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THERMAL FIELD IN AN AQUIFEROUS LAYER BEHIND A VERTICAL HEAT EXCHANGER

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ABSTRACT

Heat pumps drawing heat from a water-bearing layer with a vertical heat exchanger are used in practice. Heat is drawn from a framework of the ground and flowing in ground pores water. While designing a system it is necessary to determine the influence of heat flow drawn with a heat exchanger on the thermal field in a deposit. Existing publications do not present the solution that lets us determine this thermal field. In this publication a mathematical model describing heat flow in a deposit (an aquiferous layer) behind a vertical heat exchanger is presented. This model lets us determine the thermal field in an aquiferous layer. Starting from an energy balance equation for an elementary sector of porous surface a partial differential equation describing transient heat flow in ground medium was obtained. This equation has been solved by Laplace transformation. Obtained solution describes thermal field in an aquiferous layer behind a heat exchanger in relation to input temperature, flow of heat drawn with a heat exchanger, time and properties of the ground.

1. INTRODUCTION

In many countries heat pumps using ground energy have become commercial equipment as a part of heating systems in one-family houses, hotels and public buildings, etc.

There are used in practice heat pumps drawing heat with a vertical heat exchanger from an aquifer (Sanner, 1992). The heat is drawn from the ground framework and from flowing in ground piping water. While designing a system it is essential to determine the heat flow which can be drawn with a heat exchanger. Existing publications do not present the analytical solution that lets us estimate that thermal field.

Kujawa and Szaflik (1998), assuming some simplifications, derived a partial differential equation that describes heat propagation in ground. Solving the equation for appropriate boundary conditions, the authors estimated a relation for replacing convective heat-transfer coefficient from an aquiferous layer to a vertical heat exchanger. Coming out from the equation and changing boundary conditions it is possible to determine thermal field in the ground behind a heat exchanger.

In this publication a mathematical model is presented of heat propagation in an aquiferous layer behind a vertical heat exchanger. This model lets us determine the thermal field in the ground behind a vertical heat exchanger.

2. FORMULATION AND SOLUTION OF THE BOUNDARY PROBLEM

2.1 Differential equation

To introduce a differential equation for transient heat flow through a porous medium, there were made the following assumptions for a flow of heat in ground:

- porous space is unlimited and it does not emit heat, an overall heat-transfer coefficient λ_s, specific heat c_s and density ρ_s are constants,
- a flow rate $\dot{V}_f$, its thermal conductivity λ_f, specific heat c_f and density ρ_f are constants,
- heat flow conducted in fluid flow direction in relation to heat carried away with fluid is negligible,
- a medium is porous enough to equilibrate the temperature of the fluid filling the porous medium and the temperature of the medium very quickly and they are equal at each point of the section (Lauwerier, 1955),
- considering unit normal surface in relation to a fluid flow direction porosity determined by pores' surface P_o is constant for each section,
- a heat exchanger is infinitely long and its calorific effect to is constant $\dot{q}$.

Based on the above assumptions the problem resolves itself into a two-dimensional problem. Assuming Cartesian co-ordinate system, to simplify the problem, in the way that a water flow in ground is parallel to one of the axes (axis x). In this case giving up the heat as a result of conductance goes on perpendicular to the axis (parallel to axis y).

Starting from an energy balance equation for an elementary sector of porous surface, a partial differential equation describing transient heat flow in ground medium was obtained:

$$\frac{\lambda_s(1-P_o)+\lambda_f P_o}{c_s\rho_s(1-P_o)+c_f\rho_f P_o}\frac{\partial^2 t}{\partial y^2}-\mathrm{v}\frac{c_f\rho_f P_o}{c_s\rho_s(1-P_o)+c_f\rho_f P_o}\times\frac{\partial t}{\partial x}=\frac{\partial t}{\partial \tau}. \quad (1)$$

The velocity v from the relation (1) is determined as an average velocity in pores' space.

To simplify the problem there is introduced new variables ξ and η determined by the following relations (2) and (3):

$$\xi = \frac{c_s \rho_s (1 - P_o) + c_f \rho_f P_o}{\mathrm{v}\, c_f \rho_f P_o} x, \tag{2}$$

$$\eta = \sqrt{\frac{c_s \rho_s (1 - P_o) + c_f \rho_f P_o}{\lambda_s (1 - P_o) + \lambda_f P_o}} y. \tag{3}$$

Taking into account an average velocity v_s in equation (2)

$$\mathrm{v}_s = \mathrm{v} P_o \tag{4}$$

and Darcy formula

$$\mathrm{v}_s = k I \tag{5}$$

we obtain as follows:

$$\xi = \frac{c_s \rho_s (1 - P_o) + c_f \rho_f P_o}{k I c_f \rho_f} x, \tag{6}$$

where: k - filtration coefficient related a kind of ground, m/s; I - hydraulic gradient.

Replacing equation (1) with (3) and (6) it can be written as follows:

$$\frac{\partial^2 t}{\partial \eta^2} - \frac{\partial t}{\partial \xi} = \frac{\partial t}{\partial \tau}. \tag{7}$$

It is assumed that temperature of medium and fluid in the output moment is the same in whole volume.

2.2 Determination of thermal field behind a heat exchanger

According to one assumptions the thermal field is symmetric, a symmetry plane is parallel to direction of flow and it comes across the centre of a heat exchanger. The symmetry axis is adiathermal. (Heat does not flow through the symmetry axis.) The solution can be obtained as a sum of two boundary problems:

- one, when it is assumed that on the whole boundary of the field a negative heat source of steady expenditure equal to capacity of a heat exchanger occurs,

and

- the other when it is assumed that on the boundary, except the beginning part equal to the diameter of a heat exchanger, heat source of opposite sign and the same capacity as in first problem occurs.

Sum of above solutions fulfils the condition of adiathermal symmetry surface out of the part equal to the width of a heat exchanger. Schematic of cross-section of the field, ground heat exchanger and assumed model is presented in fig. 1.

The formulated boundary value problem resolves itself into determining a distribution of temperature for a quarter plane with the following boundary condition:

- at an output moment the temperature in a whole medium is constant and it equals to the temperature of the medium:

$$t(x, y, \tau = 0) = 0, \tag{8}$$

- the temperature on inflow surface (surface $0y$) is constant and equal to output temperature:

$$t(x = 0, y, \tau) = 0, \tag{9}$$

- the heat flux on a heating surface (of a heat exchanger) is constant

$$\dot{q} = -\lambda \frac{\mathrm{d}t(x, y = 0, \tau)}{\mathrm{d}y}. \tag{10}$$

The schematic of the system with boundary conditions, is presented in Fig. 2.

For new variables ξ, η boundary conditions are determined as follows:

$$t(\xi, \eta, \tau = 0) = 0, \tag{11}$$

$$\frac{t(\xi, \eta = 0, \tau)}{d\eta} = -\frac{\dot{q}}{\lambda a}, \tag{12}$$

where:

$$a = \sqrt{\frac{c_s \rho_s (1 - P_o) + c_f \rho_f P_o}{\lambda_s (1 - P_o) + \lambda_f P_o}}, \tag{13}$$

$$t(\xi = 0, \eta, \tau) = 0. \tag{14}$$

To solve the problem formulated above there was used Laplace'a double-transformation (Doetsch, 1961 and Korn and Korn, 1968). It was used because of variable τ, ξ. The solution of the equation in the transformed domain can be written:

$$u = \frac{\dot{q}}{\lambda a} \frac{1}{ps} \frac{e^{-\sqrt{s+p}\,\eta}}{\sqrt{s + p}}, \tag{15}$$

where u is a field of temperature and p and s are complex variables corresponding to variables ξ and τ.

Coming back to the original space the solution is written as follows:

$$t = \frac{2\dot{q}}{\lambda a} \left[U(\xi - \tau)\sqrt{\tau}\, ierfc \frac{\eta}{2\sqrt{\tau}} + U(\tau - \xi)\sqrt{\xi}\, ierfc \frac{\eta}{2\sqrt{\xi}} \right], \tag{16}$$

where function U is a unit step function and the functions *ierfc* are determined for particular variables as follows:

$$ierfc \frac{\eta}{2\sqrt{\tau}} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\eta^2}{4\tau} \right) - \frac{\eta}{2\sqrt{\tau}} erfc \frac{\eta}{2\sqrt{\tau}}, \tag{17}$$

$$ierfc \frac{\eta}{2\sqrt{\xi}} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\eta^2}{4\xi} \right) - \frac{\eta}{2\sqrt{\xi}} erfc \frac{\eta}{2\sqrt{\xi}}. \tag{18}$$

This solution describes unsteady thermal field in porous medium in which fluid flows. The field is bounded by cooling plane of steady heat flow intensity. Thermal field depends on output temperature, heat flow intensity as well as ground and time parameters.

3. DETERMINATION OF THERMAL FIELD IN GROUND BEHIND A HEAT EXCHANGER

Temperature is estimated as a sum of temperature from two solutions: given one and the other of opposite sign which is displaced in the part equal to the diameter of a heat exchanger d_o, value d_o equals value of a variable x_o :

$$\xi_o = \frac{c_s \rho_s (1-P_o) + c_f \rho_f P_o}{k\, I\, c_f \rho_f} d_o . \qquad (19)$$

System with centre in the end point of a heat exchanger was taken as a reference co-ordinate system. Thus the solution of first boundary problem is as follows:

$$t = \frac{2\dot{q}}{\lambda a}\left[U(\xi + \xi_o - \tau)\sqrt{\tau}\, ierfc \frac{\eta}{2\sqrt{\tau}} + \right. \\ \left. + U(\tau - \xi - \xi_o)\sqrt{\xi + \xi_o}\, ierfc \frac{\eta}{2\sqrt{\xi + \xi_o}} \right] \qquad (20)$$

and the other's:

$$t = -\frac{2\dot{q}}{\lambda a}\left[U(\xi - \tau)\sqrt{\tau}\, ierfc \frac{\eta}{2\sqrt{\tau}} + U(\tau - \xi)\sqrt{\xi}\, ierfc \frac{\eta}{2\sqrt{\xi}} \right] . \qquad (21)$$

Thermal field in a deposit behind a heat exchanger is determined by sum of above solution as follows:

$$t = \frac{2\dot{q}}{\lambda a}\left\{ \begin{array}{l} [U(\xi + \xi_o - \tau) - U(\xi - \tau)]\sqrt{\tau}\, ierfc \frac{\eta}{2\sqrt{\tau}} + \\ + U(\tau - \xi - \xi_o)\sqrt{\xi + \xi_o}\, ierfc \frac{\eta}{2\sqrt{\xi + \xi_o}} - \\ - U(\tau - \xi)\sqrt{\xi}\, ierfc \frac{\eta}{2\sqrt{\xi}} \end{array} \right\} . \qquad (22)$$

For some long time $(\tau \to \infty)$ it can be assumed that thermal field in the reservoir is fixed and then the temperature of reservoir is determined by the following relation:

$$t = \frac{2\dot{q}}{\lambda a}\left[\sqrt{\xi + \xi_o}\, ierfc \frac{\eta}{2\sqrt{\xi + \xi_o}} - \sqrt{\xi}\, ierfc \frac{\eta}{2\sqrt{\xi}} \right] . \qquad (23)$$

Using relation (23) temperature in a sand reservoir was calculated. The ground is characterised by the following parameters: c_s = 729 J/(kgK), λ_s = 6,048 W/(mK), ρ_s = 2800 kg/m^3. To make the calculations variable values of filtration coefficient k : 0.006 m/s, 0.012 m/s and 0.023 m/s were assumed (these values are characteristic for highly pervious ground). At assumed hydraulic gradient $I = 0.001$ an average velocity of water in ground v_s is relatively about: 0.5 m/24 hours and 1.0 m/24 hours. Parameters of the water are as follows: c_f = 4190 J/(kgK), λ_f = 0.597 W/(mK), ρ_f = 1000 kg/m^3.

The diameters of a heat exchanger d_o are relatively 0.060 m, 0.108 m and 0.219 m. A flow of heat was assumed at the level of $\dot{q}$ = 500 W/m^2.

Porosity of ground was $P_o = 0.35$, $x = 0 \div 5$ m, and $y = 0 \div 2$ m. Results of the calculations are presented in Fig. 4, Fig. 5 and Fig. 6.

4. DISCUSSION

Spatial graphs of thermal fields presented in Fig. 4, Fig. 5 and Fig. 6 show how temperature of the ground behind a vertical heat exchanger is changing towards the axis x (direction of water flow in ground) and towards the axis y. When the strength of heat flow is steady, temperature depends on assumed diameter of a heat exchanger and filtration coefficient k. Increase of value k causes decrease of temperature of the ground behind a heat exchanger. With increase of a diameter of a heat exchanger d_o temperature of the ground behind a heat exchanger increases, too.

To picture clearly the variability of temperature in ground, a selected graph was cut with planes perpendicular to axes x and y. In this way there were obtained flat diagrams presented in Fig. 3. Temperature of the ground behind a heat exchanger along the axis x (parallel to direction of water flow) reaches maximum in certain distance from the beginning of system (Fig. 3a). It results from inflow of cold ground water. In a plane perpendicular to direction of water flow (Fig. 3b) it is observed decrease of maximum and clear extension of range of interaction of temperature changes.

5. CONCLUSIONS

In this publication a relation which lets us determine the unsteady thermal field behind a vertical heat exchanger immersed in ground is deduced. Temperature depends on: intensity of heat flow drawn by a heat exchanger, the diameter of the heat exchanger, parameters describing medium filled with fluid and time. After some long time it can be assumed that temperature of a deposit is fixed. Thus deducing relation is considerably simplified. Based on obtained relations thermal field behind a vertical heat exchanger may be determined for designing systems.

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REFERENCES

Doetsch, G. (1961). *Anleitung zum praktischen Gebrauch der Laplace - Transformation.* Oldenburg, München.

Korn, G.A. and Korn, T.M. (1968). *Mathematical Handbook for Scientists and Engineers*. McGraw-Hill, New York.

Kujawa, T. and Szaflik, W. (1998). Model of Heat Exchange Between an Aquiferous Layer and a Vertical Heat Probe. In: *Mathematics of Heat Transfer*, G.E. Tupholme and A.S. Wood (Ed.)., Clarendon Press, Oxford, pp. 199-204.

Lauwerier, H.A. (1955). The transport of heat in an oil layer caused by the injection of hot fluid. *Appl. Sci. Res., Section A*, v.5.

Sanner, B. (1992). Erdgekoppelte Wärmepumpen, IZW - Tagungsbericht, 2.

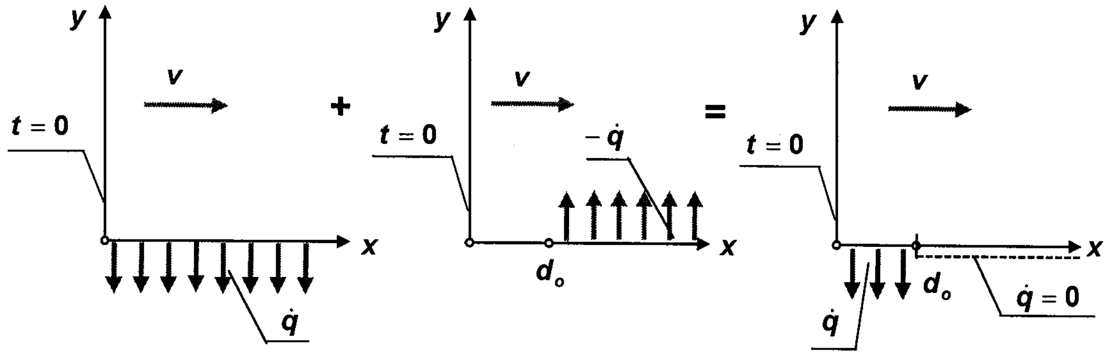

Fig. 1. The scheme of accepted model

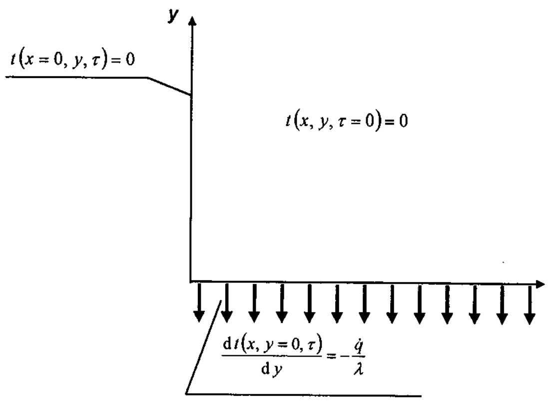

Fig. 2. The scheme of considering system with boundary conditions

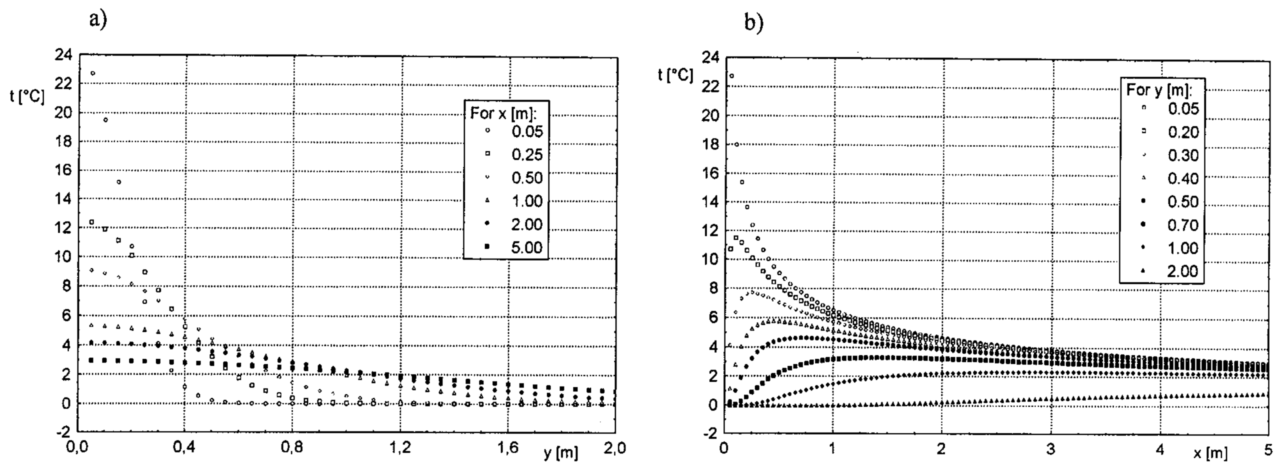

Fig. 3. Thermal field in the ground behind a heat exchanger along the axis y (a) and the axis x (b) (a diameter of a heat exchanger $d_o = 0.060\,\mathrm{m}$, a heat flow $\dot{q} = 2000\,\mathrm{W/m^2}$)

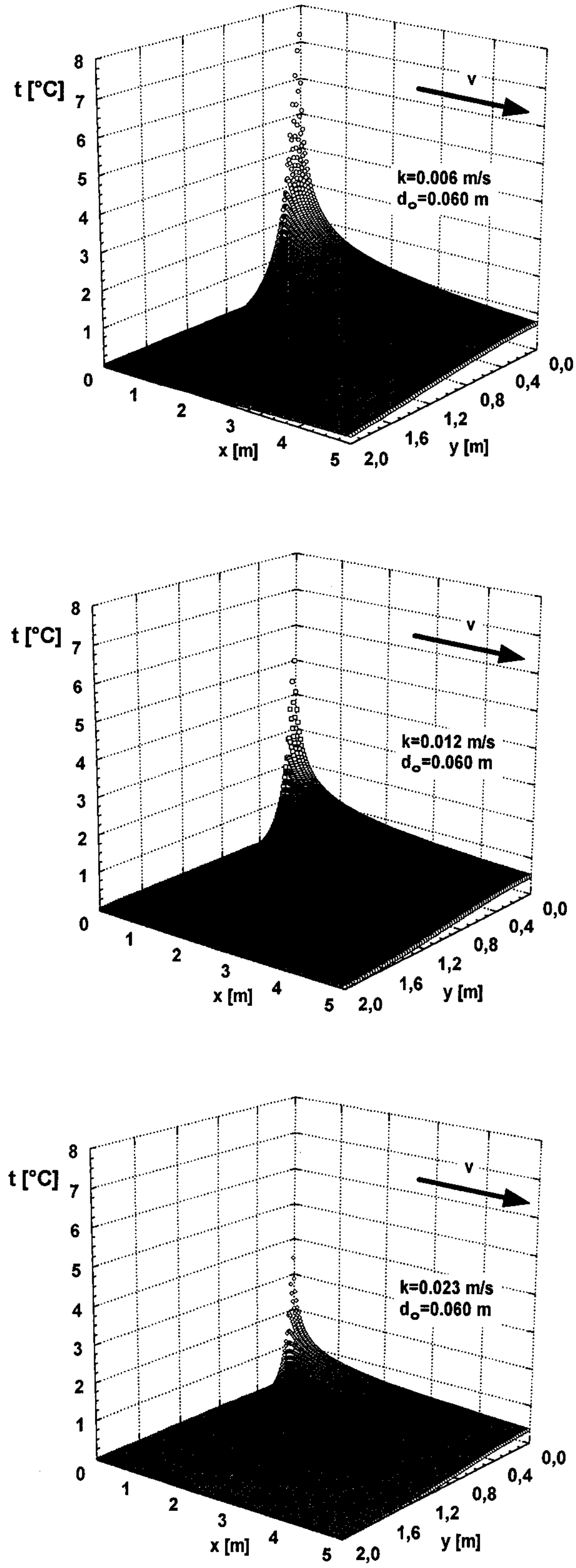

Fig. 4. Spatial graph of thermal field in the ground behind a heat exchanger of diameter $d_o = 0.060\,\mathrm{m}$ when a heat flow is $\dot{q} = 500\,\mathrm{W/m^2}$

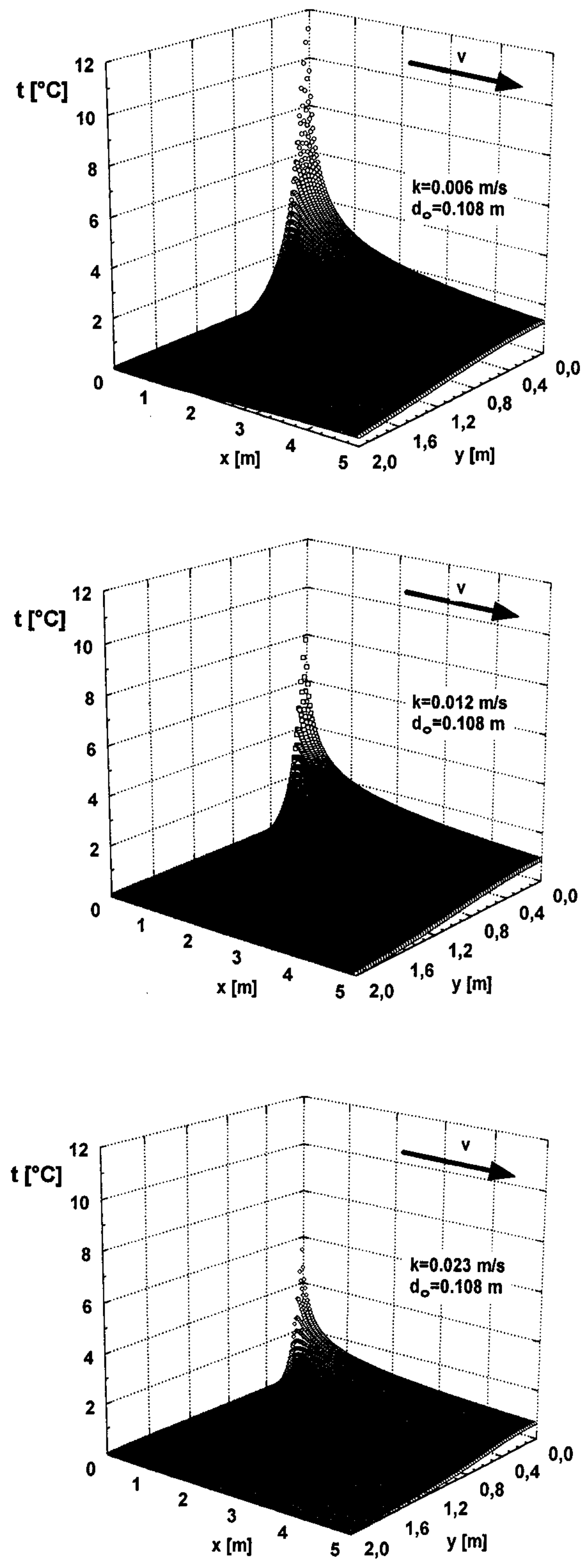

Fig. 5. Spatial graph of thermal field in the ground behind a heat exchanger of diameter $d_o = 0.108\,\mathrm{m}$ when a heat flow is $\dot{q} = 500\,\mathrm{W/m^2}$

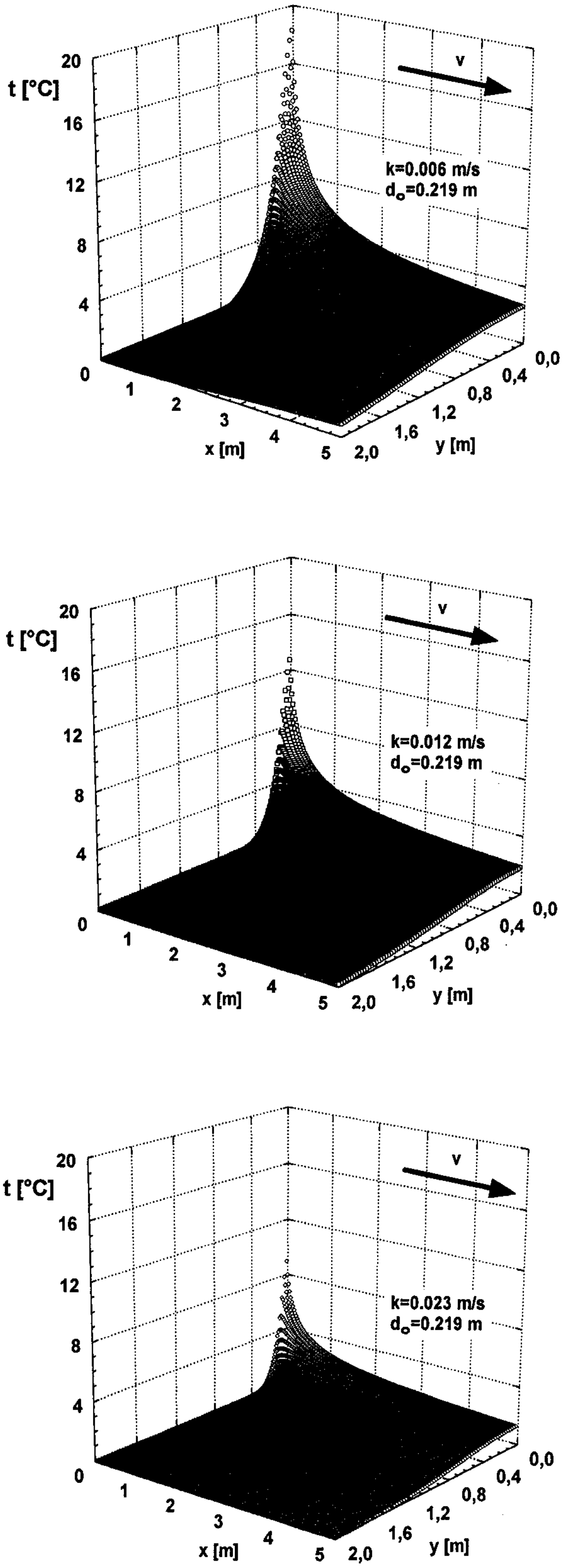

Fig. 6. Spatial graph of thermal field in the ground behind a heat exchanger of diameter $d_o = 0.219\,\mathrm{m}$ when a heat flow is $\dot{q} = 500\,\mathrm{W/m^2}$