

TIME-DEPENDENT CLOSURE AND PERMEABILITY OF A SMALL-SCALE HYDRAULIC FRACTURE UNDER CONSTANT NORMAL STRESS

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ABSTRACT

The permeability of a hydraulic fracture, which acts as an artificial heat exchanger in HDR geothermal energy extraction, may decrease with time due to time-dependent closure under compressive in-situ stresses. The decrease depends on the topography of the fracture surfaces and the rheological properties of rock. In this study, a hydraulic fracture was created in a block of granite in the laboratory. Specimens of a hollow cylinder, which contained a hydraulic fracture perpendicular to the axis, were taken from the block, and height distributions of the fracture surfaces were measured along matched paths to determine two-dimensional aperture distributions of the fracture. The power spectral density of the aperture showed that the fractures used in this study had the surfaces strongly correlated to each other. The changes in the permeability and closure of the fracture with time under constant normal stress were measured for normal stresses up to 25 MPa. The fracture permeability was given as a hydraulic aperture, which is equivalent to the aperture required by a parallel-plate model to give the same flow rate. Time-dependent closure was very small and approached a constant value. The ultimate time-dependent closure did not depend on normal stress. A fundamental mechanism of this independence was considered to lie in that the increase rate in contact area with respect to time-dependent closure of the fracture increases with normal stress. The hydraulic aperture was also very small and decreased with time to approach a constant value. Using an empirical formula between normalized hydraulic aperture and normalized mean aperture, obtained by a simulation for water flow through a hydraulic fracture in granite, the mean aperture of the fracture during time-dependent closure was estimated. The time-dependent hydraulic aperture of the fractures used in this study was less than 10 % of the mean aperture. Thus, the permeability of a hydraulic fracture never obeys the Darcy's law in the entire process of closure.

1. INTRODUCTION

In the field of geothermal energy extraction, prediction of the long-term permeability of a fracture in rock mass is essential for the evaluation of heat extraction performance. A fracture, which is closed macroscopically due to compressive normal stress, may still provide a path, called an aperture, for water flow [Walsh, 1981; Brown, 1987, 1989]. Thus, aperture distributions resulting from stresses control water flow through a fracture [Kranz et al., 1979; Raven and Gale, 1985; Witherspoon et al., 1980; Kojima et al., 1994; Kojima et al., 1995]. The two surfaces of a fracture, such as a hydraulic fracture, are more or less related to each other at long wavelengths [Brown et al., 1986; Matsuki et al., 1995; Glover et al., 1998]. That is, a hydraulic fracture is a 'mated' fracture. The aperture distributions of a mated fracture strongly depend

on the degree of the correlation between the two fracture surfaces. Thus, water flow through a fracture is governed by the surface topography of a fracture in addition to the mechanical properties of rock and rock stresses.

A fracture under constant compressive normal stress may close with time since rock is more or less viscoelastic [Jaeger and Cook, 1965]. Accordingly, the permeability of a fracture may decrease with time [Kojima et al., 1994]. For the prediction and control of water flow in a fracture in rock mass, it is essential to know the time-dependent closure and permeability of a fracture under constant compressive stresses. However, there have been very few investigations of the time-dependent closure and the associated time dependent permeability of a fracture in rocks.

In this study, a small-scale hydraulic fracture was created in the laboratory in a block of granite parallel to the rift plane. By measuring two-dimensional height distributions of the two fracture surfaces along matched paths for specimens of a hollow cylinder, which were taken from the block and contained a hydraulic fracture perpendicular to the axis, the aperture distributions were determined to obtain the statistical and spectral parameters of the aperture distributions. Experiments on time-dependent closure and permeability of a hydraulic fracture were carried out under a constant normal stress of up to 25 MPa. The fracture permeability was given as a hydraulic aperture, which is equivalent to the aperture required by a parallel-plate model to give the same flow rate. By referring to experimental results for a hydraulic fracture in granite obtained by Kojima et al. [1995] and by using simulation results for water flow obtained for the fracture by Matsuki et al. [1999], the mean aperture of a hydraulic fracture during time-dependent closure was estimated. The hydraulic aperture was much smaller than the estimated mean aperture.

2. APERTURE DISTRIBUTION

Inada granite, quarried at Ibaragi prefecture, Japan, was used in this study. The physical and mechanical properties are shown in Table 1. A hydraulic fracture was created in the laboratory in a block of granite parallel to the rift plane. First, a central hole 10mm in diameter was drilled perpendicular to the hydraulic fracture. To avoid damaging fracture surfaces, a bolt was inserted in the central hole to lightly tighten the fracture with nuts during the overcoring process. The central hole served as an outlet for water flow in the experiments on permeability. All four specimens of a hollow cylinder (Figure 1), 66 mm in outer diameter and 80 mm in height, were taken, and fixed with glue in a jig. The jig provided the reference planes to match the paths for measuring the heights of the two fracture surfaces [Matsuki et al., 1999]. Since the two fracture surfaces were not separated, i.e., the fracture existed in a so-called process zone [Atkinson, 1987], the fracture surfaces were separated completely by applying a tensile stress ranged from 0.12 MPa to 0.31 MPa, which was only 2.1 % to 5.4 % of the uniaxial tensile strength of the intact specimen. The surface heights of the fracture were measured in an area of 41 mm x 41 mm using a profilometer with a stylus of 0.025

mm in radius. The interval was 0.01 mm in the direction of the fracture growth and 0.1 mm in the perpendicular direction. The aperture was determined for about 1566000 points from the heights of the two fracture surfaces. When the two fracture surfaces are in contact at a point, we call the aperture an initial aperture. The probability density function (PDF) of the initial aperture is shown in Figure 2. These specimens were supplied to the later experiments with each normal stress. The standard deviation (SD) ranged from 0.084 mm to 0.111 mm, as shown in Table 2. These SD's were much smaller than those obtained by Matsuki et al. [1995] for a hydraulic fracture in the same granite. The PDF's were skewed and had a very sharp peak, and accordingly, were not expressed with a Gaussian distribution.

Figure 3 shows the power spectral densities (PSD's) of surface height and aperture for the four specimens. The PSD's of aperture were flattened at low spatial frequencies (at long wavelengths) while those of the surface height were almost linear in the log-log plot, suggesting that the fracture surfaces are fractal in the range of this study. For discussing the degree of correlation between the fracture surfaces for each specimen [Brown et al., 1986; Glover et al., 1998], the ratio of the aperture PSD to the surface height PSD at long wavelengths is shown in Figure 4 for specimens with each normal stress. In this figure, the results for hydraulic fractures, CR6 and CR7, used by Kojima et al. [1995] are also shown for comparison [Matsuki et al., 1995]. The ratio is two if the two surfaces are completely unrelated to each other, and is zero if the surfaces are perfectly correlated to each other. Figure 4 shows that the degree of correlation between the fracture surfaces is much greater for specimens used in this study than those used by Kojima et al. [1995]. Note that this figure is a log-log plot. Among the specimens used in this study, a specimen with a normal stress of 20 MPa has the largest degree of correlation, and a specimen with a normal stress of 10 MPa has the smallest degree of correlation. The SD of aperture may give us another measure of the degree of correlation between the surfaces of a fracture since the smaller the SD of aperture, the greater the degree of correlation (see Table 2).

3. EXPERIMENTAL METHOD

Figure 5 shows a system for measuring the closure and permeability of a fracture under a constant normal stress. Water (the electric conductivity is less than 0.1 μm) from a tank was pressurized by a pressure pump, and was supplied in a pressure vessel. The water pressure outside the specimen was kept constant to be 0.6 MPa by adjusting a relief valve and a rotational speed of the pump. The central hole was in the atmospheric pressure. Thus, water flows from the outside of the specimen into the central hole through the fracture. The volume of water flow was measured for an appropriate time using an electronic balance (the accuracy is 0.0001 g). The water flow through an intact part of the specimen was ignored since the flow conductance of an intact specimen is less than a few percent of that of a specimen with a fracture under a normal stress of 25 MPa [Kojima et al., 1995]. The permeability of the fracture was expressed as a hydraulic aperture. The hydraulic aperture e_h is the aperture of a parallel-plate model which gives the same flow rate obtained for the fracture, and is given by [Witherspoon et al., 1980],

$$e_h = \left[\frac{6\mu Q \ln(r_1/r_2)}{\pi \Delta H} \right]^{1/3}, \quad (1)$$

where μ is the coefficient of viscosity of water, r_1 and r_2 are

the outer and inner diameters of the specimen, respectively, Q is the volume flow rate and ΔH is the macroscopic pressure difference.

The displacements across the fracture were measured using two sets, called A and B, of strain-gauge type π -gauges. Each set had three π -gauges, spaced at 120° around the circumference of the fracture. The effective lengths of the set A and B were 30 mm and 60 mm, respectively. Accordingly, the difference between the average displacements measured with each set gives the displacement of the intact part 30 mm in length, which was removed from the total displacement measured with the set A to obtain the closure of the fracture. The error in the measurement of the closure was within ± 0.001 mm. The water temperature in the pressure vessel was measured using a strain gauge type temperature gauge to correct the effects of temperature on displacement.

A normal stress from 10 MPa to 25 MPa was applied using a screw type loading machine at a rate of 0.13 MPa/sec, and then kept constant. The axial load was measured with a load cell, and the fluid pressure was measured with a pressure transducer of 1 MPa in capacity. Data were recorded using a microcomputer. In the initial elastic loading, the data were measured every a half second, and after the normal stress was constant, the data were measured at appropriate intervals until no appreciable change in flow rate was observed.

4. INITIAL ELASTIC CLOSURE

Normal stress - elastic closure curves obtained in the initial elastic loading are shown in Figure 6. The origin of this figure was set when the normal stress was 196 N. When this relationship was plotted in a semi-log scale, an approximately linear relationship was obtained, showing that the relationship between the normal stress σ and the elastic closure δ of the hydraulic fracture obeys the following Goodman's formula [Goodman, 1976]:

$$\delta = A + B \log \sigma, \quad (2)$$

where A and B are constants. Similar values of constant B were obtained for these specimens. However, the closure at a certain normal stress is quite different among the specimens. The closure of the most mated fracture with a normal stress of 20 MPa is the smallest and that of the worst mated fracture with a normal stress of 10 MPa is the largest (see Figure 4). This means that the more mated fracture surfaces produce smaller closure at very small normal stresses. Accordingly, the matedness of fracture surfaces may affect the permeability of the fracture.

5. TIME-DEPENDENT CLOSURE AND PERMEABILITY

The time-dependent closure of a hydraulic fracture under different normal stresses is shown in Figure 7. The time-dependent closure was very small and reached a constant value smaller than 0.012 mm after 50 hours. The magnitude of the time-dependent closure did not depend on the normal stress, contrary to the time-dependent displacement of intact rock, which increases with stress [Jaeger and Cook, 1965]. Matsuki et al. [1996] showed that the contact area in a hydraulic fracture in granite increases linearly with the normal stress, but the true contact area is less than 1 % of the nominal one at a normal stress of 25 MPa. This means that a very small initial portion of the aperture distribution contributes to the increase of the normal stress. Figure 2 shows that the aperture PDF increases in an accelerated manner for small

apertures. Therefore, the increase rate in contact area with respect to time-dependent closure increases with normal stress. This may give us a fundamental mechanism for the independence of time-dependent closure of a fracture of normal stress. A theoretical consideration on a viscoelastic closure of a fracture with rough surfaces under constant normal stress will be described elsewhere.

The change of the hydraulic aperture with time after the normal stress was constant is shown in Figure 8 for each normal stress. For all normal stresses, the hydraulic aperture was very small and decreased with time to reach an almost constant value. In the early stage, the hydraulic aperture decreased with the normal stress, except that with a normal stress of 20 MPa, which was applied to the most mated fracture. However, this dependence disappeared in the final stage where the hydraulic aperture was almost constant. The hydraulic aperture of a fracture under a normal stress of 10 MPa, the worst mated fracture, reached the lowest value, while the normal stress was the smallest. Thus, the SD of aperture or the correlation between the two fracture surfaces affected the time-dependent flow conductance of the fracture. Several researchers [Lomize, 1951; Patir and Cheng, 1978; Brown, 1987; Matsuki et al., 1999] showed that the ratio of hydraulic aperture to the mean aperture is determined by the mean aperture normalized by the standard deviation of the surface height or aperture. In a simulation of water flow through a hydraulic fracture in granite, Matsuki et al. [1999] obtained an empirical formula for the relationship between the hydraulic aperture normalized by the SD of aperture e_h/σ_0 and the mean aperture normalized by the SD of aperture e_m/σ_0 . This formula (equation (3)) gives an average relationship for normalized hydraulic apertures greater than 0.15, and is independent of the size of the fracture.

$$\frac{e_h}{\sigma_0} = \frac{e_m}{\sigma_0} \left[1 - \frac{1.13}{1 + 0.191(2e_m/\sigma_0)^{1.93}} \right]^{\frac{1}{3}}, \quad (3)$$

This equation means that the ratio of hydraulic aperture to the mean aperture decreases as the SD of aperture increases for a certain mean aperture. Therefore, the hydraulic aperture normalized the SD of aperture must decrease more for a certain closure as the SD of aperture decreases. Note that this equation predicts that the gradient of the hydraulic aperture with respect to the mean aperture is approximately 1, regardless of the mean aperture. Figure 9 shows the time-dependent hydraulic aperture normalized by the SD of aperture. The normalized hydraulic aperture was similar in the early stage regardless of the normal stress, suggesting that the normalized mean aperture in the early stage was similar among the specimens. However, the hydraulic aperture decreased rapidly as the SD of aperture increases while the time-dependent closure was similar among the specimens (Figure 7). This is a tendency opposite to the prediction by equation (3). Therefore, this suggests that equation (3) is not applicable to very small hydraulic apertures as those obtained in this study.

Figure 10 shows the relationship between the normalized hydraulic aperture and the normalized time-dependent closure. The straight line gives a slope of -1. For comparison, the results obtained by Kojima et al. [1995] for a hydraulic fracture in the same granite were also plotted in the figure. In their study, the fractures CR6 and CR7 are a separated one, but the fracture PR2 is not separated completely, called a process zone fracture (PR2). The normal stress was applied to these fractures up to 25 MPa. Note that the closure in the

experiments by Kojima et al. is elastic one after a normal load of 245 N was applied to the fracture. Since the closure of a fracture is approximately identical to the decrease in the mean aperture, Kojima et al. concluded that the decrease in the hydraulic aperture of a hydraulic fracture with normal stress obeys the Darcy's law for a separated fracture. This is in consistence with the prediction by equation (3), as described previously. On the contrary, the flow conductance of a process zone fracture does not decrease according to the Darcy's law at all. The hydraulic apertures obtained in this study were similar to that of the process zone fracture even if the normal stress was 10 MPa. This is due to the large matedness of the fractures used in this study, as described previously.

The difficulty always arises when a closure is converted to a mean aperture because it is difficult to know the mean aperture of a fracture at the beginning of loading. It may be quite different among fractures because a few large asperities have a remarkable effect on the initial closure of the fracture. In this sense, the relationship between the mean aperture and the closure is arbitrary, except that the decrease in the mean aperture is approximately given by the increase in the closure. However, if it is assumed that the normalized mean aperture shall determine the normalized hydraulic aperture and that the normalized hydraulic aperture is given by equation (3) for normalized hydraulic apertures greater than 0.15, the normalized closure can be converted to the normalized mean aperture by the following procedure. First, the relationships between the normalized hydraulic aperture and the normalized closure are shifted along the axis of the normalized closure so that all relationships may lie on each other to obtain a unique relationship. The mean SD of aperture of the fractures CR6 and CR7 was used for the aperture SD of the process zone fracture PR2. Then, taking the negative normalized closure to obtain the mean aperture, the unique relationship is shifted so that the relationship obeys equation (3) for normalized hydraulic apertures greater than 0.15. The result obtained by this procedure is shown in Figure 11, as the relationship between e_h/σ_0 and e_m/σ_0 . The broken curve indicates the relationship given by equation (3), and the straight line indicates the relationship in which the hydraulic aperture is identical to the mean aperture. This figure suggests that the hydraulic aperture is much smaller than the mean aperture for all of the hydraulic fractures, including those used by Kojima et al.. The Darcy's law never holds in the entire process from the initial elastic loading until the ultimate time-dependent closure. By simulating elastic closure of a hydraulic fracture used by Kojima et al. according to the formula by Brown and Scholz [1985], Matsuki et al. [1996] estimated the mean aperture at a normal load of 245 N, and concluded that the hydraulic aperture was only about 30 % of the mean aperture at such a small normal load. The results obtained in this study are consistent with their conclusion, since the hydraulic aperture for the separated fractures used by Kojima et al. was estimated to be 30 % to 40 % of the mean aperture. On the other hand, the time-dependent hydraulic aperture obtained in this study was less than 10 % of the mean aperture. Thus, the flow conductance of a strongly mated fracture is very small unless the fracture is sheared to cause a shear dilatancy.

6. CONCLUSION

In this study, experiments on time-dependent closure and permeability of a hydraulic fracture created in granite in the laboratory were carried out under a constant normal stress of up to 25 MPa. The statistical and spectral characteristics of the aperture in the fracture were determined by measuring two-dimensional aperture distributions of the fracture. The hydraulic aperture was discussed in relation to the standard deviation of aperture or the matedness of the fracture. By using previous simulation results for water flow through a hydraulic fracture, the mean aperture of the fracture during the time-dependent closure was estimated. The main conclusions obtained in this study are summarized as follows:

- 1) The time-dependent closure did not strongly depend on normal stress and reached an almost constant value.
- 2) The hydraulic aperture decreased with time to reach an almost constant value. The amount of decrease in the hydraulic aperture decreased with the decrease in the standard deviation of aperture or with the increase in the matedness of the fracture.
- 3) The hydraulic aperture of a strongly mated fracture such as those used in this study was very small and similar to that of a fracture in a process zone.
- 4) The hydraulic aperture was much smaller than the mechanical mean aperture in the entire process from initial elastic loading and ultimate time-dependent closure under constant normal stress.

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Table 1. Physical and mechanical properties of Inada granite.

Uniaxial compressive strength (MPa)	Uniaxial tensile strength (MPa)	P-wave velocity (km/sec)	Specific gravity	Porosity (%)	Permeability coefficient at room temperature (darcy)
163	5.82	3.62	2.64	0.56	6.7×10^{-7}

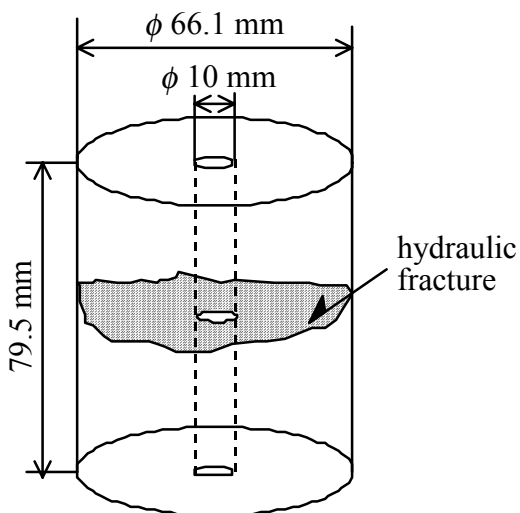


Figure 1. The specimen used in this study.

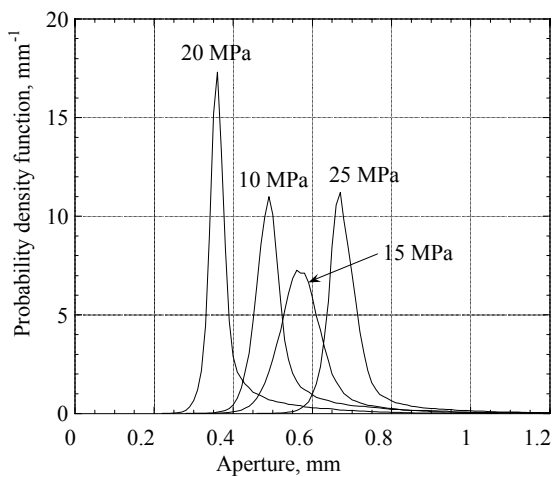


Figure 2. The probability density function of the initial aperture.

Table 2. The mean and standard deviation of initial aperture.

Normal stress (MPa)	Mean (mm)	Standard deviation (mm)
10	0.526	0.111
15	0.589	0.097
20	0.390	0.084
25	0.697	0.087

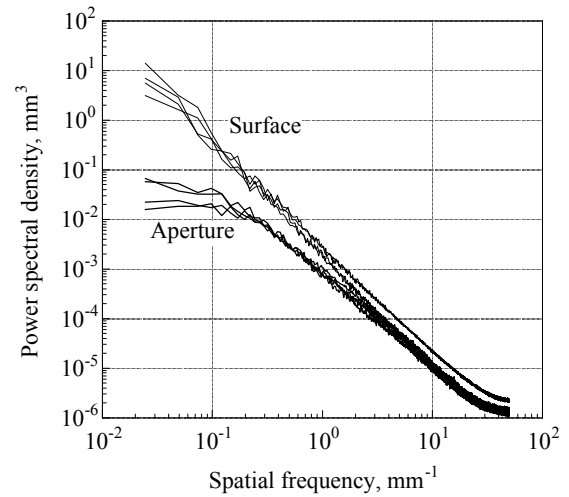


Figure 3. The power spectral densities of surface height and aperture.

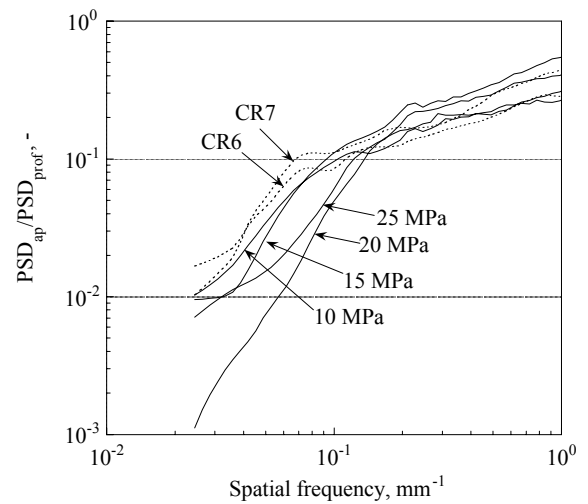


Figure 4. The ratio of the aperture PSD to the surface height PSD at long wavelengths.

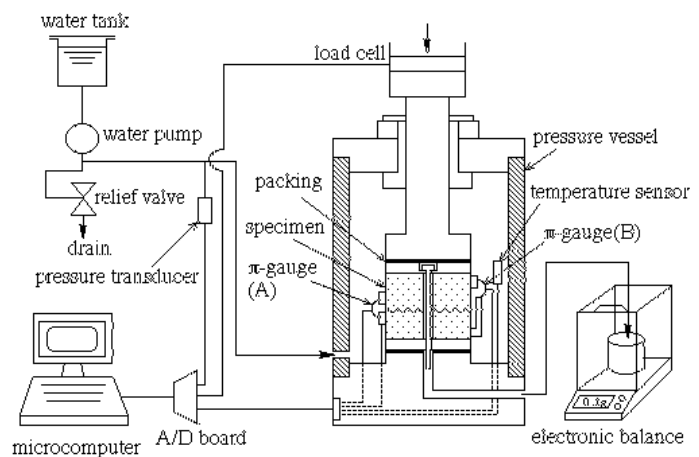


Figure 5. System for measuring closure and permeability of a fracture under constant normal stress.

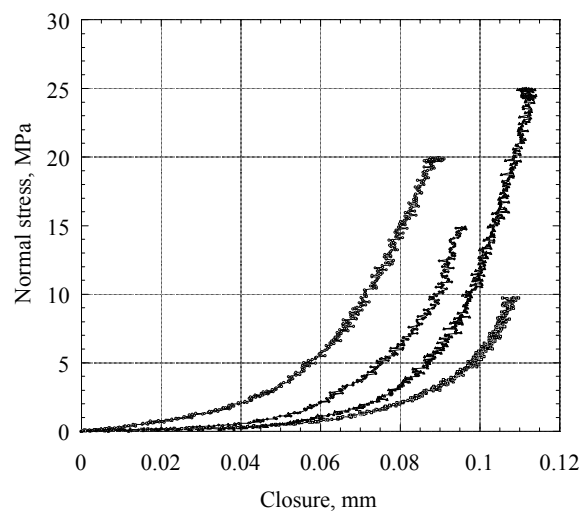


Figure 6. The normal stress – elastic closure curves in initial elastic loading.

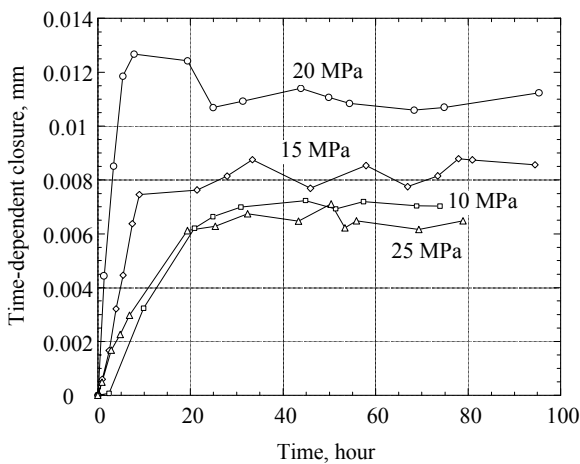


Figure 7. The time-dependent closure of a hydraulic fracture under constant normal stress.

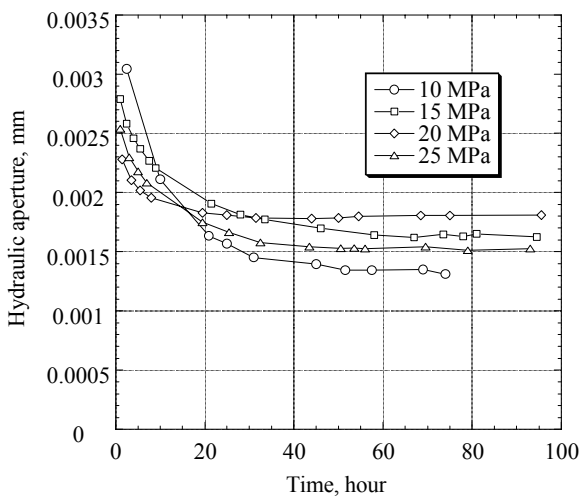


Figure 8. The time-dependent hydraulic aperture of a hydraulic fracture under constant normal stress.

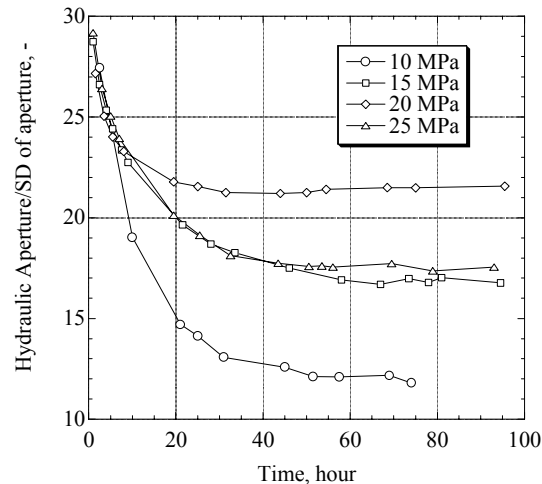


Figure 9. The time-dependent hydraulic aperture normalized by the standard deviation of aperture.

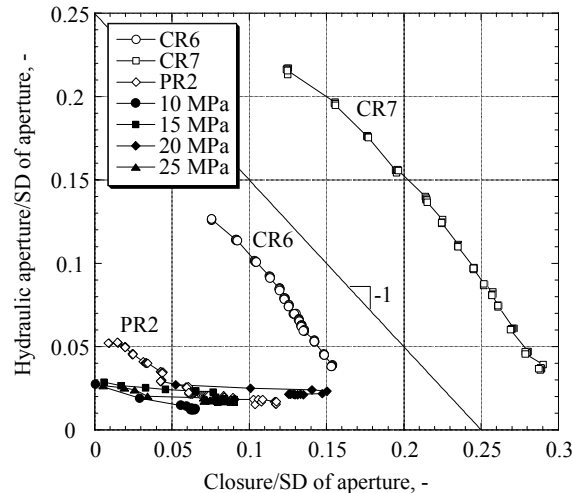


Figure 10. The relationship between normalized hydraulic aperture and normalized closure of hydraulic fractures.

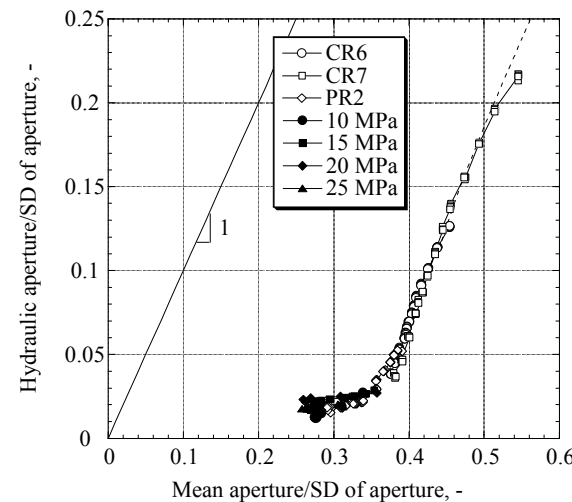


Figure 11. The relationship between normalized hydraulic aperture and normalized mean aperture of hydraulic fractures.