

# GEOHERMAL TWO-PHASE FLOW IN HORIZONTAL PIPES

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## ABSTRACT

This paper reanalysed the 255 sets of geothermal two-phase flow data of Freeston et al (1983).

A new void fraction correlation is proposed in this paper. The new correlation is derived from the analysis of two-phase flow velocity distribution using the Seventh Power Law as:

$$\frac{1-\alpha}{\alpha^{7/8}} = \left[ \left( \frac{1}{x} - 1 \right) \left( \frac{\rho_g}{\rho_f} \right) \left( \frac{\mu_f}{\mu_g} \right) \right]^{7/8}$$

To predict the two-phase pressure drop, an equivalent pseudo-single-phase flow having the same boundary layer velocity distribution is assumed. The average velocity of the equivalent single-phase flow is used to determine the wall friction factor and hence the two-phase pressure drop. This method gives very good agreement with the experimental data. The average velocity of the equivalent single-phase flow is also a very good correlating parameter for the prediction of geothermal two-phase pressure drops in a horizontal straight pipe.

## INTRODUCTION

Geothermal resources suitable for power production are mainly of the wet type that requires the separation process for use in conventional steam turbines. The separation process can take place anywhere between the wellhead and the powerhouse. Separation at the wellhead allows high separation pressure but lower steam output due to higher wellhead pressure (WHP). On the other hand, separator near the turbine has low steam pressure but more quantity of steam. Hence the separation pressure determines the turbine inlet pressure and the utilisation efficiency of the resource. Since the two-phase well fluid pressure drop in the transmission line between the wellhead and the separator affects these two important parameters, it is important to design the two-phase pipelines to transport the two-phase fluid as efficiently as possible.

James (1968) and Takahashi et al. (1970) showed that there were economic advantages using long two-phase transmission pipelines. From experience at Wairakei, this has the added advantage of solving the steam pipe corrosion problem due to the steam condensate. However, there is a minimum length of steam pipe required upstream of the steam turbines to scrub the steam of salts carryover from the separation process and for steam pressure control.

Although there are many theories and correlations available for predicting two-phase pressure drop, most of them were based on small pipe diameter (<60 mm) and high heat transfer rate conditions for single-component flows else they are two-component, two-phase flows. For geothermal application (large diameter, low heat transfer rate and single-component),

Harrison's (1975) correlation, based on geothermal data, is the most suitable.

Freeston et al. (1983) studied geothermal two-phase pressure drops in 100 mm diameter straight pipe and fittings, collected 255 sets of valuable data. However, attempts to correlate two-phase pressure gradients in a straight pipe as a function of liquid phase velocity give divergent result (see Figure 1).

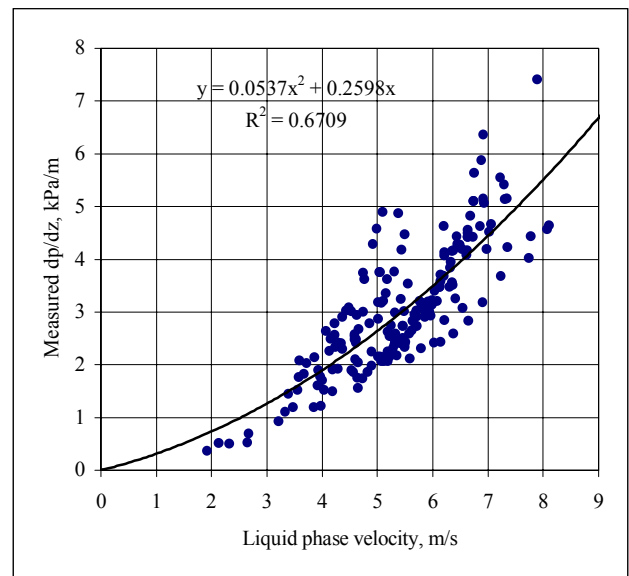


Figure 1: Measured pressure gradient versus liquid phase velocity (prior to the introduction of PC) of Freeston et al (1983).

It has to be pointed out that these results were computed prior to the availability of powerful Personal Computer (PC) and spreadsheet software. Steam-water properties were correlated from steam tables.

Mundakir (1997) reevaluated the data on a PC using a spreadsheet software, and steam-water properties were obtained from the computer software Engineering Equation Solver (EES) by Klein and Alvarado (1994). The correlation with liquid phase velocity improved significantly as shown in Figure 2. However, attempts to correlate pressure drop across pipe fittings showed no consistent results with liquid phase velocity.

Mundakir's (1997) attempt to correlate the measured pressure gradient of the straight pipe as a function of a two-phase pressure coefficient  $[(1/2\rho V^2)_{TP}]$  gave divergent results similar that of Figure 1.

This paper is an attempt to find a better correlating parameter for the two-phase pressure drop data of Freeston et al. (1983).

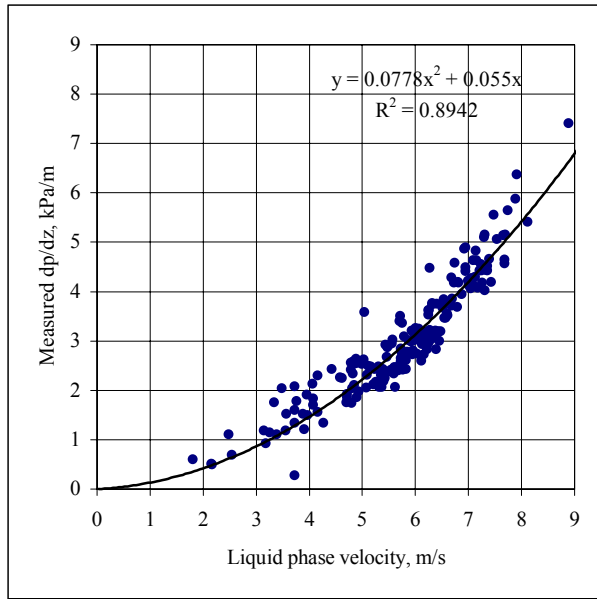


Figure 2: Measured pressure gradient versus liquid phase velocity (results of analysis by a PC and EES software) of Mundakir (1997).

## DATA REVIEW

The original data of Freeston et al (1983) were contained in Lee et al (1979). The data were gathered from an experimental rig installed at geothermal well WK207 at Wairakei (see Figure 3). Although there were a total of 255 sets of data, only 189 sets were good. The rest was either incomplete trial runs, incorrect steam orifice plate installation, or flows not at critical flow for the James Lip Pressure method to be applicable.

The data presented in Lee et al (1979) were mainly measured gauge pressures and pressure drops. The original computed results were presented in Lee and Freeston (1979).

This paper reanalysed the horizontal straight pipe data in Lee et al (1979).

## METHOD OF ANALYSIS

In two-phase flow pressure drop prediction by the separated flow model, void fraction ( $\alpha$ ) is the most important fundamental parameter. It determines other two-phase parameters such as the liquid phase velocity,  $\bar{V}_f$ , and the mean density,  $\bar{\rho}$ . These in turn determine the two-phase pressure drop.

The Seventh-Power Law velocity distribution is used to describe many fluid flow situations. It is used to derive the void fraction correlation for the two-phase flow pressure drop prediction in this paper.

The Seventh-Power Law velocity distribution for a turbulent flow can be expressed as:

$$\frac{\bar{u}}{U} = \left(1 - \frac{r}{R}\right)^{1/7} \quad (1)$$

where,

$\bar{u}$  = Velocity at radius  $r$ ,

$U$  = Central line velocity,

$r$  = Radius,

$R$  = Pipe radius.

If the void fraction for a two-phase flow is  $\alpha$ , then the gaseous phase inside core has the radius of  $\alpha^{1/2}R$  and the liquid phase flows within the outer layer of the radius from  $\alpha^{1/2}R$  to  $R$ .

By applying the Seventh-Power Law to the liquid phase layer, we get the liquid phase volume flow rate:

$$\begin{aligned} V_f &= \int_{\alpha^{1/2}R}^R U_f \cdot \left(1 - \frac{r}{R}\right)^{1/7} \cdot 2\pi r dr \\ &= \frac{49}{60} U_f \cdot \pi R^2 (1 - \sqrt{\alpha})^{8/7} \left(1 + \frac{8}{7} \sqrt{\alpha}\right) \end{aligned} \quad (2)$$

So the liquid phase mass flow rate is:

$$m_f = \rho_f V_f = \frac{49}{60} \rho_f U_f \cdot \pi R^2 (1 - \sqrt{\alpha})^{8/7} \left(1 + \frac{8}{7} \sqrt{\alpha}\right) \quad (3)$$

where,

$V_f$  = liquid phase volume flow rate,

$m_f$  = liquid phase mass flow rate,

$U_f$  = liquid phase centreline velocity assuming liquid phase only flowing through the pipe with the same boundary velocity distribution as that of the two-phase liquid layer.

From above equation, we also get:

$$\begin{aligned} \bar{V}_f &= \frac{V_f}{A_f} = \frac{\frac{49}{60} U_f \cdot \pi R^2 (1 - \sqrt{\alpha})^{8/7} \left(1 + \frac{8}{7} \sqrt{\alpha}\right)}{(1 - \alpha) \pi R^2} \\ &= \frac{49}{60} U_f \frac{(1 - \sqrt{\alpha})^{8/7} \left(1 + \frac{8}{7} \sqrt{\alpha}\right)}{(1 - \alpha)} \end{aligned} \quad (4)$$

and

$$\bar{V} = \frac{\int_0^R U_f \cdot \left(1 - \frac{r}{R}\right)^{1/7} 2\pi r dr}{\pi R^2} = \frac{49}{60} U_f \quad (5)$$

so

$$\frac{\bar{V}_f}{\bar{V}} = \frac{(1 - \sqrt{\alpha})^{8/7} \left(1 + \frac{8}{7} \sqrt{\alpha}\right)}{(1 - \alpha)} \quad (6)$$

where,

$\bar{V}_f$  = average liquid phase film velocity,

$\bar{V}$  = average velocity of the equivalent single-phase flow as defined for  $U_f$

$\bar{V}_f / \bar{V}$  = velocity ratio, a parameter which will be used for predicting pressure drop.

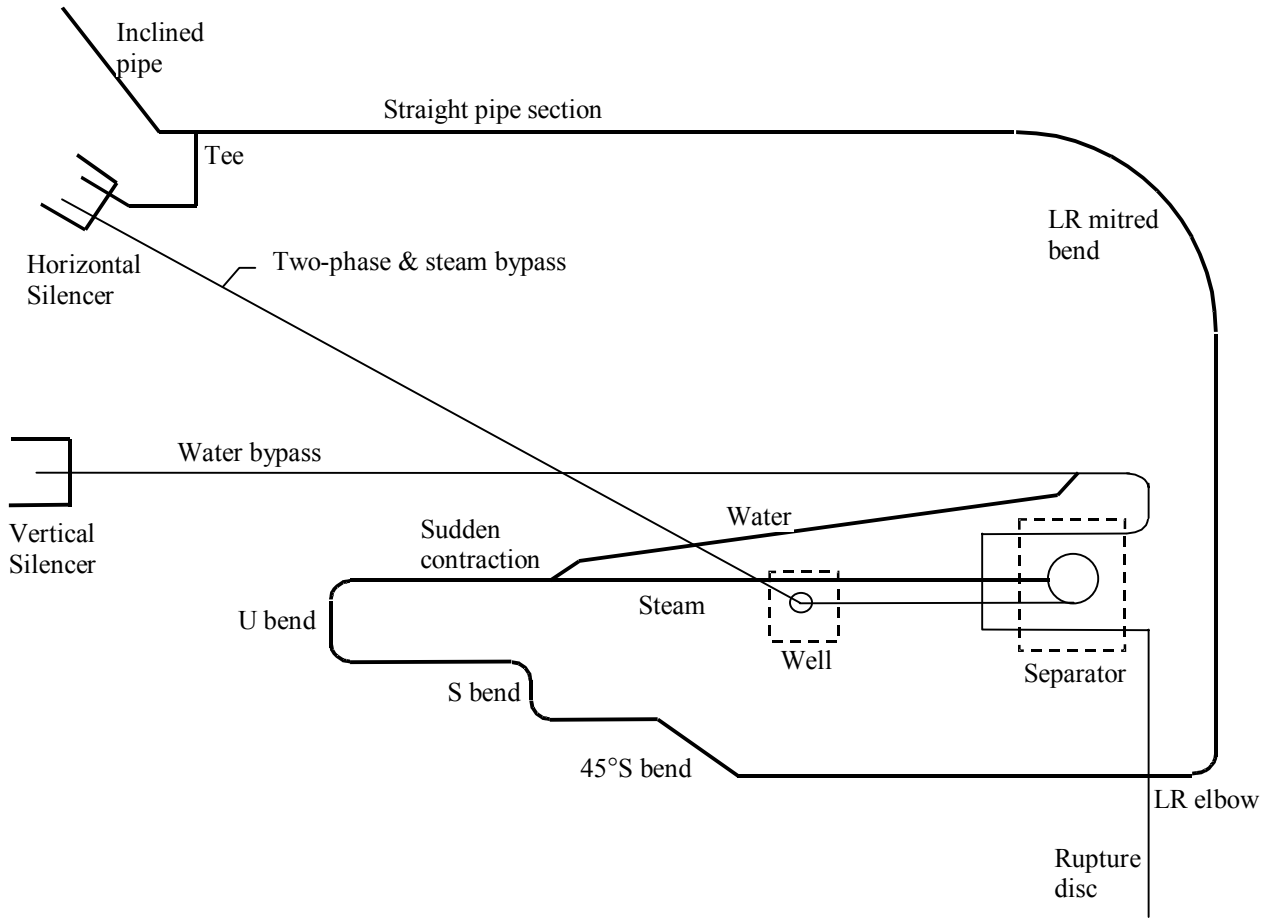


Figure 3: Schematic experimental layout of Freeston et al (1983).

By applying the Seventh-Power Law to gaseous phase, the velocity distribution of the gaseous core is:

$$\bar{u} = \bar{u}_{IP} + U_g \left( 1 - \frac{r}{\sqrt{\alpha} R} \right)^{1/7} \quad (7)$$

where,

- $\bar{u}$  = Velocity at radius  $r$ ,
- $\bar{u}_{IP}$  = Inter-phase velocity,
- $U_g$  = Gaseous phase centreline velocity,
- $r$  = Radius,
- $R$  = Pipe radius.

Here,  $\bar{u}_{IP}$  is only approximately 7% of  $U_g$ . In order to simplify the problem, it is considered to be negligible. It is found later that the assumption only introduces 3% error to the final value of void fraction,  $\alpha$ . Furthermore, the accuracy is always of the same trend, so that it can be corrected by a factor later. So the gaseous phase volume flow rate can be expressed as:

$$V_g = \int_0^{\sqrt{\alpha} R} U_g \cdot \left( 1 - \frac{r}{\sqrt{\alpha} R} \right)^{1/7} \cdot 2\pi r dr$$

$$= \frac{49}{60} U_g \cdot \pi R^2 \alpha \quad (8)$$

So gaseous phase mass flow rate is:

$$m_g = \frac{49}{60} \rho_g U_g \pi R^2 \alpha \quad (9)$$

After getting the mass flow rates for two phases, we can relate them to the steam quality of the two-phase flow system:

$$x = \frac{m_g}{m_g + m_f}$$

$$= \frac{\alpha \rho_g}{\alpha \rho_g + \left( \frac{U_f}{U_g} \right) (1 - \sqrt{\alpha})^{8/7} \left( 1 + \frac{8}{7} \sqrt{\alpha} \right) \cdot \rho_f} \quad (10)$$

Here assuming  $U_f/U_g = \mu_g/\mu_f$ , which is a reasonable assumption for a turbulent flow in this situation, then after simplification the formula becomes:

$$\left[ \left( \frac{1}{x} - 1 \right) \left( \frac{\rho_g}{\rho_f} \right) \left( \frac{\mu_f}{\mu_g} \right) \right]^{7/8} = \frac{(1 - \sqrt{\alpha}) \left( 1 + \frac{8}{7} \sqrt{\alpha} \right)^{7/8}}{\alpha^{7/8}} \quad (11)$$

According to Wallis (1969),

$$(1 - \sqrt{\alpha}) \left( 1 + \frac{8}{7} \sqrt{\alpha} \right)^{7/8} \approx 1 - \alpha$$

It is found later that this approximation only introduces an error of <1% to void fraction,  $\alpha > 0.88$ , and <5% to  $\alpha > 0.61$ .

The final void fraction correlation is:

$$\frac{1-\alpha}{\alpha^{7/8}} = \left[ \left( \frac{1}{x} - 1 \right) \left( \frac{\rho_g}{\rho_f} \right) \left( \frac{\mu_f}{\mu_g} \right) \right]^{7/8} \quad (12)$$

The assumptions made in deriving the void fraction correlation can be summarised as:

1. Gaseous phase centreline velocity  $U_g$  is not sensitive to flow diameter, i.e. for flow diameter of  $2R$  and  $2\alpha^{1/2}R$ , the  $U_g$  value is the same.
2. Inter-phase velocity effect on gaseous phase is negligible.
3. No liquid is in gaseous phase core, i.e. no entrainment.

### PREDICTION OF PRESSURE DROP

Harrison (1975) used average liquid phase velocity  $\bar{V}_f$  and superficial liquid phase wall friction factor  $(C_f)_{fs}$  to predict two-phase pressure gradient. However, the idea of using friction factor and velocity to predict pressure gradient comes from single-phase flow system. Therefore, it seems a better idea to relate two-phase system to a pseudo-single-phase system which has the same pressure gradient. Then two-phase pressure gradient is evaluated through the equivalent single-phase flow system. Such an equivalent single-phase system is considered to be the one which has the same boundary layer velocity distribution as the two-phase flow liquid layer.

Although equation (12) is not in the simplest form, it can still be solved by “trial and error”. The equation is solved by Engineering Equation Solver (EES) computer program.

The average liquid phase film velocity can be expressed as:

$$\bar{V}_f = 1.1(1-x) \frac{W(1-x)v_f}{(1-\alpha)A} \quad (13)$$

where,

- W = Total mass flow rate,
- x = Steam quality,
- $v_f$  = Liquid phase specific volume,
- $1.1(1-x)$  = Correction factor mainly for the entrainment.

At this stage, a correction factor is introduced to account for the entrainment effect and the simplification made in deriving void fraction correction. It can be explained as  $1.1(1-x)$  fraction of the liquid phase is left in the liquid phase boundary layer. The other fraction is entrained inside the gaseous phase as water droplets. When the steam quality decreases, the gaseous phase can carry less liquid. This means a higher percentage of the liquid is left in the boundary layer. The choice of factor is mainly to give a good result rather than having a rigorous theoretical justification.

Equation (6) shows the relationship between the average liquid phase film velocity and the average velocity of the equivalent single-phase flow. The average velocity of the equivalent flow,  $\bar{V}$ , can then be calculated. A sample calculation is given below.

### SAMPLE CALCULATION

#### Measured data (Run 99)

Pipe inner diameter, D	= 0.1023 m
Pipe wall roughness, $\epsilon$	= 0.00015 m
Pressure, p	= 2.925 bar abs
Water flow, $m_f$	= 3.868 kg/s
Steam flow, $m_g$	= 1.625 kg/s
Pressure gradient, $dp/dz$	= 4.867 kPa/m

#### Predicted pressure gradient

Pipe cross-sectional area, A	= 0.008215 m <sup>2</sup>
Total mass flow, m	= 5.493 kg/s
Dryness, x	= 0.2958
Fluid specific enthalpy, h	= 1198 kJ/kg
Void fraction, $\alpha$ , (Eq 12)	= 0.9164
Liquid phase velocity, $V_f$ (Eq 13)	= 4.676 m/s
Equivalent single phase velocity, V (Eq 6)	= 6.857 m/s
Re based on V	= 3.14E6
Friction factor, $\lambda$	= .0217
Liquid density, $\rho_f$	= 932 kg/m <sup>3</sup>
Specific steam volume, $v_g$	= 0.620 m <sup>3</sup> /kg
Wall shear stress, $\tau_w = \lambda \rho_f V^2 / 8$	= 118.9 Pa/m <sup>2</sup>
Pressure gradient, $dp/dz = 4\tau_w/D$ (neglecting acceleration effect)	= 4.649 kPa/m
Acceleration correction, $AC = m_g^2 v_g / (pA^2 \alpha)$	= 0.0905
Pressure gradient, $dp/dz = 4\tau_w/[D(1-AC)]$	= 5.112 kPa/m

### RESULTS AND DISCUSSION

Figure 4 shows the result of the new method of calculation. Comparing to Figure 5 of Harrison's prediction method, Figure 4 gives less scatter of data points.

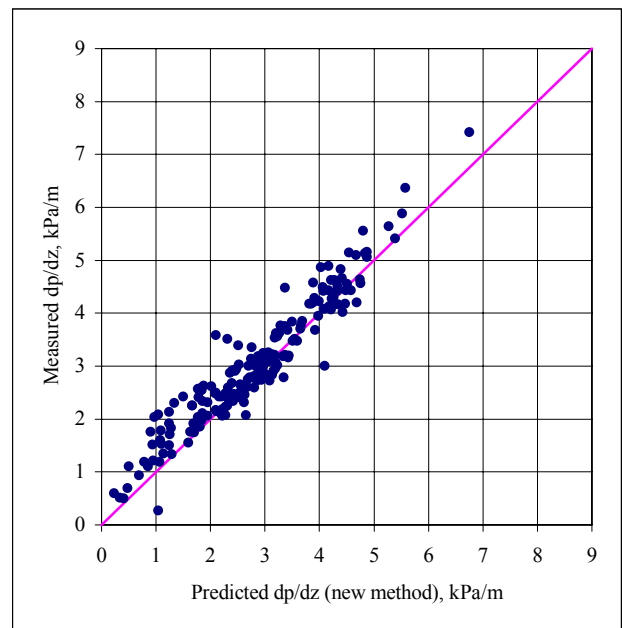


Figure 4: Measured versus predicted geothermal two-phase flow pressure gradient by the new method.

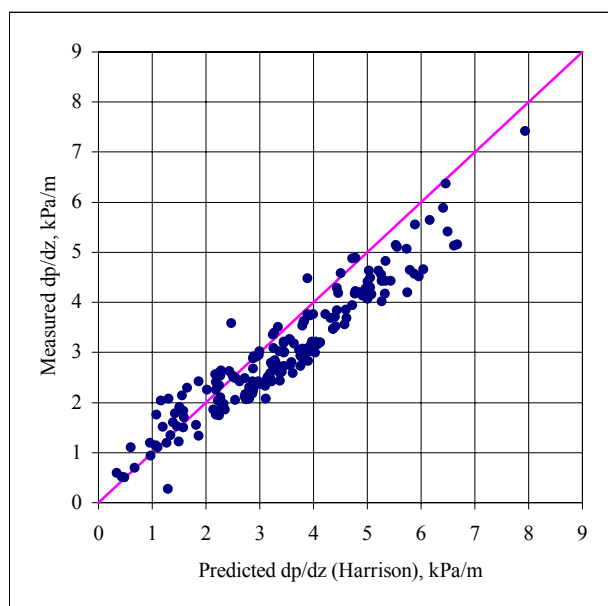


Figure 5: Measured versus predicted geothermal two-phase flow pressure gradients by Harrison's method (1975).

In Figure 6, the measured pressure drops are plotted as a function of the average velocity of the equivalent single-phase flow,  $\bar{V}$ . It shows a marginally improved correlating parameter than the liquid phase velocity in Figure 2 of Mundakir (1997). However, it also appears to be a better correlating parameter for pressure drop across pipe-fittings than liquid phase velocity but this is inconclusive at this point in time.

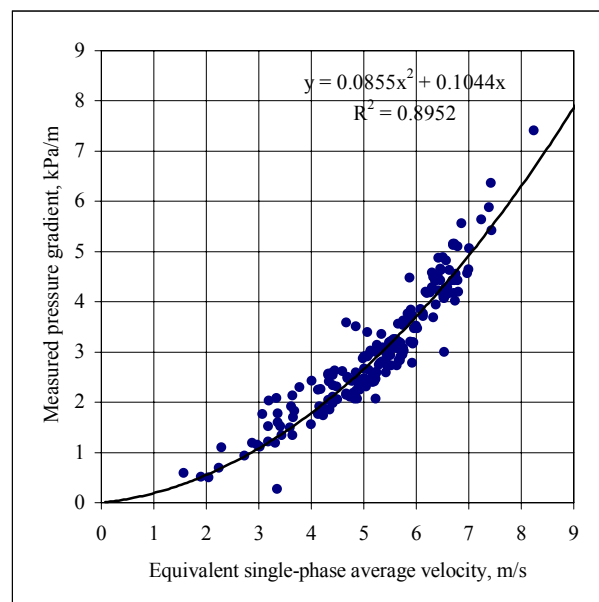


Figure 6: Measured pressure gradient versus equivalent single-phase average velocity.

## CONCLUSIONS

A new void fraction correlation derived from the Seventh Power Law gives good agreement between the measured and the predicted two-phase pressure drops in geothermal steam-water flow in large pipes.

The average velocity of the equivalent single-phase flow,  $\bar{V}$ , is a good correlating parameter of the geothermal two-phase pressure drops in straight horizontal pipes.

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