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MODELLING GEOTHERMAL HISTORY

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ABSTRACT

This paper focuses on the application of two mathematical techniques, dimensional analysis and linear stability theory, to modelling the evolution of geothermal systems at geological time scales. Dimensional analysis is used to verify conceptual models and to identify key physical processes that lead to the development of various steady states. Linear stability theory is applied to explain how one steady state transforms into another. We discuss three most significant non-dimensional numbers for geothermal modelling: the Reynolds number, the Rayleigh number and dimensionless heat flux. The Reynolds and Rayleigh numbers are important for all sub-surface water flow problems, whereas dimensionless heat flux is particularly useful for modelling two-phase water-steam systems. The whole approach is based on appreciation of the time-varying nature of a geothermal system. A numerical example is presented.

1. INTRODUCTION

Modelling geothermal history is important for understanding maturation, migration and accumulation of petroleum and ore deposits. Many world ore deposits, for instance the Australian gold deposits, are located at the sites of ancient geothermal systems. The past time groundwater temperatures and pressures governed ore deposition. The maturation and migration of petroleum is also related to the past geothermal activities. With the help of mathematical modelling it is possible to look at a distant past of a geothermal system. It is also possible to predict the system response to various natural and industrial processes. The latter is important for sustainable exploitation of geothermal resources.

The purpose of mathematical modelling is to develop a computer model that reflects essential features of the phenomenon considered. As the final product of the modelling process, a computer model includes all simplifications and assumptions made at the previous steps, particularly at the starting step when empirical data is conceptualised. Sinc there is always uncertainty in empirical data, conceptual models may become a major source of errors. Dimensional analysis has proven efficient for validating conceptual models and for identifying key physical processes.

A geothermal system is never at steady-state but is undergoing various physical processes. Most of these processes, even relatively fast processes such as hydrothermal eruptions, fall into the category of slow processes as defined in Pestov (1998). If a real geothermal process is slow, it can be approximated by a quasi-static path. The evolution of a real system can then be modelled as a succession of equilibrium states, i.e. steady states. How one steady state transforms into another can be determined from linearised stability studies.

What follows is an illustration of how dimensional analysis and linear stability theory can be used in geothermal modelling.

2. DIMENSIONAL ANALYSIS

Dimensional analysis is particularly useful for modelling natural systems with large numbers of parameters. The dimensional analysis procedure consists of several steps: *scaling, non-dimensionalisation, estimating orders of magnitude* and, finally, *selecting dominant and deleting non-dominant effects/processes.* This can be applied when the governing equations are already known, which is the case for a geothermal system.

First, for each independent and dependent variable a characteristic quantity (scale) must be introduced. Next, all variables in the governing equations and boundary conditions must be replaced by the dimensionless ratios between variables and corresponding scales. The obtained non-dimensional equations will include non-dimensional groupings of the system parameters in front of each term. These non-dimensional numbers determine the importance of each term in the governing equations.

We shall discuss three most significant non-dimensional numbers for geothermal modelling: the Reynolds number Re, the Rayleigh number Ra and dimensionless heat flux $\tilde{Q}$.

2.1 The Reynolds Number

The Reynolds number is the most important non-dimensional number for fluid flow problems. By definition, the Reynolds number is

$$Re = \frac{\rho v H}{\mu}, \qquad (1)$$

where ρ and μ are fluid density and dynamic viscosity respectively, v is fluid velocity and H is characteristic length.

The Reynolds number determines a flow regime and, hence, a choice of the momentum equation. When $Re < 10$, Darcy's law can be used to describe the fluid flow through permeable rocks (Nield & Bejan, 1992). When $Re > 10$, other forms of the momentum equation must be used.

Most geothermal reservoir simulators solve the momentum equation in the form of Darcy's law. Let us calculate the Reynolds number for flow in a reservoir. Typically, fluid density $\rho \sim 10^3\, kg/m^3$, dynamic viscosity $\mu \sim 10^{-4}\, Ns/m^2$ and velocity in the absence of production $v \sim 10^{-6}\, m/s$. (Here and in the following symbol "$\sim$" stands for "is of the order of".) If the pore/fracture dimension H does not exceed $10^{-1}\, m$, then formula (1) gives Re much less than 10. Therefore, Darcy's law is applicable to the fluid flow in a reservoir under natural conditions.

Let us consider flow near a deep geothermal well. According to Ikeuchi *et al.* (1997), fluid velocities at the well bottom can be 1 to $100\, m/s$. Then for the fracture cross-sectional dimension $H \sim 10^{-3}\, m$ formula (1) gives Reynolds numbers up to 10^6. At such large Reynolds numbers turbulent boundary layer equations must be used to simulate flow from a fracture to a well. An attempt to use the Darcy's law based reservoir simulators will give non-convergent numerical iterations.

2.2 The Rayleigh Number

The Rayleigh number is used to determine driving mechanism for the fluid flow. In practical terms, it can help to differentiate between groundwater and geothermal systems and to make the right choice of numerical software for reservoir simulations.

In a homogeneous porous medium the Rayleigh number is defined as follows:

$$Ra = \frac{\rho g \beta k H \Delta T}{\mu \alpha}, \qquad (2)$$

where g is gravitational acceleration, β is thermal expansion coefficient for water, k is rock permeability, α is thermal diffusivity of the saturated rock, and ΔT is the temperature difference over a distance H. When $Ra > 4\pi^2 \approx 39.48$, thermal convection starts in the system and temperature gradients become the main driving force for the fluid flow (Nield & Bejan, 1992).

The applicability of the above criterion is limited to the most idealised case of a homogeneous isotropic reservoir. In more realistic cases laboratory or computer experiments are required. However, as a preliminary step it is always useful to calculate the Rayleigh number given by equation (2). The latter will help to identify geothermal sites within the system. Geothermal reservoir simulators should be applied to parts of the system with $Ra > 4\pi^2$.

Similar non-dimensional criteria have been developed for the onset of two-phase water-steam convection (boiling) in porous media (Nield & Bejan, 1992).

2.3 Dimensionless Heat Flux

Dimensionless heat flux is the main governing parameter for two-phase water-steam convective flows. This type of convective flows is particularly important for geothermal modelling since many geothermal systems contain two-phase convective regions. Examples include Wairakei in New Zealand, The Geysers in California, Larderello in Italy and Kawah Kamojang in Indonesia. The fluid flow in the two-phase regions of these systems exists in the form of the two-phase convection: the lighter phase (vapour) flows up as it is displaced by the heavier phase (liquid). Two-phase convection can be vapour or liquid-dominated with vapour or liquid being the most mobile phase respectively.

Pestov (1997) gives the following definition of dimensionless heat flux:

$$\tilde{Q} = \frac{\mu_l Q}{k \rho_v \rho_l g l}, \qquad (3)$$

where Q is vertical heat flux at the base of the system, l is latent heat and subscripts l and v stand for liquid and vapour phases respectively. In formula (3) the phase densities, ρ_v and ρ_l, are calculated at the upper reservoir boundary.

According to formula (3), $\tilde{Q}$ is proportional to the ratio between vertical heat flux Q and reservoir permeability k. The importance of the ratio between Q and k has been pointed out by many modellers (cf. McGuinness, 1996). As shown theoretically (Pestov, 1997) and confirmed in numerous computer experiments (McGuinness, 1996; Pestov, 1999), the phase distribution within the two-phase region depends on $\tilde{Q}$. In the vapour-dominated case, the liquid phase saturation is an increasing function of $\tilde{Q}$, that is a decrease in $\tilde{Q}$ produces a drier two-phase zone. In the liquid-dominated case, the liquid phase saturation in a decreasing function of $\tilde{Q}$. Hence, a decrease in $\tilde{Q}$ leads to higher liquid phase saturations within the two-phase zone. If the total amount of water in the reservoir remains constant (which is the case in many practical situations), changes in $\tilde{Q}$ could produce a single-phase layer of either water or steam in the system.

After examining formula (3), the following key geothermal processes can be identified:

- increase/decrease in conductive heating at the base of the system (i.e. changing Q);
- deposition/dissolution of chemicals in the rock matrix (i.e. changing k).

Indeed, the above processes change $\tilde{Q}$ trough Q and k and, hence, the phase distribution in the two-phase zone leading to the development of new steady states.

3. LINEAR STABILITY THEORY

How one steady state transforms into another can be determined with the help of linear stability theory. Linear stability theory studies periods of time during which the systems returns to equilibrium after being disturbed by an infinitesimal force. Disturbances in the system parameters are assumed to be so small that their second and higher order products can be neglected in the governing equations. With the above assumption, the governing equations can be linearised and solved, in many cases even analytically. Analytical solutions are always preferable as they give explicit relationships between unknown variables and system parameters.

In reality, however, there is no infinitesimal force and changes in the system are always finite. This difficulty can be overcome by introducing the concept of a quasi-static process.

3.1 Quasi-Static Process

A slow geothermal process can be approximated by a quasi-static path as shown in Figure 1. Suppose that some parameter of the system k changes over a period of time $t_n - t_0$ which is long compared to the characteristic time of the system t^*. Note that the change of k over $[t_0, t_n]$ can be finite. Assume that at $t < t_i$, $(i = 0, 1, ..., n)$ the system is at the state of thermodynamic equilibrium. At time $t = t_i$, called *the event time*, we change parameter k instantly by a small amount and wait until the system equilibrates to a new steady state. The time τ during which a steady state can be restored, is called the relaxation time of the system. If for any i $(i = 0, ..., n-1)$ we have $\tau \ll t_{i+1} - t_i$, then process $k(t)$ can be replaced by a quasi-static process ($K(t)$ in Figure 1), and for any $t \in [t_i, t_{i+1}]$ $i = 0, ..., n-1$ linearised governing equations can be applied.

3.2 Relaxation Times

Important information on the system development over real time can be obtained from linearised governing equations. In particular, it is possible to calculate the relaxation time using linearised equations. There are two relaxation times associated with flows through permeable rocks: the time for pressure to return to equilibrium and the time during which water thermodynamic properties equilibrate. When water-steam geothermal systems are considered, the latter relaxation time is replaced by the time during which phase saturations restore equilibrium.

According to Pestov (1998), in a vapour-dominated system pressure disturbance, p', decays exponentially, i.e.

$$p' = p_\circ e^{-\lambda t}, \tag{4}$$

where λ is the eigenvalue for the dominant mode determined from linearised equations, t is time and $p_\circ$ is the initial disturbance at $t = 0$. In equation (4) all quantities are non-dimensional. From equation (4), p' reduces by a factor of $e \approx 2.72$ after time $1/\lambda$. Hence, $\tau_p = 1/\lambda$ can be used as the relaxation time for pressure.

It is natural to take $\tau_s = 1/s$ as the relaxation time for phase saturations. Here s is the non-dimensional wave speed of small saturation waves originating in the porous medium in response to small pressure disturbances (Pestov, 1999). The wave speed s can be calculated from linearised governing equations, too.

Table 1 shows eigenvalue λ and non-dimensional wave speed s calculated for various reservoir permeabilities and for $Q = 500\,mW/m^2$. The temperature of the upper boundary is taken to be $240°C$. Corresponding values of dimensionless heat flux $\tilde{Q}$ are also calculated. Only for very small reservoir permeability s does exceed λ. In most practical cases $\tau_p \gg \tau_s$ and the pressure field equilibrates faster than the saturation field. Thus, τ_s does define which steady state is likely to develop during the evolution of a geothermal system.

3.3 Numerical Example

As an example we consider a vapour-dominated geothermal reservoir with all boundaries impermeable to fluid flow so that the amount of water trapped in the pore space is constant at all times. A uniform heat flux is imposed at the bottom boundary and a constant temperature of $240°C$ is prescribed at the upper boundary. Rock material is assumed to be homogeneous and isotropic. For our purposes it is sufficient to use a one-dimensional vertical numerical grid.

We assume that the reservoir permeability k decreases with time due to deposition of chemicals while vertical heat flux Q remains unchanged at $500\,mW/m^2$. This process corresponds to an increase in dimensionless heat flux $\tilde{Q}$ given by formula (3). At every step of this process we decrease k by a small amount and wait until a new steady state develops. As shown in our numerical experiments, this process leads to the formation of a two-layer structure with a single-phase vapour layer underlying a two-phase convective zone. Figure 2 and Figure 3 show temperature and pressure distributions respectively. Temperature and pressure profiles for higher values of k represent distributions of the corresponding quantities at earlier times. In the above example we used the EOS module 1 of the numerical simulator TOUGH2 (Pruess, 1991).

In agreement with formula (3), we obtained the structure with a vapour layer underlying a vapour-dominated zone when heat flux Q was increased from $500\,mW/m^2$ to $2000\,mW/m^2$ and k was kept constant. Calculated temperature and pressure distributions closely matched those shown in Figure 2 and Figure 3.

4. CONCLUSIONS

Dimensional analysis and linear stability theory are ubiquitous in mathematical modelling. This paper illustrates the application of these mathematical techniques to modelling geothermal reservoir history.

Dimensional analysis is particularly useful at the starting step of the modelling process when empirical information is conceptualised. It helps to validate conceptual models and to make the right choice of numerical software. When choosing a numerical program for reservoir simulations, it is always useful to begin with calculating the Reynolds number (1) and the Rayleigh number (2). If the Reynolds number exceeds 10, Darcy's law and most geothermal reservoir simulators cannot be used to describe the geothermal flow. If the Rayleigh number exceeds $4\pi^2$, a numerical program that includes coupling between heat and mass transfer processes may be needed.

Dimensionless heat flux (3) is another important non-dimensional number for geothermal modelling. It governs the phase distribution within the two-phase water-steam region. Dimensionless heat flux is proporsional to the ratio between vertical heat flux and reservoir permeability. Therefore, fluctuations in the conductive heating at the base of a reservoir and deposition/dissolution of chemicals in the rock matrix change the phase distribution within the two-phase zone and lead to the development of distinct steady states.

Linear stability theory determines why one steady state transforms into some other particular steady state. Linear stability theory studies relaxation time periods during which the system returns to equilibrium. There are two relaxation times associated with flows through permeable rocks: the time for pressure to equilibrate and the time for water thermodynamic properties to restore equilibrium. When water-steam geothermal systems are considered, the latter relaxation time is replaced by the time for phase saturations to return to equilibrium. Both relaxation times can be calculated from linearised governing equations. Since the pressure field equilibrates much faster than the saturation field, the relaxation time for saturation defines the evolution of a geothermal system.

Both dimensional analysis and linear stability theory provide guidance for setting up numerical experiments in the most efficient and computationally inexpensive way and enable scientists and engineers to study complex natural systems at geological time scales.

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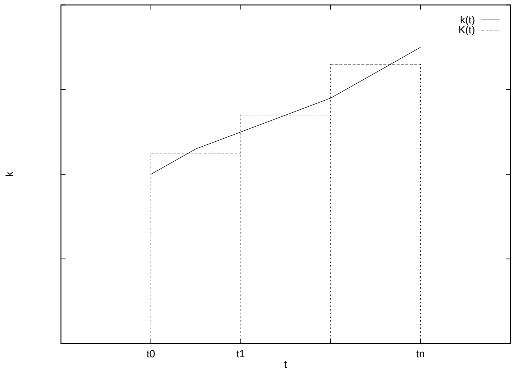

Figure 1: Quasi-static approximation.

$\tilde{Q}$	$k\ (m^2)$	λ	s
0.01	0.5×10^{-14}	660.81	47.7
0.05	2.5×10^{-14}	135.02	31.6
0.1	0.5×10^{-15}	69.03	26.3
0.5	2.5×10^{-15}	16.19	16.1
1.0	0.5×10^{-16}	9.62	12.4

Table 1: Eigenvalue and wave speed.

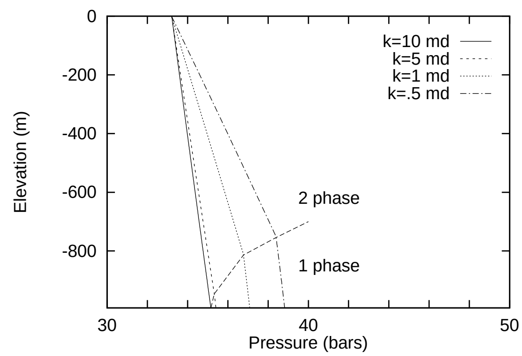

Figure 3: Pressure versus depth.

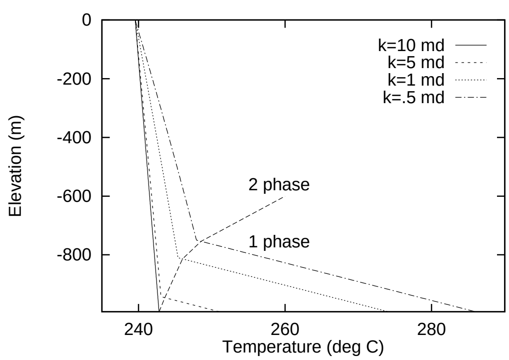

Figure 2: Temperature versus depth.