Proceedings World Geothermal Congress 2000
Kyushu - Tohoku, Japan, May 28 - June 10, 2000

SHALLOW AND DEEP VERTICAL GEOTHERMAL HEAT EXCHANGERS AS LOW TEMPERATURE SOURCES FOR HEAT PUMPS

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Key Words: shallow vertical heat exchangers, deep vertical heat exchangers, heat energy, heat pump, resource temperature

ABSTRACT

The authors present mathematical models of shallow and deep vertical heat exchangers. Based on the formulated mathematical models, the effects of: 1) fluid mass flow rate, 2) the temperature of a resource, and 3) the geometry of a heat exchanger are determined both on the temperature of circulating water at the outlet of the heat exchanger and on the amount of heat energy extracted from the ground as well as the amount of heat energy available at the surface.

1. INTRODUCTION

Geothermal energy is used, among other things, as a heat source for ground-coupled heat pumps. When horizontal heat exchangers shallowly situated under ground and vertical heat exchangers of rather small depth are used, geothermal energy extracted for small heat load applications (mostly one-family houses). Increasing the vertical heat exchanger depth makes it possible not only to extract more heat but also to further raise the temperature of the circulating fluid. The depth setting highly influences the choice of a heat pump and its efficiency. Accessible publications do not provide simple methods of performance calculations for vertical heat exchangers. Therefore, the authors developed simplified mathematical models for both shallow and deep vertical heat exchangers (Fig. 1 and Fig. 6).

In the mathematical models of vertical heat exchangers, the relationship between overall heat transfer coefficient and characteristics of the resource were used.

Based on the formulated mathematical models, the effects of heat transfer fluid mass flow rate, the temperature of the deposit, and the geometry of the heat exchanger are determined on the temperature of the circulating water at the outlet of the heat exchanger and the amount heat energy extracted from the ground and available at the surface. Using the heat exchanger as a low temperature heat source, the models were used to develop a system consisting of a vertical heat exchanger and a heat pump for heating. When deep vertical heat exchangers are used, it is possible to extract more geothermal energy and supply a larger heat load. For this case, the depth of the heat exchanger and the effect of insulating the top of the production string were determined relative to the choice of heat pump, including consideration of the amount of heat that the load requires.

2. SHALLOW VERTICAL HEAT EXCHANGERS

2.1 A shallow vertical heat exchanger with concentric flow

The principle of operation of a geothermal heat exchanger with concentric flow is presented in Figure 1a. In this case, the fluid drawing heat from the deposit flows through the annulus of concentric pipes. Warm fluid returns to the surface through the inner pipe. Figure 2 defines the variables used in the model of the heat exchanger, the temperature profiles of the fluid, and a differential element of the heat exchanger.
The balance equations for fluid in the annulus when the equations of heat exchange are taken into consideration, can be written as follows:

$$\dot{W}\,\mathrm{d}T_1 = K_w\left(T_2 - T_1\right)\mathrm{d}X + K_z\left(T_o - T_1\right)\mathrm{d}X\,, \qquad (1)$$

$$\dot{W}\,\mathrm{d}T_2 = K_w\left(T_2 - T_1\right)\mathrm{d}X\,, \qquad (2)$$

where: $X = x/L_o$, $\dot{W} = \dot{m}c_p$, $K_z = k_z\pi D_z L_o$, $K_w = k_w\pi D_w L_o$, $\dot{W}$ – thermal capacity of heat flow [W/K], c_p – specific heat at constant pressure [J/(kgK)], $\dot{m}$ – mass flow rate [kg/s], L_o – length of exchanger [m], x – co-ordinate [m], X – reduced co-ordinate, D_z, D_w – diameter [m], T_1, T_2 – temperature of input fluid and output fluid, T_o – temperature of rock massif, k_z – substitute overall heat transfer coefficient [W/(m^{2}K)], k_w – overall heat transfer coefficient with inside surface [W/(m^{2}K)].
Introducing additional relations

$$\Theta_1 = T_o - T_1\,,\ \Theta_2 = T_o - T_2\,,\ a = K_w/\dot{W}\,,\ b = \left(K_w + K_z\right)/\dot{W}\,,$$

where: Θ – difference of temperature [K], a, b – constants, equations (1) and (2) can be expressed by:

$$\mathrm{d}\Theta_1/\mathrm{d}X = a\Theta_2 - b\Theta_1\,, \qquad (3)$$

$$\mathrm{d}\Theta_2/\mathrm{d}X = a\Theta_2 - a\Theta_1\,. \qquad (4)$$

This system of equations is solved using d'Alambert's method (Lykow and Michajlow, 1963). Multiplying (3) by p and (4) by q , to obtain after rearranging

$$\mathrm{d}\left(p\Theta_1 + q\Theta_2\right)/\mathrm{d}X = \left(pb + qa\right)\Theta_1 + a\left(p + q\right)\Theta_2\,. \qquad (5)$$

If the condition is fulfilled

$$-\left(pb + qa\right)/p = a\left(p + q\right)/q = n^2\,, \qquad (6)$$

the relation (5) can be written as

$$\mathrm{d}\left(p\Theta_1 + q\Theta_2\right)/\mathrm{d}X = n^2\left(p\Theta_1 + q\Theta_2\right)\,. \qquad (7)$$

After introducing a new function

$$Z_i = p_i\Theta_1 + q_i\Theta_2 \quad (i=1,2), \tag{8}$$

the relation (7) will be formed in a linear equation

$$\mathrm{d}Z_i/\mathrm{d}X = n_i^2 Z_i \quad (i=1,2), \tag{9}$$

for which the following functions are the solution

$$Z_i = C_i \exp\left(n_i^2 X\right) \quad (i=1,2). \tag{10}$$

From (6) we derive that n_i^2 is an algebraic root of

$$n_i^4 + (b-a)n_i^2 - a(b-a) = 0 \tag{11}$$

and given by

$$n_i^2 = 0.5\left(K_z/\dot{W}\right)\left(-1 \pm \sqrt{1+4K_w/K_z}\right) \quad (i=1,2). \tag{12}$$

From (6) we get the other expressions:

$$(p_i/q_i) = \left(n_i^2 - a\right)/a \quad (i=1,2), \tag{13}$$

$$(q_i/p_i) = -\left(n_i^2 + b\right)/a \quad (i=1,2). \tag{14}$$

From relations (8) and (10), after the appropriate transformation and introducing the new integration constants C_1^* and C_2^* functions Θ_1 and Θ_2 will be formed into:

$$\Theta_1 = C_1^* \exp\left(n_1^2 X\right) - C_2^* \exp\left(n_2^2 X\right), \tag{15}$$

$$\Theta_2 = C_2^*(q_2/p_2)\exp\left(n_2^2 X\right) - C_1^*(q_1/p_1)\exp\left(n_1^2 X\right), \tag{16}$$

Based on the boundary conditions

$$\Theta_1(X=0) = \Theta_1', \tag{17}$$

$$\Theta_1(X=1) = \Theta_2(X=1), \tag{18}$$

the integration constants C_1^* and C_2^* have been determined as follows

$$C_2^* = \frac{\Theta_1'(1+q_1/p_1)\exp\left(n_1^2\right)}{-(1+q_1/p_1)\exp\left(n_1^2\right) + (1+q_2/p_2)\exp\left(n_2^2\right)}, \tag{19}$$

$$C_1^* = \Theta_1' + C_2^*. \tag{20}$$

Based on the following relationships, (15), (16), (19) and (20) the values of temperature difference $\Theta_2(X=0) = \Theta_2''$ and a heat flow collected by the fluid from a resource can be calculated from the relation:

$$\dot{Q} = \dot{W}\left(T_2'' - T_1'\right) = \dot{W}\left(\Theta_1' - \Theta_2''\right) \quad \text{[W]}. \tag{21}$$

2.2. A shallow vertical heat exchanger with a counter-flow

The principle of operation of a shallow geothermal heat exchanger with counter current flow is presented in Figure 1b. In this case, fluid flows in both directions through half-circular cross-sections and draws heat from a deposit in both flow directions. Heat is transferred between the entering heating flows through a flat dam with surface area DL_o.

Using a similar method as in the previous variant and based on balance and heat transmission equations, the system of equations can be written as follows:

$$\mathrm{d}\Theta_1/\mathrm{d}X = a^*\Theta_2 - b^*\Theta_1, \tag{22}$$

$$\mathrm{d}\Theta_2/\mathrm{d}X = b^*\Theta_2 - a^*\Theta_1, \tag{23}$$

where: $X = x/L_o$, $\dot{W} = \dot{m}c_p$, $K_z^* = 0{,}5\, k_z \pi D L_o$, $K_w^* = k_w DL_o$, $a^* = K_w^*/\dot{W}$.

The general solution of the system of equations (22) and (23) is the same as the solution of (3) and (4) and it is determined by the relations (15 through 20). The difference is in the different method of calculating the following relations:

$$n_i^{*2} = \pm\left(K_z^*/\dot{W}\right)\sqrt{1+4\left(K_w^*/K_z^*\right)} \quad (i=1,2). \tag{25}$$

$$\left(p_i^*/q_i^*\right) = \left(n_i^{*2} - b^*\right)/a^* \quad (i=1,2), \tag{26}$$

$$\left(q_i^*/p_i^*\right) = -\left(n_i^{*2} + b^*\right)/a^* \quad (i=1,2). \tag{27}$$

Based on the relations above, universal graphs for both variants were made (Fig. 3 and Fig. 4). Their form, $\Theta_2''/\Theta_1' = f\left(K_w/K_z, K_z/\dot{W}\right)$ or $\Theta_2''/\Theta_1' = f\left(K_w^*/K_z^*, K_z^*/\dot{W}\right)$, allows one to determine the net heat flow into the fluid according to the formula, $\dot{Q} = \dot{W}\Theta_1'\left(1 - \Theta_2''/\Theta_1'\right)$. K_w/K_z or K_w^*/K_z^* has evident influence on Θ_2''/Θ_1' at higher values of $K_z/\dot{W}$ $\left(K_z^*/\dot{W}\right)$. The most advantageous case, when $K_w = 0$ $\left(K_w^* = 0\right)$, in the form of $\dot{Q} = f\left(\dot{W}, K_z\right)$ for a chosen difference of temperature, is presented in a graph (Figure 5) for both variants.

3. A DEEP GEOTHERMAL VERTICAL HEAT EXCHANGER FIELD TYPE

In this section, an approximate computational model of the geothermal Field type heat exchanger is presented. The exchanger is located in an impervious rock massif with temperature increasing linearly with depth. The principle of operation is shown in Figure 6. To determine the quantity of heat transmitted between the rock massif and water flowing in the annulus, a substitute overall heat transfer coefficient k_z is used. The value of the coefficient can change in time. It can be estimated based on formulas presented by Diad'kin and Giendler, 1985. The centre pipe is insulated to minimise heat transfer from the fluid in the annulus.

Figure 7 presents the thermal field of fluid and rock massif for the ground exchanger presented in Figure 6. In this variant it is assumed that the temperature of the water entering the heat exchanger is higher than the temperature of the deposit at the depth $x=0$. As a result, fluid in the first phase of its flow in the heat exchanger transfers heat into the ground. After this first phase, heat is transferred into the fluid. Fluid reaches the initial temperature that it had upon first entering the heat exchanger after flowing a distance x_o^*. If conditions allow it, fluid reaches temperature T''. On the basis of elementary balance equations of a part of an exchanger of length $\mathrm{d}x$, and taking into account equations of heat exchange, the equation describing the fluid temperature is as follows:

$$\dot{m} c_p T = k_z (T - T_x)\mathrm{d}A + \dot{m} c_p \left(T + \frac{\mathrm{d}T}{\mathrm{d}A}\mathrm{d}A \right), \tag{28}$$

where: $A = \pi DX$, $X \in (0, L)$.
After a series of transformations and introducing additional signs

$$x = X/L, \; \dot{W} = \dot{m} c_p, \; k_z^* = (k_z \pi DL)/\dot{W}$$

equation (1) is as follows:

$$\mathrm{d}T/\mathrm{d}x = -k_z^* (T - T_x). \tag{29}$$

A linear variation of deposit temperature is written in the following relation:

$$T_x = E^* x + F \tag{30}$$

where: $E^* = EL$.

The temperature difference between fluid and a deposit may be written as follows:

$$\Theta = T - T_x. \tag{31}$$

Replacing T_x in relation (31) with relation (30) and differentiating relation (31) in relation to x we obtain:

$$\mathrm{d}T/\mathrm{d}x = \mathrm{d}\Theta/\mathrm{d}x + E^*. \tag{32}$$

Substituting relation (29) into equation (32) we obtain the first-order differential equation:

$$\mathrm{d}\Theta/\mathrm{d}x + k_z^* \Theta + E^* = 0. \tag{33}$$

The solution is determined by the following relation:

$$\Theta = -E^*/k_z^* + C\mathrm{e}^{-k_z^* x} \tag{34}$$

Using the boundary condition $\Theta = \Theta'$ for $x=0$, the constant C is determined by the following relation:

$$C = \Theta' + E^*/k_z^*. \tag{35}$$

Substituting *C* from (35) into (34) we obtain:

$$\Theta = -E^*/k_z^* + (\Theta' + E^*/k_z^*)\mathrm{e}^{-k_z^* x} \tag{36}$$

The temperature of the water exiting the heat exchanger at the surface is calculated from the above relation: for $x=1$ the difference of temperature is $\Theta = \Theta''$. Relation (36) is as follows:

$$\Theta'' = -E^*/k_z^* + (\Theta' + E^*/k_z^*)\mathrm{e}^{-k_z^*} \tag{37}$$

or:

$$T'' = E^* + F - E^*/k_z^* + (\Theta' + E^*/k_z^*)\mathrm{e}^{-k_z^*}. \tag{38}$$

The flow of heat transmitted from a deposit is determined by the following relation:

$$\dot{Q} = \dot{W}(T'' - T') \quad \text{[W]}. \tag{39}$$

Using relations (28 through 38) and after a series of transformations it is possible to determine:

- length x_o at which the temperature of the fluid equals the temperature of the ground

$$x_o = \ln\left[(\Theta'/E^*)/(1/k_z^*) + 1\right]^{(1/k_z^*)}, \tag{40}$$

- length x_o^* at which the temperature of the fluid reaches the same value as at $x=0$

$$x_o^* = (\Theta'/E^* + 1/k_z^*)\left(1 - \mathrm{e}^{-k_z^* x_o^*}\right). \tag{41}$$

These derived relations were used to prepare the examples shown in Figs. 8-11. The Figures may be used to estimate heat exchanger performance given the conditions stated in the figures.

4. DISCUSSION

Figure 8 shows how the net heat transfer rate is changing with the change of parameter E^* (this parameter is responsible for the value of temperature of a resource at the depth of $x=0$, k_z^* and Θ'). When Θ' increases, the net heat transfer rate into the fluid decreases but heat transfer rate increases when parameter k_z^* increases. When Θ' equals 10K and 15K for certain field of E^* and k_z^*, the heat transfer rate has negative values. That means the temperature of water leaving the ground exchanger is lower than temperature of the water entering; heat is pumped into the ground instead of being drawn out.

Figure 9 presents how the amount of extracted heat varies. It also shows the change of fluid temperature as a function of k_z^* for chosen values of Θ' (at $E^* = 54$ °C and $\dot{W} = 30000$ W/K). Note that at steady Θ', increasing k_z^* above 20 does not result in further increases in heat output. Estimation of the flow of extracted heat (temperature on output of an exchanger) is possible after analysis: Figure 10 shows the variability of length x_o^* at which temperature of fluid reaches the same value as at $x=0$ and Fig. 11 shows the length x_o at which temperature of fluid equals temperature of ground. The smaller the values of x_o^* and x_o, the longer the path $(1-x_o^*)$ of extracting heat of water pumped from ground . When k_ζ^* is steady, the increase of relation Θ'/E^* disadvantageously influences the performance of a ground-source heat exchanger.

5. CONCLUSIONS

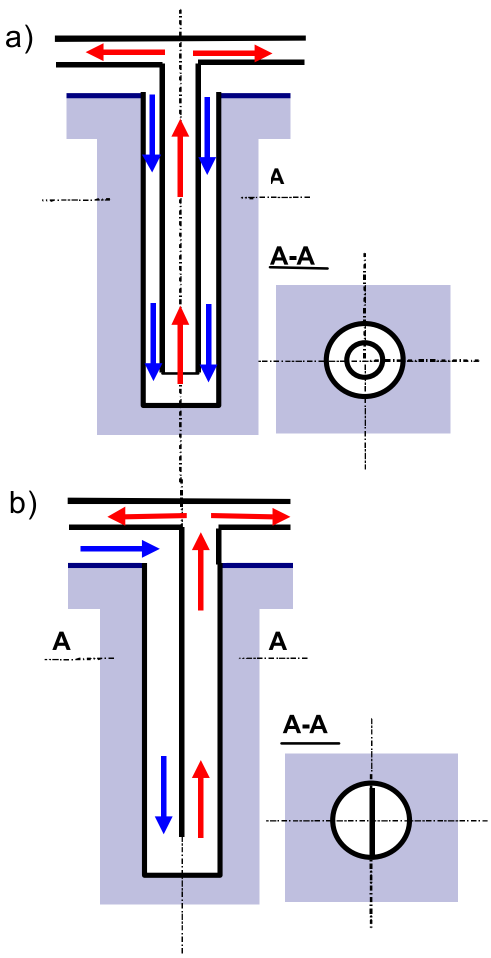

Fig. 1. Model of shallow vertical heat exchanger with: a) Concentric flow, b)counter-current flow.

This work presents two mathematical models of calculations for vertical heat exchangers. The models allow determination of the fluid's thermal field and the amount of extracted geothermal energy. When shallow vertical heat exchangers are used (up to 35 meters), one can use the models to calculate and draw graphs of estimates, of the change of fluid temperature and extracted heat energy for two types of heat exchangers: 1)a heat exchanger with concentric flow and 2) a heat exchanger with counter current flow. The formulated mathematical model of deep vertical heat exchangers allows determination of the fluid's thermal field and amount of drawn out geothermal energy. For each considered case calculations must be done for selected output parameters so that it would be possible to optimise the design of the heat exchanger. The model presented may also be used when the top part of a heat exchanger is insulated. The model is also useful to estimate accuracy of an approximate method of calculations for shallow heat exchangers as well.

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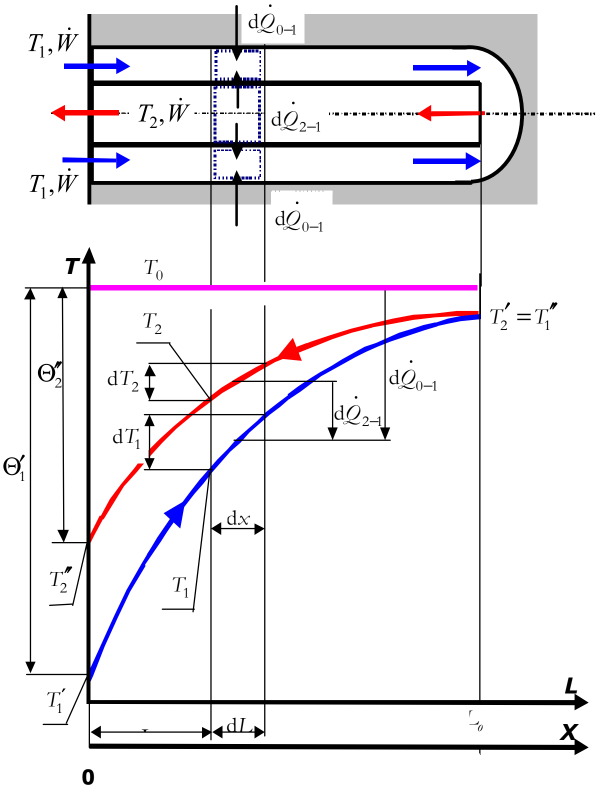

Fig. 2. Thermal field of a fluid in the considered mathematical model of a shallow vertical heat exchanger with concentric flow.

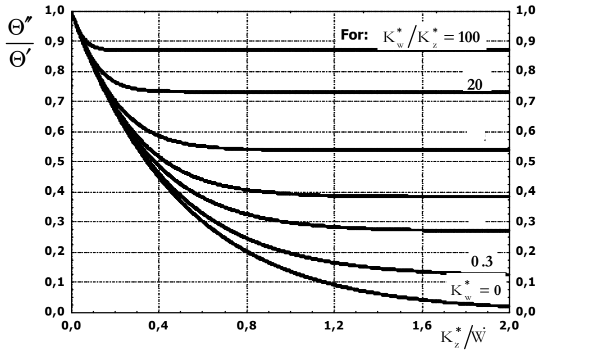

Fig. 3. $\Theta''_2/\Theta'_1 = f\left(K^*_w/K^*_z,\ K^*_z/\dot{W}\right)$ for a shallow vertical heat exchanger with a counter-flow

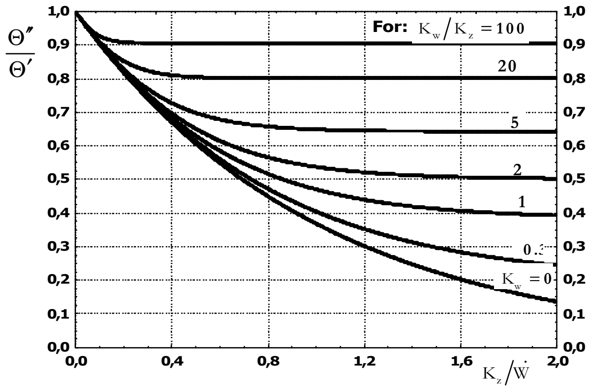

Fig. 4. $\Theta''_2/\Theta'_1 = f\left(K_w/K_z,\ K_z/\dot{W}\right)$ for a shallow vertical heat exchanger with a concentric flow

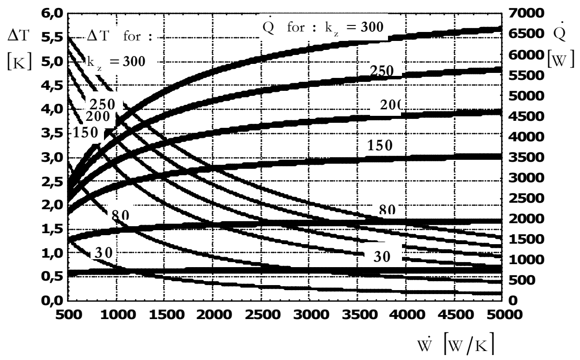

Fig. 5. $\Delta T, \dot{Q} = f\left(\dot{W},\ k_z\right)$ for both variants shallow vertical heat exchanger

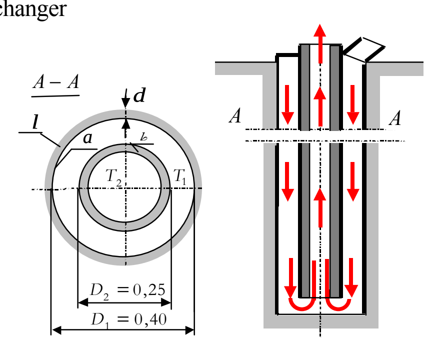

Fig. 6. Model of geothermal deep vertical heat exchanger Field type with insulation

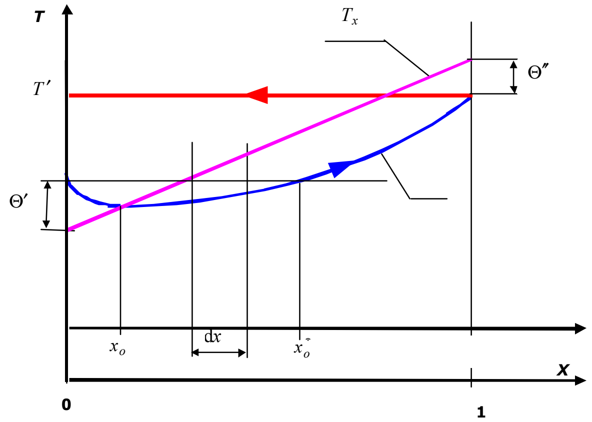

Fig. 7. Thermal field of a fluid in the considered mathematical model of a deep geothermal vertical heat exchanger Field type

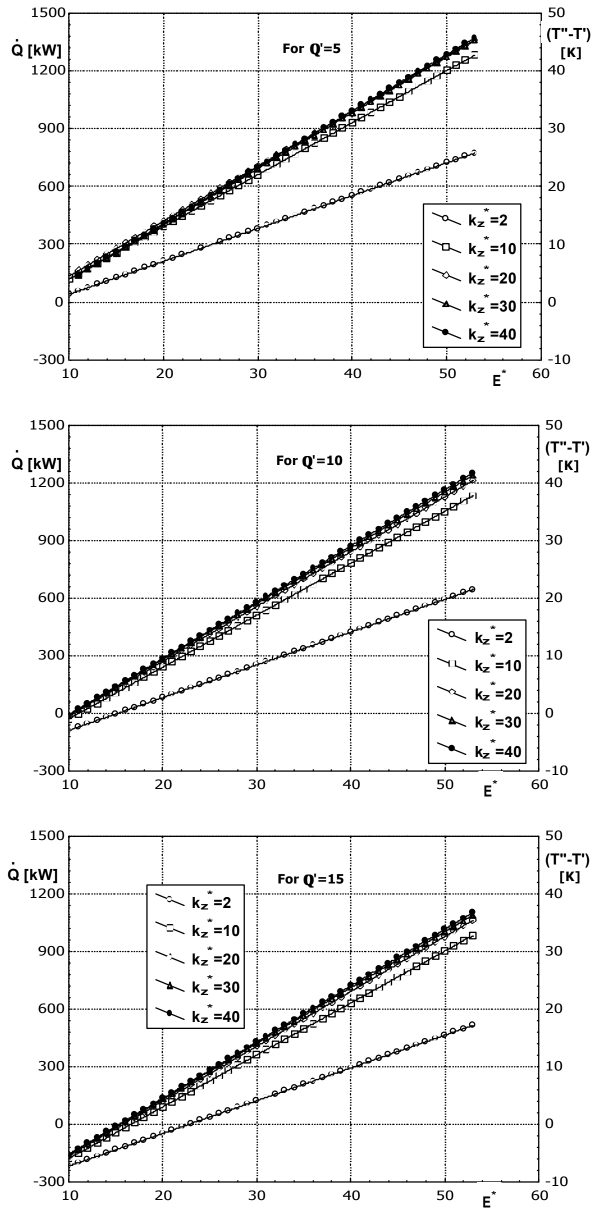

Fig. 8. Influence of parameter E^* on heat capacity of chosen model of exchanger Field type (for $\dot{W} = 30000$ W/K)

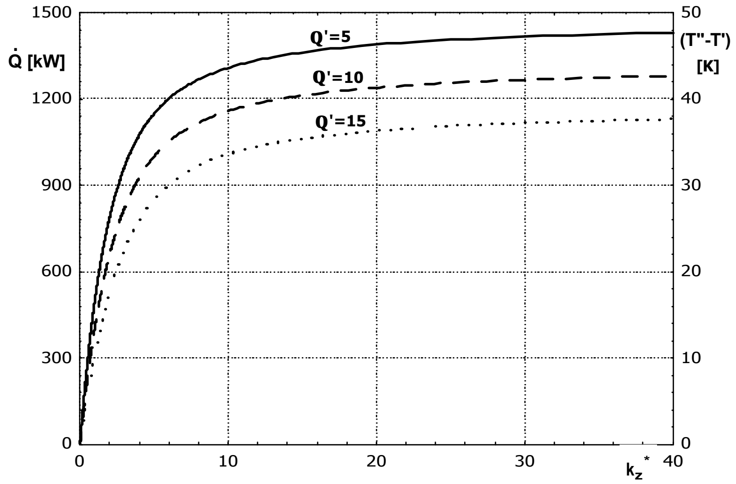

Fig. 9. Influence of parameter k_z^* on heat capacity of chosen model of exchanger Field type for chosen values Θ' ($\dot{W} = 30000$ W/K, $E^* = 54$ °C)

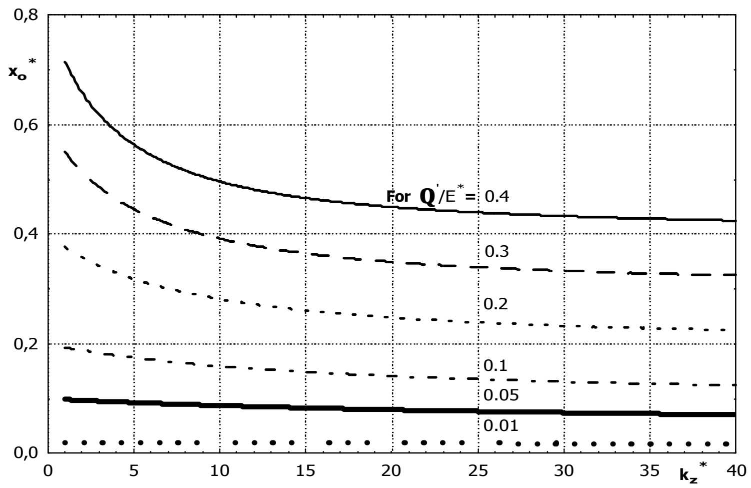

Fig. 10. Influence of parameter k_z^* on length x_o^* at which temperature of fluid reaches the same value as at length $x = 0$

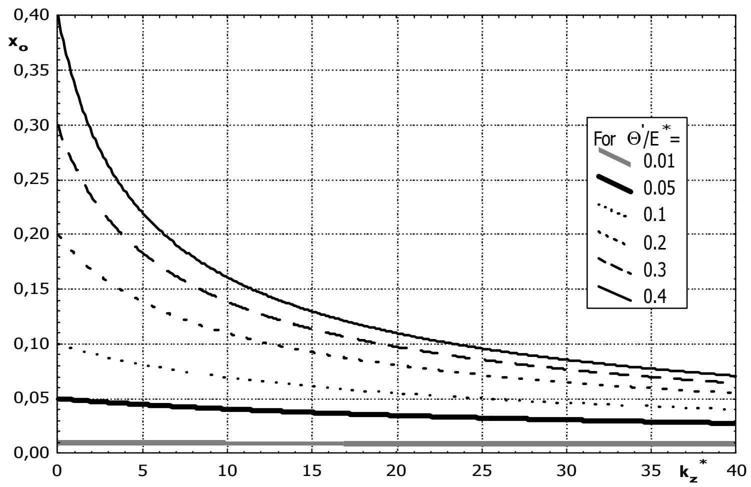

Fig. 11. Influence of parameter k_z^* on length x_o at which temperature of fluid equals temperature of ground (to reach value x_o injected water transfers heat into ground, and then water removes heat from the ground)