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PRESSURE TRANSIENT TESTING INVERSION FOR FLUID FLOW MODELING IN FRACTURED ROCKS USING SIMULATED ANNEALING: -THREE DIMENSIONAL SYNTHETIC CASES -

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ABSTRACT

The inversion of pressure transient tests by cluster variable aperture (CVA) simulated annealing is used as an inversion technique for developing models of fluid flow in fractured formations. A three-dimensional (3D) fracture network system is represented as a filled regular lattice of fracture elements. The algorithm iteratively changes element apertures for a cluster of fracture elements, which are chosen randomly from a list of discrete apertures, in order to improve the match to observed pressure transients. In this technique, the finite element code TRINET is used as a subroutine to solve for the pressure distribution at each iteration. Aperture size is chosen randomly from a list of discrete apertures. The cluster size is held constant throughout the iterations. This technique is applied to a 3D synthetic model to invert a series of three injection tests. The inversion result shows that the high transmissivity distribution is clearly reconstructed by CVA simulated annealing.

1. INTRODUCTION

For geothermal reservoir engineering, it is quite important to evaluate the transmissivity distribution within a geothermal reservoir in order to predict the future performance of the reservoir. Pressure interference testing using multiple wells is thought to be a useful and direct method to collect information on the transmissivities of the system. Once pressure transient data are obtained, an inverse method can be applied to match the observed data and hence to construct a hydrological model.

Simulated annealing (SA) is a stochastic search method and has attracted attention as a scheme for optimizing an objective function. SA has been used in a variety of optimization problems (Kirkpatrick et al., 1983), and also been applied to hydrology to develop a groundwater management strategy (Dougherty and Marryott, 1991). The major disadvantage of SA is that the algorithm may require large amounts of computation time, because SA often requires more iterations of the model than other conventional search methods. In spite of the disadvantages, SA has an advantage over conventional gradient methods in that the algorithm selects parameter values outside the neighborhood of a local minimum. This prevents SA from being forced to converge toward a nearby local minimum, no matter what a starting model is.

In hydrological applications (hydrology of fractured rocks), Lawrence Berkeley National Laboratory (LBNL) has been developing an inverse method for well test data using SA (Mauldon et al., 1993). In their work, a partially-filled lattice represents a fracture network and an individual fracture element is changed randomly from conductive to nonconductive or vice versa. The effect of this change is examined by numerically simulating well tests and comparing simulations with field test data. In this technique, the finite element code TRINET (Karasaki, 1987) was used to solve for the pressure distribution at each iteration. Jacobsen (1993), and Najita and Karasaki (1995) developed a SA method called cluster variable aperture (CVA) annealing that uses a distribution of fracture apertures. The algorithm changes element transmissivities by changing element apertures following the cubic law. In addition, the algorithm will change the property of a cluster of elements instead of a single element. The cluster size and shape, which represent simplified fracture sets, are the parameters specified in the inversion algorithm. Sensitivity studies, investigating optimal cluster size using 2D-synthetic models with spatially correlated transmissivity, showed that the optimal cluster size seems to be 20 ~ 40 % of the practical range of spatial correlation for transmissivity distribution (Nakao et al., 1999).

For geothermal application, there is a powerful inverse modeling code, ITOUGH2 (Finsterle, 1993), which implements the TOUGH2 simulator (Pruess, 1991) as the forward modeling tool for modeling multi-phase, multi-component fluid and heat flow. ITOUGH2 also has an option to use SA as the optimization algorithm. However, it may be infeasible to use ITOUGH2 for a 3D inverse modeling with SA to solve for fracture geometry in that the SA often requires large amount of computation time (iteration), when TOUGH2 is already a computationally demanding. In this paper we will describe application of SA for well testing inversion using the TRINET code, and present the inversion results of a 3D synthetic model.

2. ANALYSIS METHOD

2.1. Simulated Annealing in General

After briefly describing the method, application to hydraulic well tests will be presented. SA has its origins in thermodynamics and the manner in which liquid metals cool and anneal. In physical annealing, a metal is heated and allowed to cool very slowly to obtain a regular molecular configuration having the lowest possible energy state. If the temperature T is held constant, the system approaches thermal equilibrium and the probability distribution for the configuration with energy E approaches the Boltzmann probability [$Pr(E) = \exp(-E/kT)$; where k is the Boltzmann constant]. Metropolis et al. (1953) first introduced a simple algorithm to incorporate these ideas into numerical calculations. The following criterion (Metropolis algorithm) is applied in determining whether a transition to another configuration occurs at the current temperature. For the $n-1^{st}$ configuration X_{n-1} and the n^{th} configuration X_n with energies

E_{n-1} and E_n respectively, the transition probability at the system temperature T is given by

$$\Pr(X_{n-1} \rightarrow X_n) = \begin{cases} 1 & \text{if } E_n - E_{n-1} < 0 \\ \exp\left[-(E_n - E_{n-1})/T\right] & \text{if } E_n - E_{n-1} > 0. \end{cases} \quad (1)$$

This criterion always allows a transition to a configuration if system energy is decreased and sometimes allows a transition to a configuration with higher energy. This stochastic relaxation step allows SA to search the space of possible configurations without always converging to the nearest local minimum. In SA, the objective function for an optimization problem is analogous to energy state and the set of free parameters (configuration) is analogous to the arrangement of molecules. "Temperature" is simply a control parameter in a given optimization problem. We will refer to the objective function as "energy" in the following text.

In general, the SA algorithm consists of the following tasks: 1) generate or randomly change system configuration, 2) calculate values of the objective function (energy), 3) perform the Metropolis algorithm to determine whether a new configuration is accepted or not, and 4) adjust the temperature according to the annealing schedule.

Several choices of annealing schedule are possible. A computationally practical schedule is the widely used decrement rule. Given an initial temperature T_0, assign

$$T_k = T_0\,\alpha^k; \;\; k=0,1,2,3 \ldots, \quad (2)$$

where α is between 0 and 1. This general form has been implemented by others with values of α ranging from 0.5 to 0.99 (e.g., van Laarhoven and Aarts, 1987). In this schedule, the current control parameter T_k is kept fixed until a finite number of transitions, L_k, have been accepted, then the parameter T is lowered to T_{k+1}.

2.2. Implementation to Well Test Inversion

In order to use SA for the inversion of well tests, we consider a 3D-fracture network model as shown in Figure 1. Each element between two nodes represents a simplified fracture that controls the transmissivity. Transmissivity (m^2/sec), specific storage (m^{-1}), aperture b (m), unit thickness (1 m) and length (spacing) L are assigned to each element. Well locations are specified at node points and the pressure transients (hydraulic head changes) due to pumping or injection are calculated at each node using the finite element code TRINET (Karasaki, 1987). Although TRINET was originally developed to study a single-phase and isothermal fluid flow and transport in fracture networks, its lattice structure is equivalent to a block model, and TRINET can also be used to model porous media (Doughty, 1995; Najita and Doughty, 1997). TRINET is well suited for use in a hydrologic inversion for several reasons: 1) the one-dimensional finite element formulation is efficient computationally, enabling many forward calculation to be done for both 2D and 3D models, 2) the lattice structure can be used effectively to represent highly channeled flow typical of fractured media as well as to represent a heterogeneous porous medium. It is assumed that the transmissivity of any fracture element follows the cubic law ($T = \rho g b^3/12\mu$, ρ: density of water, g: acceleration due to gravity, μ: dynamic viscosity of water, b: aperture). Although this assumption may be too simple under all circumstances, we think that it is the best way to represent the fractures if there is not enough information about the relationship between aperture and transmissivity.

In CVA annealing, the objective is to find a near-optimal fracture network geometry by modifying clusters of element apertures (hence transmissivities) and calculating pressure transient curves in order to simultaneously match observed head data at multiple observation wells. At each step of the algorithm, a cluster of elements is randomly selected and their apertures are changed to an aperture chosen at random from a discretized distribution. This step is analogous to the perturbation step in the general SA algorithm. The number of elements in the cluster is limited by the maximum cluster size that is specified by a user. The cluster shape, whether it is isotropic (random shapes) or anisotropic (elliptical), is also a user-specified parameter. Following the perturbation step, well tests are simulated on the fracture network and the calculated pressure transients are compared with observed data. The objective function (energy) to be minimized is the sum of the squared differences between the calculated and observed pressure transients due to production or injection. At the i^{th} iteration:

$$E_i = \sum_{j=1}^{\#wells} \sum_{k=1}^{ntimes[j]} (o_{jk} - p_{ijk})^2 \quad (3)$$

where o_{jk} refers to the observed pressure transient at the k^{th} time step for j^{th} well. p_{ijk} refers to the simulated pressure transient at the i^{th} iteration for the j^{th} well and k^{th} observed time.

The Metropolis algorithm is applied to determine whether the current fracture configuration is acceptable based on the equation (1). When L_k acceptances at control parameter T_k have been achieved, the parameter is reduced to T_{k+1} via equation (2) with α = 0.9. T_{k+1} stays constant until L_{k+1} transitions have been accepted at T_{k+1}. This process continues until the annealing schedule is exhausted or the number of iterations has reached a user-specified maximum. At this point, the model with the lowest energy found thus far is expected to be close to the global minimum. In the following, we refer to "minimum energy", however, this is used to describe the lowest energy found at the end of the inversion.

3. INVERSION RESULT OF 3D SYNTHETIC MODEL

In order to examine the potential of CVA annealing as an inversion method for 3D modeling, the following synthetic case has been conducted. As shown in Figure 2, consider the 100 m x 100 m x 20 m 3D-region filled with fractures of 10 m spacing, consisting of 363 nodes and 902 fracture elements. It is a rather simple fracture network model where the transmissivity of the center elliptic region is 1.0 x 10^{-4} (m^2/sec), that of the middle layer is 1.0 x 10^{-5} (m^2/sec), and background transmissivity is 1.0 x 10^{-6} (m^2/sec). We assign constant pressure conditions on the horizontal boundaries. Both upper and lower boundaries are impermeable and insulated. Total number of the observation wells is eight including three injection wells. Three injection (and falloff) tests are simulated in series to generate synthetic observed data based on the 3D-fracture network solution (Fig. 2). Injection wells are indicated by filled circles in Figure 3. The injection rates are constant, 30 l/min for each injection. The initial (starting) model used here is a homogeneous transmissivity distribution with 10^{-3} (m^2/sec). The equivalent specific storage of the fracture elements are set to be 0.01 m^{-1}

and remain unchanged throughout the inversion. For simplicity three replacement apertures are specified. These are the apertures corresponding to the transmissivities of 10^{-4}, 10^{-5} and 10^{-6} after applying the cubic low. The cluster size and shapes specified in the inversion are 20 and isotropic in shapes.

The inversion result (case 1) shows that the minimum energy, 50.33, is reached at iteration of 4290, starting from the initial energy of 1.48 x 10^5. Comparison of the pressure transients between the observed (synthetic) data and the model prediction using the minimizing configuration is illustrated in Figure 4. In this figure, The observed (synthetic) data are indicated by plotting symbols and lines refer to the calculated pressure transients. A good fit of pressure transients is achieved. The inversion result (Fig. 5) is close to the true configuration in that the high transmissivity region is clearly reconstructed, although the shape of the region is a little different from the real configuration. This is caused by the inherent nature of pressure transient testing, where a large number of equivalent models exist that can match one set of observed data (non-uniqueness). This high transmissivity region is also clearly defined by the inversion (Fig. 6), when most of the wells are located in the middle layer (case 2).

4. CONCLUSIONS

The inversion of pressure transient tests by cluster variable aperture (CVA) simulated annealing is applied as an inversion technique for developing models of fluid flow in fractured formations. On the basis of the cases considered, it appears that the CVA simulated annealing can be effective in matching the observed pressure transient data from the 3D-fracture network model. For future study, the application to 3D geothermal reservoir modeling using field test data is planned.

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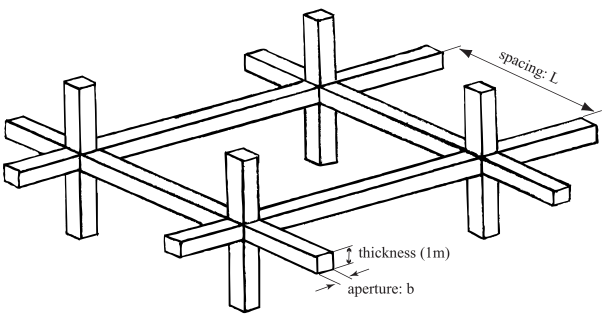

Figure 1. Schematic diagram of a 3D TRINET lattice.

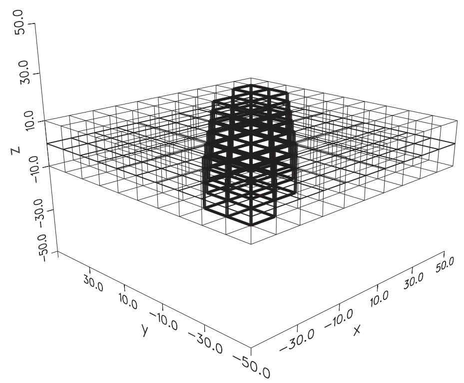

Figure 2. The synthetic 3D fracture network model.

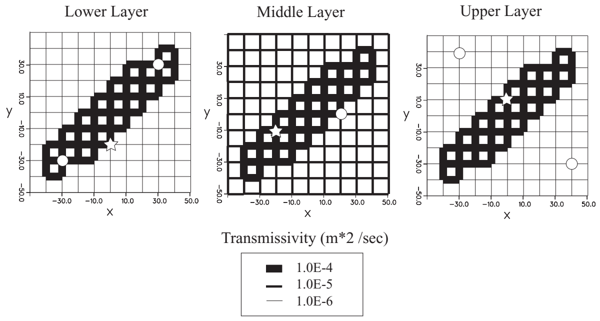

Figure 3. The plan of each layer for the synthetic 3D fracture network model: Asterisks and circles represent injection wells and observation wells, respectively for the case 1.

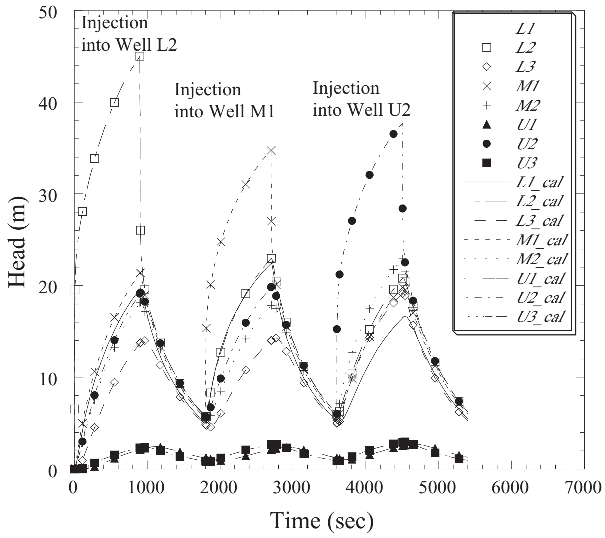

Figure 4. Pressure transient match for minimizing configuration shown in Fig. 5 (case 1).

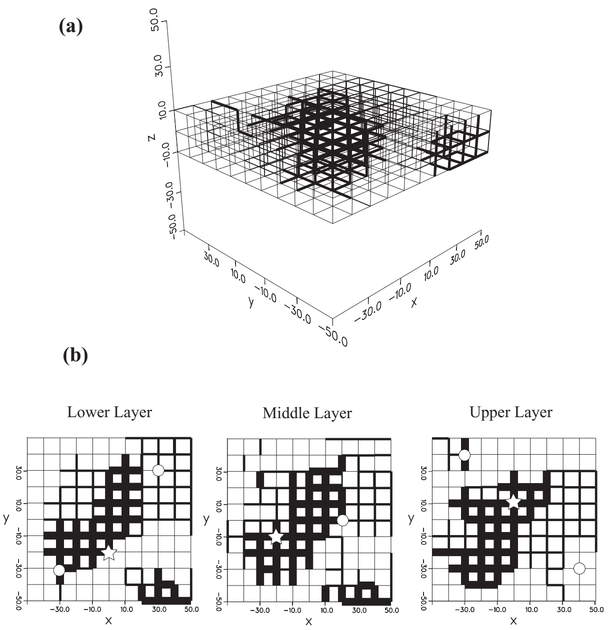

Figure 5. The inversion result for case 1: (a) perspective view and (b) plan of the each layer.

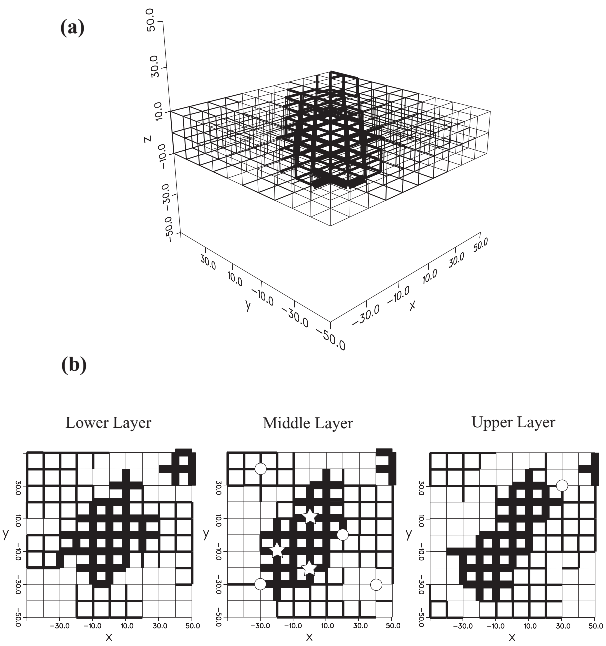

Figure 6. The inversion result for case 2: (a) perspective view and (b) plan of the each layer.