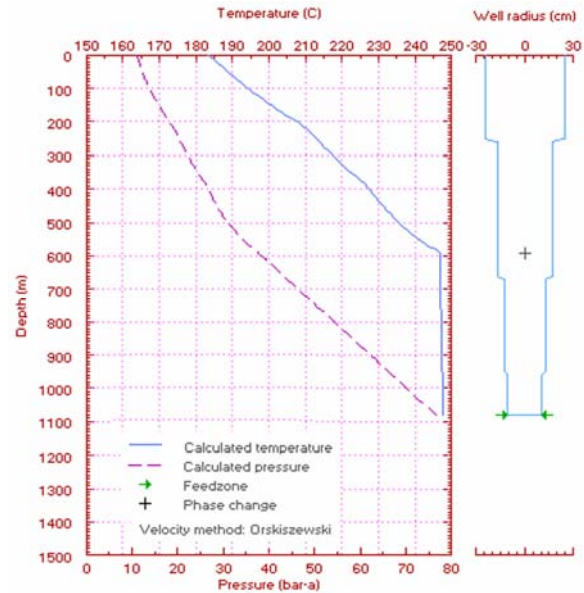

Figure 3: The pressure and temperature profiles in the well M24 located at the Dachny site of Mutnovsky geothermal field, plotted by HOLA-simulator.

The pressure and temperature profiles are represented for some wells located at the Dachny site of Mutnovsky geothermal field at Kamchatka. Because of intensive development of the field the modeling of its response to exploitation is of great importance. This modeling includes the estimation of conditions in the wells, which is to be joined to complex multi-parameter simulator like TOUGH2.

Table 1
Inpute parameters for wells M01 and M24
(Assaulov and Assaulova, 2000)

Well	Depth (m)	Flowrate (kg/s)	Wellhead enthalpy (kJ/kg)	Wellhead pressure (bar-a)
M01	1155.00	52.40	1358.00	10.00
M24	1080.00	22.90	1064.00	10.90

The location of the wells of Mutnovsky field is shown in Figure 1, and the results of plotting pressure and temperature profiles for wells M01 and M24 are presented in Figures 2, 3. The input data is shown in table 1. For each well one feedzone is proposed and well depth is considered until feedzone. The breaking point of temperature curve for well M24 corresponds to phase change; below this point the fluid in the well is in single-phase (liquid) state. In the well M01 the fluid is in two-phase condition; dryness is equal to 18.7% at the wellhead and decreases with depth almost uniformly. The parameters of feedzones for wells M24 and M01 are shown in table 2.

Table 2
Feedzone parameters for wells M01 and M24
calculated by HOLA-simulator

Well	Enthalpy (kJ/kg)	Pressure (bar-a)
M01	1369.16	20.41
M24	1074.60	76.47

3. MODELS FOR CALCULATING THE CRITICAL FLOWING PARAMETERS

In the development and exploitation of underground fluids critical well flowing can occur, which may result in heavy technical and ecological consequences. It is especially actual for deep-seated hydrocarbon fields of multi-component composition with a great content of H_2S and CO_2 as well as for high-temperature hydrothermal systems, since their exploitation may be accompanied by phase transformations both in a seam and in a borehole.

Estimations of critical flow rate, wellhead and well-bottom pressure are necessary for choice of the manner of blowout damping and require special calculation model. The follows are models designed for calculating critical flow rates of gas-condensate wells.

3.1. Non-isothermal heterogeneous (two-rate) model for gas-condensate well

In (Basniyev et al., 1991) the following one-dimensional two-rate model for calculating the critical flow rates of gas-condensate wells is presented. This model involves following equations of stationary two-phase multi-component flow in the vertical well under exploitation:

mass conservation equation:

$$M_m = M_1 + M_2 = const \qquad (6)$$

momentum change equation:

$$\frac{dp}{dz} = g\rho_m + \frac{\lambda_m}{2DS^2}\left(\frac{M_1^2}{(1-\varphi)\rho_1} + \frac{M_2^2}{\varphi\rho_2}\right) - \frac{1}{S^2} * \frac{d}{dz}\left(\frac{M_1^2}{(1-\varphi)\rho_1} + \frac{M_2^2}{\varphi\rho_2}\right) \qquad (7)$$

energy conservation equation:

$$\frac{d}{dz}\left[M_1\left(i_1 + \frac{w_1^2}{2}\right) + M_2\left(i_2 + \frac{w_2^2}{2}\right)\right] = gM_m - \pi DK(T_R - T) \qquad (8)$$

The following equations are need to enclose the equations set (6) – (8):

state equations for liquid and gas (steam) phases:

$$\rho_g = \rho_g(P,T) \qquad \rho_l = \rho_l(P,T) \qquad (9)$$

empirical formulae for real gas (steam) content and fluid hydraulic resistance coefficient which are different at various fluid flow regimes (Basniyev et al., 1991):

$$\varphi = \varphi\left(\beta, \frac{\mu_2}{\mu_1}, \frac{\rho_2}{\rho_1}, Fr_m, Re_m, We_m\right) \qquad (10)$$

$$\lambda = \lambda\left(\varphi, \frac{\mu_2}{\mu_1}, \frac{\rho_2}{\rho_1}, \varepsilon, Fr_m, Re_m, We_m\right) \qquad (11)$$

formulae for Reynolds, Froud, and Weber numbers, and relations resulting from definitions of two-phase flow parameters.

Temperature distribution (initial temperature) in reservoir is defined by expression:

$$T_R = T_{nl} + G(z - z_{nl}) \qquad (12)$$

Interaction between productive bed and well is described by the two-term inflow equation:

$$P_k^2 - P_3^2 = A\mu_{cp}*z_{cp}M_c + Bz_{cp}M_c^2 \qquad (13)$$

$$P_R^2 - P_b^2 = AQ_m + BQ_m^2 \qquad (13a)$$

A and *B* designate filtration resistance coefficients defined by well testing under stationary regimes (Basniyev et al., 1991).

The following additional functions are introduced for creating the algorithm:

$$\Phi_1 = (1/S^2)[M_g^2/\varphi\rho_g + M_l^2/(1-\varphi)\rho_l] \qquad \Phi_2 = (\lambda_c/2D)\Phi_1 + g\rho_c$$

$$\Phi_3 = M_g(i_g + w_g^2/2) + M_1(i_l + w_l^2/2) \qquad \Phi_4 = gM_c$$
$$-\pi DK(T_0 - T) \qquad (14)$$

After substituting (14) into equations (7) - (8) we have:

$$dp/dz = (1/\Delta)[\Phi_4\partial\Phi_1/\partial T - \Phi_2\partial\Phi_3/\partial T] \qquad (15)$$

$$dT/dz = -(1/\Delta)[\Phi_4(1 + \partial\Phi_1/\partial p) - \Phi_2\partial\Phi_3/\partial T] \qquad (16)$$

where

$$\Delta = \partial\Phi_1/\partial T \, \partial\Phi_3/\partial p - (1 + \partial\Phi_1/\partial p)\,\partial\Phi_3/\partial T \qquad (17)$$

Set of equations (15)-(16), (6), (9)-(12) at given boundary conditions (bottom-hole pressure and temperature) allows us to calculate distribution of pressure, temperature and other flow characteristics in the wellbore. Reservoir properties are considered by equation (13) allowing us to define bottom-hole pressure depending on mass flow rate.

Critical flow rate is defined from pressure and temperature jump on the wellhead. Mathematically it means that pressure and temperature derivatives at wellhead convert to infinity:

$$dp/dz = \infty \qquad dT/dz = \infty \qquad (18)$$

This requires denominator in (15) - (16) (or expression (17)) to be equal to zero:

$$\Delta = 0 \qquad (19)$$

Wellhead flow rate defined by (19) is called "critical flow rate" (or, "free flow rate"). It is maximal possible flow rate of the well.

3.2. Isothermal homogeneous model for gas-condensate well

The paper (Polyak (Vereina), 1996) presented calculating critical parameters (flow rate and wellhead pressure) for gas-condensate wells using a simpler, isothermal homogeneous hydrodynamic model proposed in (Astrakhan and Rozenberg, 1988). The use of such simplified model is justified because the overwhelming majority of critical blowouts occurs during the field prospecting (at the stage of the drilling and inflow testing) and at initial stages of the field exploitation. Under these conditions, all necessary data on structure of the field, fluid properties and composition, features of fluid flowing within a bore hole and phase transformations (gas-liquid) are unknown beforehand, so that use of complicated hydromechanical methods for the description of critical well flowing is inexpedient.

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The simplified model considers joint flowing in reservoir-well system and based on following assumptions: flowing is state and one-dimensional, homogeneous (single-rate), and isothermal. Moreover, the liquid phase is assumed incompressible and gas phase is described by Clapeyron equation. Two-term filtration equation is assumed, and the flowing direction (i.e. upflow) is assumed as positive.

According to these assumptions pressure gradient for one-dimensional homogeneous flowing can be formulated as:

$$-\frac{dP}{dz} = \frac{\frac{\lambda}{2D}\frac{M^2}{S^2}\left(\frac{1-x}{\rho_1}+\frac{x}{\rho_2}\right)+\frac{g}{\frac{(1-x)}{\rho_1}+\frac{x}{\rho_2}}}{1-\frac{cM^2}{P^2}} \quad (20)$$

where

$$c = \frac{xRT}{S^2} \quad (21)$$

Condition of critical flowing is conversion of denominator in (20) into zero at z=0. After substitution the Clapeyron formula:

$$\rho_2 = \frac{P}{RT} \quad (22)$$

and taking into account expression (21), equation (20) for critical flowing is written as:

$$-\frac{dp}{dz} = \frac{\frac{\lambda}{2D}p_{cr}^2\left(\frac{1}{p}+\frac{1-x}{xRT\rho_1}\right)+\frac{g}{xRT/p+(1-x)/\rho_1}}{1-\frac{p_{cr}^2}{p^2}} \quad (23)$$

Bottom-hole pressure is defined from equation (13a) describing filtration in reservoir.

Critical flow rate is defined as:

$$Q_{cr} = \frac{M_{cr}}{\rho_{mat}} \quad (24)$$

Using expressions (22) and (24), the equation (13) can be written as:

$$P_R^2 - P_{b\,cr}^2 = A^{**}P_{cr} + B^{**}P_{cr}^2 \quad (25)$$

where

$$A^{**} = \frac{AS}{\rho_{m\,at}\sqrt{xRT}} \qquad B^{**} = \frac{BS^2}{\rho_{m\,at}^2\,xRT} \quad (26)$$

Set of equations (23) and (25) define wellhead and well bottom pressure at critical flowing. Critical flow rate is defined from equation, derived from (22) and (24):

$$Q_{cr} = \frac{S}{\rho_{mat}\sqrt{xRT}}\,p_{cr} \quad (27)$$

The paper (Polyak (Vereina), 1996) also offers modification of isothermal homogeneous model for the case of variable sectional area of the well. Boreholes may be distorted by contracting and pulling stresses, which are acting in rocks; thus, the sectional area of the well becomes unequal in various depth intervals. These fluctuations are to be considered in model for critical parameters calculation.
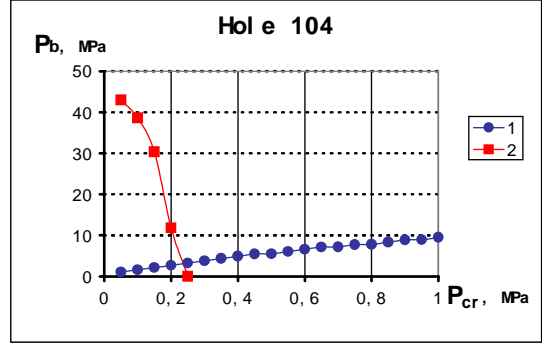

Figure 4: Calculation of critical parameters for well 104 of Karachaganak gas-condensate field (Polyak (Vereina), 1996).
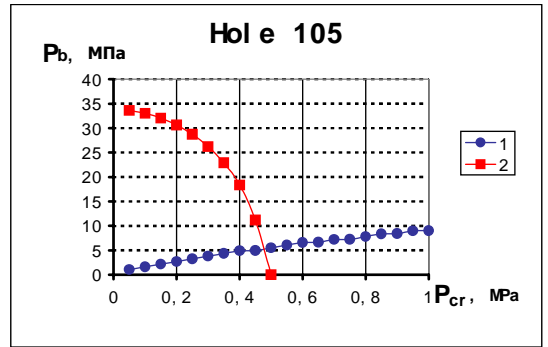

Figure 5: Calculation of critical parameters for well 105 of Karachaganak gas-condensate field (Polyak (Vereina), 1996).

3.3. Critical flow rate calculation: comparing the results for different models

Two variants of calculation are presented in (Polyak (Vereina), 1996):

1. If gas content is considered to be constant, then equation (23) is solved analytically. In this case its solution is written as:

$$\Phi(p_з,p_{кр}) - \Phi(p_{кр},p_{кр}) = H \quad (28)$$

where:

$$\Phi(p,p_{kp}) = \int \frac{\left(1-\frac{p_{kp}^2}{p^2}\right)dp}{\frac{\lambda}{2D_m}p_{kp}^2\left(\frac{1}{p}+\frac{1-x}{xRT\,\rho_1}\right)+\frac{g}{xRT/p+(1-x)/\rho_1}}$$

So, wellhead and well bottom critical pressure are defined from equations (25) and (28). Critical flow rate is defined from equation (27).

2. If gas content depends on pressure, equation (23) is solved by numeric methods, and graph is plotted for bottomhole pressure depend on wellhead critical pressure. Another graph is plotted according to equation (25), and cross-section point is to be found. This point corresponds to solution (see Figs. 4, 5). Critical flow rate is defined by the same way as in previous case.

4

In (Basniyev et al., 1991) and (Polyak (Vereina), 1996) calculations of critical parameters were performed for the wells of Karachaganak gas-condensate field (located to the north of Kaspian Sea). Comparison of results for different models has shown that the simplified (isothermal homogeneous) model can be used to estimate free flow rates for the wells under consideration. Maximum discrepancy didn't exceed 16 %. It may be explained by validity of simplified model at enough high both gas content and flow rate of the fluid in the wells, which is typical for the given field. However, the simplified model is inadequate for calculating critical wellhead pressure, since it results in extremely overstated estimations (the maximum difference between values obtained from complicated and simpler model reaches 55 %). Nevertheless, for single-phase fluid (pure gas) the estimations both critical flow rate and critical wellhead pressure using the simplified model well agree with the results obtained using more complex model (the maximal discrepancy is of 11 and 13 %, respectively).

4. CONCLUSIONS

The development and exploitation of geothermal and gas-condensate fields need the model for calculating the main parameters of flowing in the productive and re-injection wells. The current paper considers existent models for description of well flowing in both cases of fluid and some calculation results are presented. The following inferences can be made:

1. Considering the mathematical models for the flowing in geothermal and hydrocarbon wells we can see that they are similar. Both them include the basic equations of mass, momentum and energy balance for flowing in the well and equations of inflow (or outflow), i.e. equations of well-reservoir interaction. The main difference is the description of fluid properties. Moreover, unlike the flow in geothermal well, the flow in gas-condensate well may be considered isothermal and gas phase content may be assumed constant.

2. The multi-feedzone geothermal wellbore simulator HOLA is considered, and some results of plotting pressure and temperature profiles are presented. Such model describing the flow in wellbore is necessary for modeling the field response to exploitation, and is to be joined with complex multi-parameter simulator, like TOUGH2.

3. Two models for calculating the critical flow parameters for hydrocarbon wells, non-isothermal two-rate and isothermal homogeneous, developed in previous works, are examined as well as comparison of their results is presented. It was shown that simplified (isothermal single rate) model allows us to predict critical flow rates for hydrocarbon wells.

4. Critical flowing may occur during exploitation of geothermal fields too, but this specific case need the special calculation model.

NOMENCLATURE

A, B = filtration coefficients
A^*, B^* = reduced filtration coefficients
D = hole diameter
E – energy flux
G = geothermal gradient
g = acceleration of gravity
H = well depth
i = enthalpy
K = heat transfer coefficient
M = mass flow rate
P = pressure
PI = productivity index
Q = flow rate
R = gas constant
S = well sectional area,
s = volumetric steam saturation
T = temperature
w = average velocity
x = mass gas (steam) content
z = vertical coordinate

Greek letters

β = volumetric gas (steam) content
λ = hydraulic resistance
ρ = fluid density
φ = real gas (steam) content

Subscript

0 = standard conditions
1 = gas (steam) phase
2 = liquid (water) phase
at = atmospheric conditions
b = well bottom
cr = critical
feed = feedzone
h = wellhead
m = mixture (two-phase fluid)
nl = neutral layer
r = reservoir
w = well

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