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THERMAL CALCULATIONS OF GEOTHERMAL HEAT UTILISING ONE-WELL SYSTEMS WITH BOTH INJECTION AND PRODUCTION

Tomasz Kujawa[1] and Wladyslaw Nowak[1]
[1]Department of Heat Engineering, Technical University of Szczecin, Al. Piastow 19, 70-310 Szczecin, Poland

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ABSTRACT

A method of heat calculation and an analysis of the influence of the annular injected fluid on the inner-liner produced fluid for two kinds of heat exchangers in a one-well system is presented. The exchangers are partly immersed in a geothermal reservoir from which the heat of the geothermal water is drawn. The first part of the heat exchanger is located in the impervious caprock where temperature changes linearly. Graphs of changes of fluid temperature for various production and reservoir conditions are also presented.

1. INTRODUCTION

Geothermal heat plants and power stations generally use two-well systems with both injection and production wells. Two-well systems are used for pumping warm, mineralised geothermal water. In systems of this kind, the temperature of geothermal water extracted to the earth surface may be estimated using known computational models in a relatively simple way. But the high expenditure of well drilling in relation to the total capital cost of projects is the negative aspect of using this method of winning thermal energy.

The capital cost may be reduced by using (for highly mineralised geothermal water) one-hole injection-production systems. The use of existing single wells drilled during oil and gas exploration is the most economical proposition for heat extraction. In this case the conditions of heat exchange are entirely different than for two-well systems. For the same geothermal reservoir, the temperature of extracted water in a one-well system is lower than for a two-well system.

Existing publications do not provide analytical solutions to the problem of this kind. Mathematical models describing heat transmission in one-well systems are needed. Previously published computational models of chosen geothermal heat exchangers (Kujawa and Nowak, 1998) are further developed in this paper with an analysis of insulation in the top part of the exchanger:

- a vertical geothermal heat exchanger Fields type (Fig. 1),
- a double-pipe heat exchanger with spiral-tube exchanger (Fig. 2).

One portion of the heat exchanger is located in impervious caprock in which temperature is changes linearly with depth. This portion is insulated both outside and inside the pipe. The other portion of heat exchanger is immersed in a geothermal reservoir from which it extracts heat. The temperature of the geothermal water also changes linearly with depth. The models allow an estimate of the temperature of output water as a function of mass flow and other parameters, which describe a geothermal reservoir.

2. MATHEMATICAL MODEL: VERTICAL GEO-THERMAL HEAT EXCHANGER FIELDS TYPE

A Fields type vertical heat exchanger is illustrated in Figure 1. An approximate computational model of the exchanger is presented in this section. The lower portion of the heat exchanger is immersed in a geothermal reservoir from where heat is extracted. This portion of the exchanger determines how much heat is transmitted. Overall heat transfer coefficient, k_z^* at the outside surface and k at the inside surface, are calculated from known classical formulae. The shallower portion of an exchanger is located in caprock with a linearly increasing temperature with depth. Temperature in the geothermal reservoir also increases with depth. The amount of transmitted heat between the water and the formation is estimated using overall heat transfer coefficient k_z, which changes with time. Values of the overall heat transfer coefficient may be estimated based on formulae presented by Diad'kin and Giendler, 1985.

Consider the Field exchanger divided into two portions, according to different conditions of heat exchange on the outside surface. The first part for $0 \le X \le X_1 = L/\left(L+L^*\right)$ is located in impervious rock (see Figure 3). The other portion, $X_1 \le X \le 1$, is immersed in a geothermal reservoir. The fluid's thermal field is shown in Figure 3. The following relations are introduced: $\dot{W} = \dot{m} c_p$, $K = k\pi D_2\left(L+L^*\right)$, $K_z = k_z \pi D_1\left(L+L^*\right)$, $X = x/\left(L+L^*\right)$, where $\dot{W}$ – thermal capacity of heat flow [W/K], c_p – specific heat at constant pressure [J/(kgK)], $\dot{m}$ – mass flow rate [kg/s], x – co-ordinate [m], X – reduced co-ordinate, D_1, D_2 – diameter [m].

Based on the balance equations of the first part of the exchanger for fluid flow in an annular space and an inner liner, when the equations of heat exchange are taken into account and introducing $\Theta_1 = T_1 - T_x$, $\Theta_2 = T_2 - T_x$, $a = K/\dot{W}$, $b = \left(K + K_z\right)/\dot{W}$, we obtain the following system of differential first-order equations:

$$\mathrm{d}\Theta_1/\mathrm{d}X = a\Theta_2 - b\Theta_1 - E\,, \qquad (1)$$

$$\mathrm{d}\Theta_2/\mathrm{d}X = a\Theta_2 - a\Theta_1 - E\,. \qquad (2)$$

for $0 \le X \le X_1$, where T_1, T_2 – temperature of input fluid and output fluid, T_x – temperature of the caprock.

The above equations have been solved using d'Alambert's method (Lykow and Michajlow, 1963) and can be written as follows (Kujawa and Nowak, 1998):

$$\Theta_1 = -\frac{1}{R}\left[\frac{C_2}{q_2}\exp\left(\mathbf{n}_2^2 X\right) - \frac{C_1}{q_1}\exp\left(\mathbf{n}_1^2 X\right)\right], \qquad (3)$$

$$\Theta_2 = \frac{1}{R}\left[\frac{C_2}{p_2}\exp\left(\mathbf{n}_2^2 X\right) - \frac{C_1}{p_1}\exp\left(\mathbf{n}_1^2 X\right) + \frac{R}{a}E\right], \qquad (4)$$

where

$$\mathbf{n}_i^2 = 0.5\left(K_z/\dot{W}\right)\left(-1 \pm \sqrt{1+4\,K/K_z}\right), \left(p_i/q_i\right) = \left(\mathbf{n}_i^2 - a\right)/a\ ,$$

$$R = \sqrt{\left(K_z/K\right)^2 + 4\left(K_z/K\right)}\ ,\ \ \left(q_i/p_i\right) = -\left(\mathbf{n}_i^2 + b\right)/a\ .$$

Based on relations (1) and (2) and introducing $\Theta_1^* = T_1^* - T_x$, $K_z^* = k_z^* \mathbf{p}\, D_1\left(L + L^*\right)$, $\Theta_2^* = T_2^* - T_x$, $b^* = \left(K + K_z^*\right)/\dot{W}$, the equations describing heat exchange in the second part of the exchanger may be written as follows:

$$\Theta_1^* = -\frac{1}{R^*}\left[\frac{C_2^*}{q_2^*}\exp\left(\mathbf{n}_2^{*2} X\right) - \frac{C_1^*}{q_1^*}\exp\left(\mathbf{n}_1^{*2} X\right)\right], \qquad (5)$$

$$\Theta_2^* = \frac{1}{R^*}\left[\frac{C_2^*}{p_2^*}\exp\left(\mathbf{n}_2^{*2} X\right) - \frac{C_1^*}{p_1^*}\exp\left(\mathbf{n}_1^{*2} X\right) + \frac{R^*}{a}E\right], \qquad (6)$$

for $X_1 \le X \le 1$ where:

$$\mathbf{n}_i^{*2} = 0.5\left(K_z^*/\dot{W}\right)\left(-1 \pm \sqrt{1 + 4\,K/K_z^*}\right), \left(p_i^*/q_i^*\right) = \left(\mathbf{n}_i^{*2} - a\right)/a$$

$$R^* = \sqrt{\left(K_z^*/K\right)^2 + 4\left(K_z^*/K\right)}\ ,\ \left(q_i^*/p_i^*\right) = -\left(\mathbf{n}_i^{*2} + b\right)/a\ .$$

To obtain an explicit description of the thermal field in both parts of the exchanger, it is necessary to determine the four integration constants occurring in (3), (4), (5) and (6), using four boundary conditions:

$$\Theta_1\left(X=0\right) = \Theta_1'\ ,\quad \Theta_1\left(X = X_1\right) = \Theta_1^*\left(X = X_1\right),$$

$$\Theta_2\left(X = X_1\right) = \Theta_2^*\left(X = X_1\right),\ \ \Theta_2^*\left(X=1\right) = \Theta_1^*\left(X=1\right)$$

The method of successive approximation requires an initial assumption of the values of temperature difference $\Theta_2^*\left(X=1\right) > \Theta_2'^*$ for the first calculation cycle. In the following cycles, the numerical value $\Theta_2^*\left(X=1\right)$ is corrected so that in the end $\Theta_1^*\left(X=1\right) = \Theta_2^*\left(X=1\right) = \Theta_2'^* = \Theta_1''^*$. Using the above mentioned boundary conditions, taking into account functions determined by relations (3), (4), (5) and (6), introducing signs given in Table 1 and solving four algebraic equations, integration constants may be written as follows:

$$\frac{C_2}{q_2} = \frac{g - \dfrac{p_2^*}{q_2^*}\dfrac{b}{b_1}g_1}{a - \dfrac{q_2}{p_2}\dfrac{p_2^*}{q_2^*}\dfrac{b}{b_1}a_1}\ ,\qquad \frac{C_2^*}{q_2^*} = \frac{\dfrac{p_2}{q_2}\dfrac{a}{a_1}g_1 - g}{rb - \dfrac{p_2}{q_2}\dfrac{q_2^*}{p_2^*}\dfrac{a}{a_1}rb_1}\ ,$$

$$C_1/q_1 = R\Theta_1' + C_2/q_2\ ,$$

$$\frac{C_1^*}{q_1^*} = \frac{p_1^*}{q_1^*}\left[\begin{array}{r}\left(C_2^*/p_2^*\right)\exp\left(\mathbf{n}_2^{*2} - \mathbf{n}_1^{*2}\right) + z^*\exp\left(-\mathbf{n}_1^{*2}\right) - \\ - R^*\Theta_2'^*\exp\left(-\mathbf{n}_1^{*2}\right)\end{array}\right].$$

The integration constants allow us to determine the thermal fields for both parts of the heat exchanger. Temperature of the output of a geothermal exchanger is determined by the following relation:

$$\Theta_2\left(X=0\right) = \left(C_2/p_2 - C_1/p_1 + RE/a\right)/R\ . \qquad (7)$$

A flow of heat collected by a Field heat exchanger is determined by the relation:

$$\dot{Q} = \dot{W}\left[\Theta_2\left(X=0\right) - \Theta_1\left(X=0\right)\right] = \dot{W}\left(\Theta_2'' - \Theta_1'\right). \qquad (8)$$

3. MATHEMATICAL MODEL: VERTICAL GEOTHERMAL HEAT EXCHANGER DOUBLE-PIPE TYPE

A vertical geothermal heat exchanger consisting of a spiral tube immersed in a geothermal reservoir and a double-pipe exchanger located in an impervious caprock is shown in Figure 2. The top part of the exchanger is insulated. Fluid is injected through the annulus. Initially, heat is drawn from the formation and then fluid flowing through a spiral-tube exchanger draws heat from the geothermal reservoir. Next warmed fluid is pumped through the insulated inner production liner and is pumped out to the earth's surface. To determine the difference in fluid temperature on the earth surface (between input and output) the heat exchange function with depth for both kinds of exchanger should be calculated.

The temperature field of a fluid (is presented in Figure 4) for both heat exchangers. For a double-pipe exchanger with boundary conditions

$$\Theta_1\left(X=0\right) = \Theta_1'\ , \qquad (9)$$

$$\Theta_2\left(X=1\right) = \Theta_2' \qquad (10)$$

difference of temperature $\Theta_1\left(X=1\right) = \Theta_1''$ and $\Theta_2\left(X=0\right) = \Theta_2''$ can be calculated. Temperatures T_1'' and T_2'' can also be calculated. Based on a simple computational model (Kujawa and Nowak, 1998), temperature on the output of a spiral tube heat exchanger is determined by the relation:

$$\Theta_1''^* = \dot{W}E^*/K_s^* + \left(\Theta_1'^* - \dot{W}E^*/K_s^*\right)e^{-\left(K_s^*/\dot{W}E^*\right)}, \qquad (11)$$

$$T_1''^* = \left(E^* + F^*\right) - \Theta_1''^*\ , \qquad (12)$$

or:

$$T_1'''^{*} = \left(E^* + F^*\right) - \dot{W}E^*/K_s^* - \left(\Theta_1''^{*} - \dot{W}E^*/K_s^*\right)e^{-\left(K_s^*/\dot{W}E^*\right)}$$

To calculate the total heat exchanged, the method of successive approximations was used. Assuming $\Theta_2' = \Theta_2(X=1)$ and $\Theta_1' = \Theta_1(X=1)$ for the first calculation cycle, $\Theta_1'' = \Theta_1(X=1)$ is determined from relation (3). Given that $\Theta_1'' = \Theta_1'^{*}\left(T_1'' = T_1'^{*}\right)$, equations (11) and (12) let us calculate temperature $T_1'''^{*}$. In subsequent calculation cycles, the numerical value $T_2'(\Theta_2')$ is corrected by assuming $T_1'^{*} = T_1''$. The flow of heat drawn by the exchanger is determined by relation (8).

4. CALCULATION RESULTS

On the basis of the presented mathematical model of heat exchange, we derived a computational program to calculate the temperature field and heat flux within a geothermal reservoir using the following assumptions:

- total depth, L_c, is equal to the sum of the thickness of a rock layer L and the thickness of a geothermal layer L^*, $L_c = L + L^* =$ 1600 m,
- the temperature of geothermal water at 1600 m is 64°C,
- the temperature at the surface is 10°C,
- the temperature of geothermal water injected from the surface to a deposit is 10°C, 15°C and 20°C,
- thermal capacity of heat flow $\dot{W} =$ 30000 W/K ($\dot{m} = 25.78$ m^3/h) and $\dot{W} =$ 300000 W/K ($\dot{m} = 257.76$ m^3/h),
- inside diameter of the outer pipe is $D_1 = 0.40$ m,
- outside diameter of the inner pipe is $D_2 = 0.25$ m,
- the ratio of heat transfer coefficient $k_z^*/k =$ 0.1; 1; 3; 5; 10; 20 for chosen quantities k and k_z,
- the thickness of the geothermal layer $L^* = 200$ m,
- the insulation length changes from $L_o = 100 \rightarrow 1200$ m,
- the relative thickness of the geothermal layer varies from $L^*/\left[(L - L_o) + L^*\right] = 0.133 \rightarrow 0.5$.

The calculated results are presented in Figures 4 and 5.

5. DISCUSSION

Modelling results (Fig. 5 and Fig. 6) suggest that the flow of heat depends on the temperature of injected water, the length of the impervious caprock and the thickness of the geothermal reservoir penetration. Insulation is useful only to a certain depth, after which , the flow of heat decreases and insulation does not provide a needed effect.

Comparing these results with those for identical models without insulation (presented in publications – Kujawa *et al.*, 1999, Kujawa and Nowak, 1998) it is found that using insulation causes the efficiency of one-hole systems to increase – the flow of heat is greater.

6. CONCLUSIONS

In a one-hole system with an inner production liner, the heat flux $\dot{Q}_{n=1}$ is smaller than for a two-hole system operating under similar conditions. The temperature of water returned to the earth's surface for a one-hole system is always lower than a two-hole installation. From modelled observations $\dot{Q}_{n=1} < 0.6\dot{Q}_{n=2}$. The temperature of injection water, the thickness of the geothermal layer and the heat transfer coefficients k_z^*/k all influence the quantity of input heat flow, for chosen quantities k and k_z. There exists a boundary relation $\left(k_z^*/k\right)_{gr}$ for which the flow of heat does not depend on the thickness of the geothermal layer. The quantity $\left(k_z^*/k\right)_{gr}$ decreases as the thermal capacity of the fluid increases. If $\left(k_z^*/k\right) < \left(k_z^*/k\right)_{gr}$ an increase in the thickness of the geothermal layer causes a drop in heat flow. But if $\left(k_z^*/k\right) > \left(k_z^*/k\right)_{gr}$, increasing the thickness of the geothermal layer causes an increase in the quantity of heat flow. In this case the higher increment is in the zone of the relatively thin geothermal reservoir penetration. Reducing the heat transfer coefficients k and k_z has a positive impact on the heat flow.

The computational models of geothermal heat exchangers presented allow one to choose, at an early project stage, the most effective solution for imposed geothermal conditions.

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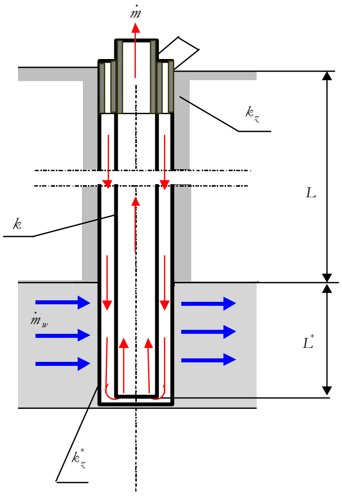


Fig. 1. The scheme of vertical heat exchanger Fields type

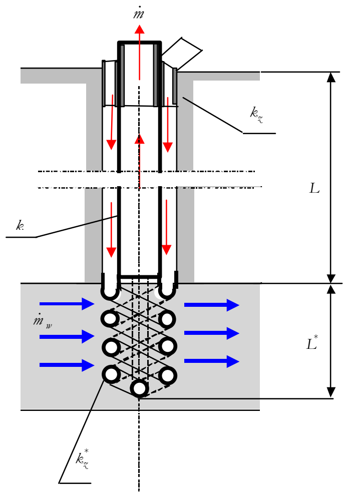


Fig. 2. The scheme of the double-pipe heat exchanger with spiral-tube heat exchanger

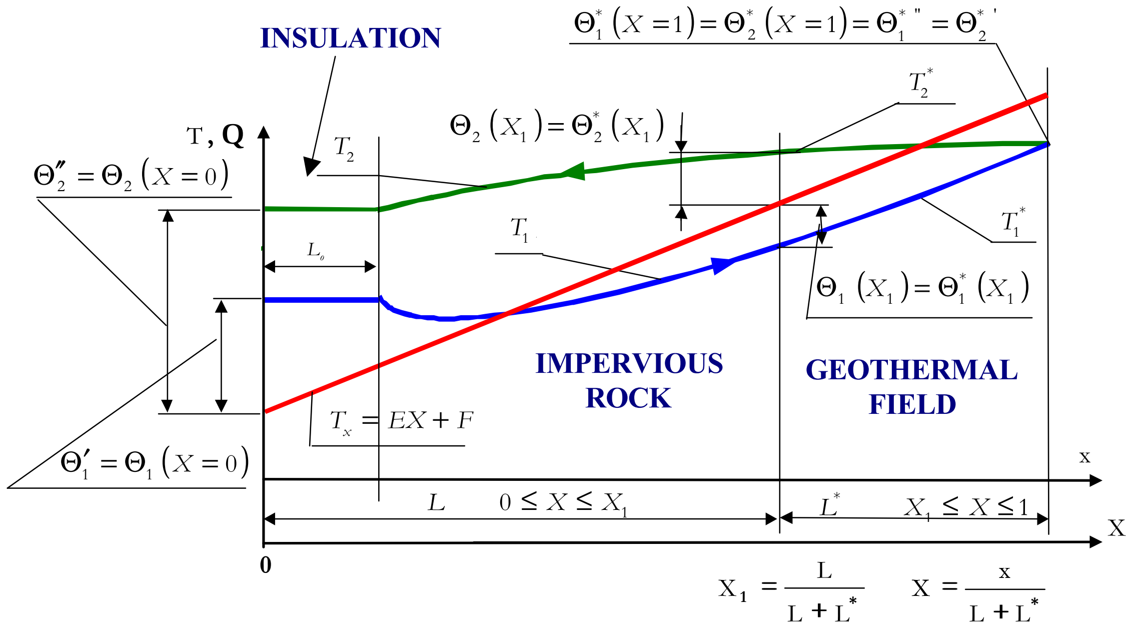


Fig. 3. The temperature field of a fluid with Field exchanger

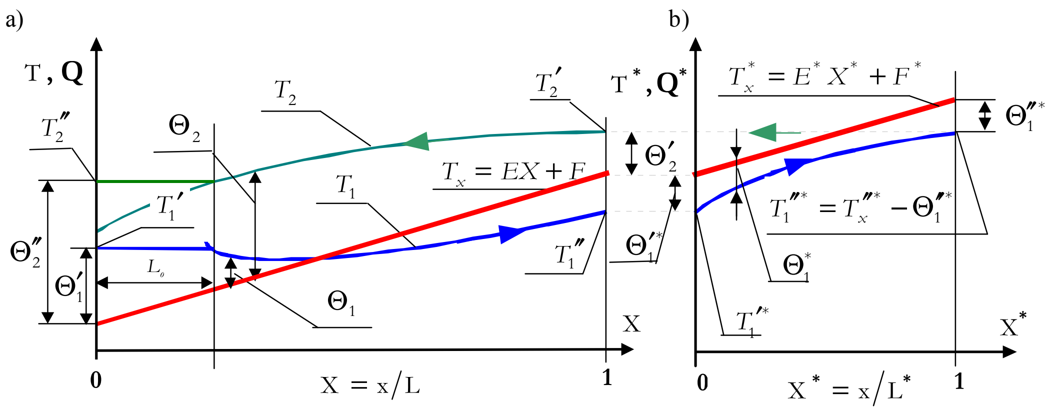
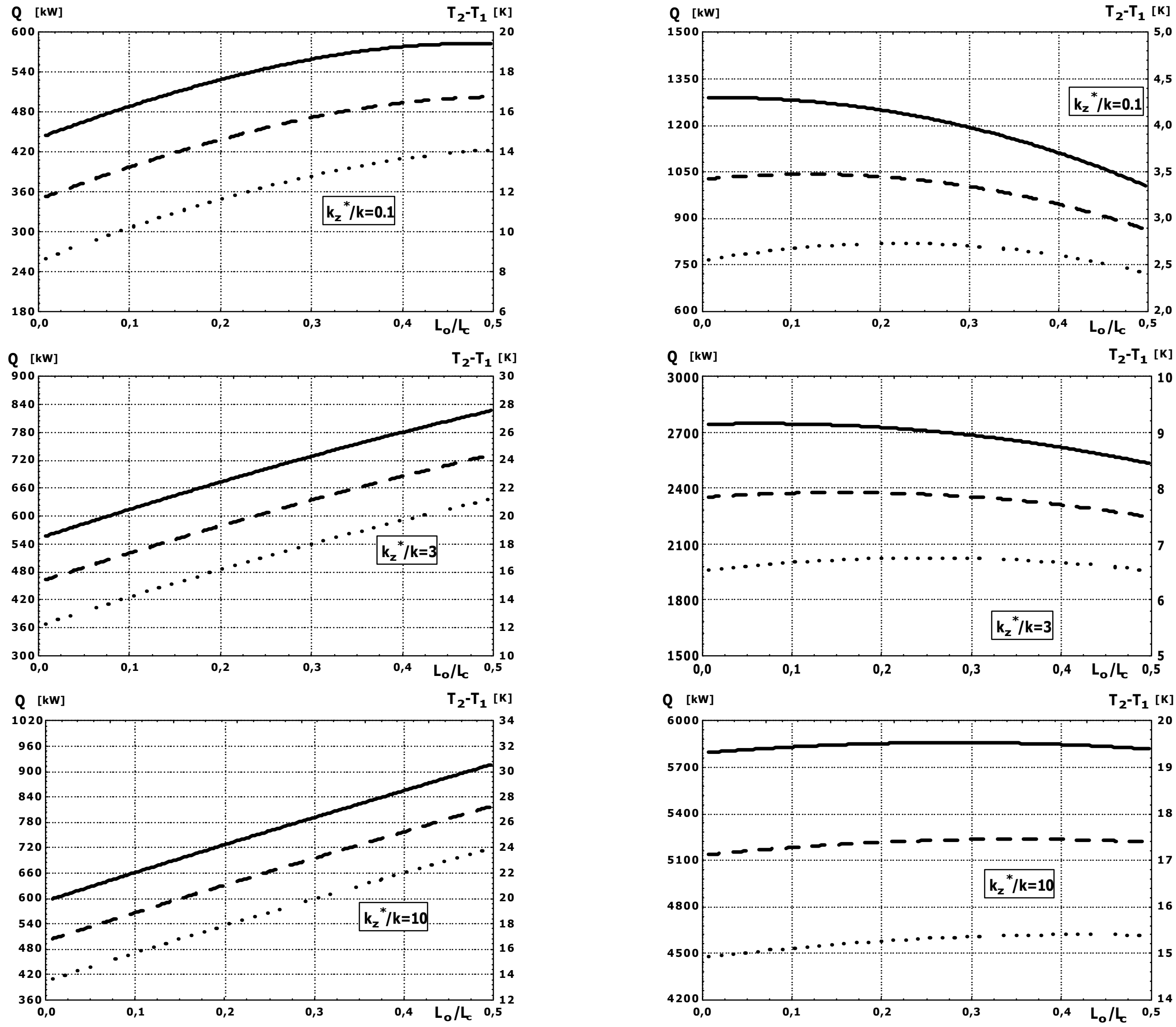


Fig. 4. The temperature field of a fluid with Field exchanger for vertical probe, presented in Figure 2: a) double-pipe heat exchanger, b) spiral-tube heat exchanger

Table 1. Auxiliary quantities

Auxiliary quantity	Expression
a	$\exp\left(n_2^2 X_1\right) - \exp\left(n_1^2 X_1\right)$
a_1	$\exp\left(n_2^2 X_1\right) - \left(q_1 p_2 / p_1 q_2\right)\exp\left(n_1^2 X_1\right)$
b	$\exp\left(n_2^{*2} X_1\right) - \left(q_2^* p_1^* / p_2^* q_1^*\right)\exp\left(n_1^{*2} X_1 + n_2^{*2} - n_1^{*2}\right)$
b_1	$\exp\left(n_2^{*2} X_1\right) - \exp\left(n_1^{*2} X_1 + n_2^{*2} - n_1^{*2}\right)$
g	$R\Theta_1' \exp\left(n_1^2 X_1\right) - r\left(p_1^* / q_1^*\right) z^* \exp\left(n_1^{*2} X_1 - n_1^{*2}\right) + r\left(p_1^* / q_1^*\right) R^* \Theta_2'^* \exp\left(n_1^{*2} X_1 - n_1^{*2}\right)$
g_1	$rR^* \Theta_2'^* \exp\left(n_1^{*2} X_1 - n_1^{*2}\right) + \left(q_1 / p_1\right) R\Theta_1' \exp\left(n_1^2 X_1\right) - rz^* \exp\left(n_1^{*2} X_1 - n_1^{*2}\right) + rz^* - z$
where: $r = R/R^*$; $z = (RE)/a$; $z^* = (R^* E)/a$	

a) $\dot{W} = 30\ 000$ W/K ($\dot{m} = 25.78$ m^3/h)   b) $\dot{W} = 300\ 000$ W/K ($\dot{m} = 257.76$ m^3/h)

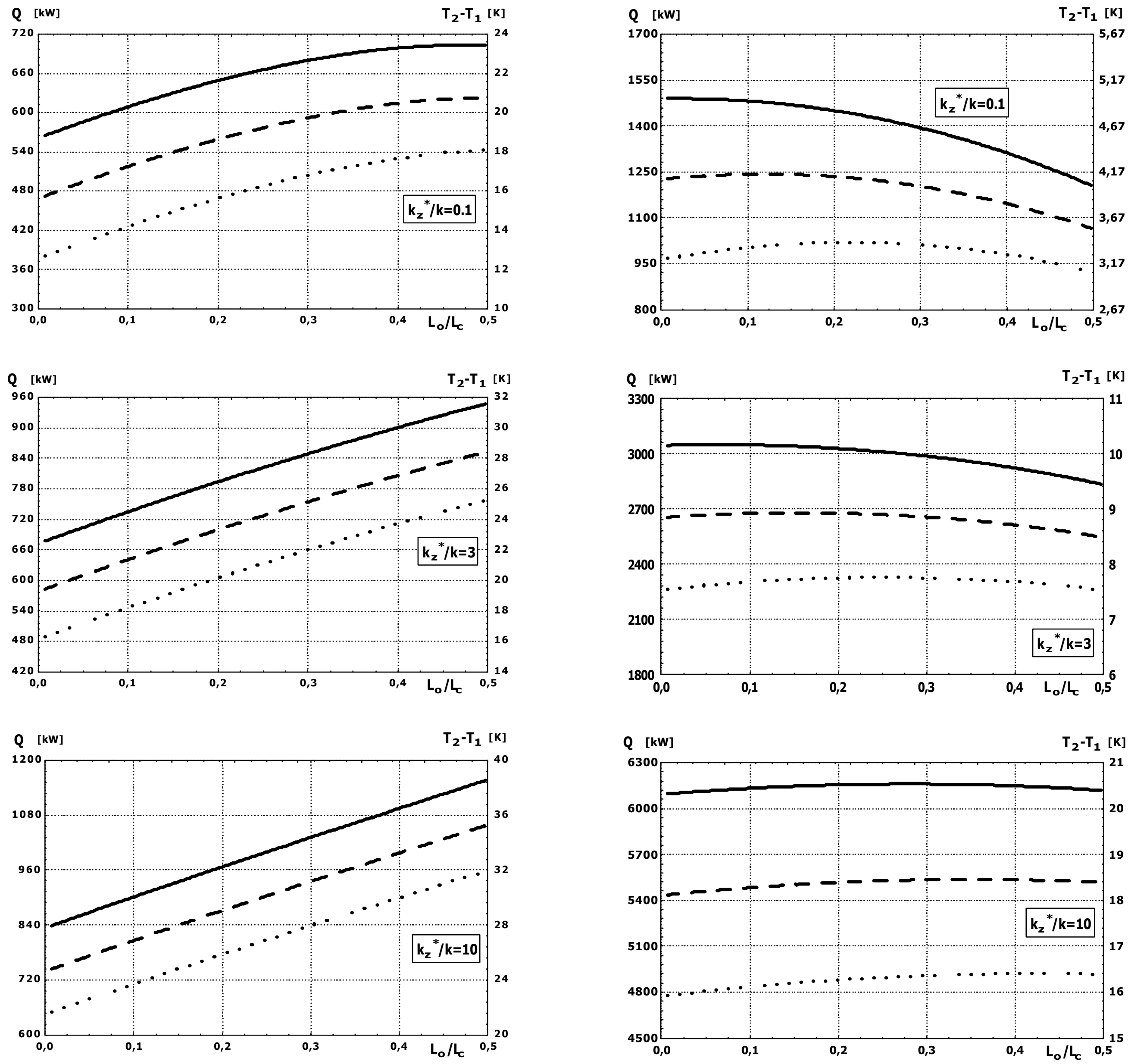


Temperature of geothermal water injected from the surface to a deposit:

for $\Theta_1' = 0$ K, for $\Theta_1' = 5$ K, for $\Theta_1' = 10$ K

Fig. 5. Heat flow in a geothermal deposit and temperature field within fluid as a function of the relative thickness of geothermal layer (heat exchanger Fields type)

a) $\dot{W} = 30\ 000$ W/K ($\dot{m} = 25.78$ m^3/h) b) $\dot{W} = 300\ 000$ W/K ($\dot{m} = 257.76$ m^3/h)



Temperature of geothermal water injected from the surface to a deposit:

for $\Theta_1' = 0$ K, for $\Theta_1' = 5$ K, for $\Theta_1' = 10$ K

Fig. 6. Heat flow in a geothermal deposit and temperature field within fluid as a function of the relative thickness of geothermal layer (double-pipe heat exchanger with spiral-tube heat exchanger)