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A MATHEMATICAL MODEL COUPLING HEAT AND MASS FLOW AND EXTENSION RATE IN THE TAUPO VOLCANIC ZONE, NEW ZEALAND

Graham Weir
Applied Mathematics, Industrial Research Limited, PO Box 31-310, Lower Hutt, New Zealand

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ABSTRACT

A new mathematical model is presented for heat flow from the Taupo Volcanic Zone (TVZ). The model consists of three layers, each with constant density, down to a depth of about 25 km, where the pressure is assumed constant. A constant rate of eruption of material onto Layer 1 is assumed, which implies a constant rate of spreading (extension), and a constant rate of volumetric creation in Layers 2 and 3. Heat flow calculations are performed, by assuming conductive heat transport below 8 km, where the rocks are assumed to be in a ductile state. The top of Layer 3, which moves upwards with time, is assumed to be at a temperature of about 1150°C. Some examples are provided which match the present total heat output from the TVZ of about 4200 MW, but these either have extension rates greater than the low values of about 8 ± 4 mm/a being reported from GPS measurements, or else consider extension rates in the TVZ to have varied over time.

1. INTRODUCTION

The majority of geothermal activity in New Zealand is concentrated in the Taupo Volcanic Zone (TVZ). Hochstein (1995) has emphasised that the long term eruption rate of rhyolites from the TVZ is the highest on earth for a volcanic arc setting, and also that the total crustal heat transfer is arguably the highest on earth for an arc setting. The geological, geochemical and geophysical properties of the TVZ have been summarised in a Special Issue of Journal of Volcanology and Geothermal Research (Simmons and Weavers, 1995) on the TVZ. Large-scale reservoir engineering properties of the TVZ are summarised by McNabb (1975), and in the book by Elder (1976).

The location of the TVZ is shown in Fig. 1, and the corresponding location of the 23 known geothermal fields in the TVZ are plotted in Fig. 2.

To date, six models have been constructed in attempts to explain the geophysical properties of the TVZ. The **spreading** model assumes that the TVZ is spreading apart, and the volume created is occupied by magma. Meteoric water then cools the intruded magma, establishing convective plumes which are the geothermal fields in the TVZ. These ideas are widely accepted, and so will probably be a component in most models of the TVZ. To distinguish it from the rifting model below, in this paper it is assumed that in the spreading model, the volume displaced by Layer 3 in Fig. 3 has largely moved into Layers 1 and 2.

The **rifting** model assumes that the original greywacke rocks exist at depth, having been stretched by the opening of the TVZ. The volume of greywacke then exists as a continuous unit, below which mantle or other material enters to conserve volume and maintain isostacy. The volume occupied by Layer 3 in Fig. 3 in the rifting model has been formed by rifting, and not by mobilisation of greywacke by upward moving magma.

The **plasticity** model was formulated by Hochstein (1995) in order to produce an additional heat flow over that imagined in the spreading model. The plasticity model assumes that the entire lithosphere beneath the brittle crust is opening, and that the mechanical work performed in this opening is converted into heat, which is then available to fuel the TVZ.

The **hot-plate** model considers a plate fragment (perhaps 200 km by 100 km by 10 km) from the Pacific Plate in the Bay of Plenty region to be torn by the back-arc spreading between 5 - 10 Ma ago. Melt from this descending plate fragment then forms magma which rises under the region of Mt Ruapehu, and then moves roughly northward, in a thin sill-like flow, at a depth of about 8 km, yielding heat to the TVZ region, with the magma then descending in the region of about Kawerau. This model (McNabb, 1992, McNabb and McKibbin, 1998) imagines heat to leave the flowing magma conductively. A plate fragment was chosen to provide the high chloride flows typical of the TVZ region.

The **froth** model (Giggenbach, 1995) imagines a magma and water foam or froth to rise up in the east of the TVZ from the descending Pacific Plate. The lighter more volatile components then rise preferentially to form the gas-rich eastern geothermal fields, while the residual magma flows to the west, to form the chloride-rich, gas-poor western geothermal fields. Heat input is considered to occur from cooling of intrusives by circulating meteoric water, so the froth model has many similarities to the spreading model.

The **transtensional** model (Cole, 1990) assumes that the rapid opening of the TVZ is the sum of back-arc spreading, induced by the NNE Taupo - Hikurangi arc-trench system, together with stresses from a combined NNW North Island Shear Belt and a Mayor Island Fault Belt system. Rhyolitic volcanism is assumed to result from partial melting of andesitic volcanics from the Coromandel Volcanic Zone, assumed to underlie the TVZ region, to produce the predominantly metaluminous rhyolitic and ignimbritic magmas observed in the TVZ, rather than the peraluminous magma which are expected from partial melting of a greywacke-argillite basement. Magma flow is induced by the rapid thinning of the TVZ, and so in this model the TVZ is presumably fuelled by a mantle flow.

A mathematical analysis has been completed for the hot-plate model by McNabb (1992) and McNabb and McKibbin (1998). The aim of this paper is to provide a mathematical analysis for the rifting model, and to derive a contraint on the heat flow from such a model, for a given extension rate. This work extends that in recent papers (Weir, 1998, 1998a). The key assumptions needed in developing such models are the rate of volcanism, the density structure with depth, and the stress state with depth in the TVZ. These assumptions have been discussed by Weir (1998a), who suggested that an approximately constant rate of eruption be used (of about 2×10^{-6} m^2/s for volumetric eruption rate per unit length of the TVZ), and for simplicity that the density in each layer be taken as a constant. These assumptions are made below.

2. TRANSIENT RIFTING MODEL

The aim of this section is to develop approximations to the evolution of the three regions in Fig 3 for a transient rifting model. If z is depth, then z_1 is the base of the pyroclastic infill region, z_2 the top of the mantle rise, and z_3 the base of the mantle rise, assumed fixed at 25 km depth. The height of the upper (land) surface in the TVZ region will alter with time, but this surface $(z = 0)$ is assumed to be fixed in space, because its motion is small relative to the motion of the other surfaces, and little is known about its motion.

Cole (Fig. 14, 1990) suggests the middle layer $(z_1 \le z \le z_2)$ region is formed by partial melting of volcanic and plutonic rocks, and high-level reservoirs of rhyolitic magma. The upper layer $(0 \le z \le z_1)$ is the infill layer, although this region will comprise of both infill, mostly from rhyolitic volcanism, and intrusions. For example, a diorite intrusion was discovered at Ngatamariki (Wilson et al., 1995) in the infill region. The lower region $(z_2 \le z \le z_3)$ is the mantle region, although it is possible that its composition may differ from the mantle to the west, due to the incorporation of greywacke and earlier intrusions into the rising mantle plume, and to the entry of magma from below. Cole (Fig. 14, 1990) suggests the mantle-rise region is formed by partial melting of peridotite at the top of the mantle to produce high-alumina basalt.

The pressure at a depth of 25 km $(z = z_3)$ is assumed constant. Then ρ_1, ρ_2, ρ_3 are the densities (assumed constant) of rock in Layers 1 $(0 < z < z_1)$, $2(z_1 < z < z_2)$, $3(z_2 < z < z_3)$,

$$\rho_3(z_3 - z_2) + \rho_2(z_2 - z_1) + \rho_1 z_1 = \rho_2 z_3 \qquad (1)$$

where it is also assumed that the initial density of the rock is the density of the middle layer. These two assumptions are not exactly true, as a pressure difference above lithostatic is needed to drive rock upwards. This pressure difference may be relatively small, given the transient time scale for such processes is of the order of 0.01 Ma (Weir, 1998), whereas the rifting processes in the TVZ have been occurring for 1.6 Ma. Also, mixing of greywacke and igneous intrusions, for example, will alter the density in the middle layer.

Rearranging (1) shows that

$$(\rho_2 - \rho_1)z_1 = (\rho_3 - \rho_2)(z_3 - z_2) \qquad (2)$$

and so the thickness of pyroclastic infill is proportional to the height of the mantle rise. In Fig. 2 the width of the pyroclastic infill is assumed proportional to the width of the mantle rise, so (2) also implies the volume of total extruded volcanics in the TVZ is proportional to the volume of the mantle rise. This theoretical argument is then analogous, though different, to the observation that there is an approximate proportionality between magma volumes and extrusives at some locations. It is shown below that the transient rifting model also has the property of a constant ratio between geothermal volumetric magma flows and erupted volumes.

The assumption above that the density of rock remains constant in each layer imposes strong constraints on the nature of the intrusions in the TVZ which can be considered in such a model. If a significant amount of rhyolite is intruded into the upper layer, then the density could vary over time. Below it is assumed that geothermal heating mostly results from magmatic intrusions into the middle layer, because most of the heat associated with the formation of the upper layer will be lost to the atmosphere on being deposited onto the upper surface after eruption.

Chemical and isotopic analyses suggest (Graham et al., 1995) that TVZ rhyolite appears to be mostly of andesitic composition, with only a small amount of greywacke present. This suggests that the rock originally occupying Layer 3 was mostly andesite, as suggested by Cole (1990), and this requires an immense amount of andesite to initially underlie the TVZ, and also a large deep heat source. One way to circumvent these difficulties is by allowing the volume occupied by Layer 3 to be formed by rifting Layers 1 and 2 apart. Layer 3 would then be formed from andesite and other components, some of which would flow up into Layers 1 and 2, being slightly contaminated by greywacke, to form rhyolite. Some of this rhyolite must remain within Layer 2 to provide the heat and chemicals for the geothermal fields in the TVZ.

For simplicity, it is assumed that greywacke originally occupies the volume of Layer 3, and that rifting occurs, so that no greywacke is present in Layer 3. Then conservation of greywacke requires

$$\rho_2(1 - \lambda + \varepsilon\lambda)(z_2 - z_1)w + \rho_1 \varepsilon z_1 w = \rho_2 z_3 w_0 \qquad (3)$$

where ε is the mass fraction of greywacke in rhyolite, λ is the volume fraction of rhyolite in Layer 2, w is the average width of the TVZ, and that this increases with time, starting from an initial average width of w_0. Layer 1 is assumed to be fully occupied by rhyolite, and so the c. 5% of other eruptive types have been ignored in Layer 1.

Defining

$$\theta = \frac{\lambda(z_2 - z_1)}{z_1} \qquad (4)$$

or θ is the volume ratio between stored rhyolite in Layer 2 to that in Layer 1, and using (2), (3) and (4) yields

$$z_3 - z_2 = z_{3\infty}\left(1 - \frac{w_0}{w}\right) \qquad (5)$$

where

$$z_{3\infty} = \frac{z_3}{\left[1 + \frac{(\rho_3 - \rho_2)}{(\rho_2 - \rho_1)}\left(1 + \theta(1-\varepsilon) - \frac{\rho_1 \varepsilon}{\rho_2}\right)\right]} \quad (6)$$

In the limit $\theta = 2, \varepsilon = 0, (\rho_2 - \rho_1)/(\rho_3 - \rho_2) = 4$, then $z_{3\infty} = 4z_3/7$. The asymptotic thicknesses of Layers 1 and 2 are then $z_{1\infty} = z_3/7$ and $z_{2\infty} = 2z_3/7$, respectively. The asymptotic value of λ is one, from (4), for the selected values above.

The choice of $\theta = 2$ is chosen as typical. The present value of $z_3 - z_2$ is about 10 km, and so from (5), $z_{3\infty}$ is greater than 10 km. Then (6) implies $0 < \theta < 5$. Hence $\theta = 2$ is roughly typical and will be assumed in the rest of this paper.

The density values above are taken from Weir (1998a). Provided θ is constant, this rifting model has constant rates of increase of Layer areas, and so the motion of the interfaces satisfy

$$z_1 = \frac{\dot{A}_1 t}{w} = \frac{Rt}{w} \quad (7)$$

$$z_2 - z_1 = \frac{\dot{A}_2 t + z_3 w_0}{w} \quad (8)$$

$$z_3 - z_2 = \frac{\dot{A}_3 t}{fw} \quad (9)$$

where $\dot{A}_1$, $\dot{A}_2$ and $\dot{A}_3$ are the rates of increase of area for Layer 1, 2 and 3 respectively. Adding (7), (8) and (9) yields

$$z_3 v = \dot{A}_1 + \dot{A}_2 + \frac{\dot{A}_3}{f} \quad (10)$$

From (7) - (9), the asymptotic values of the Layer thicknesses $z_{1\infty}$, $z_{2\infty}$ and $z_{3\infty}$ equal $\dot{A}_1/v$, $\dot{A}_2/v$ and $\dot{A}_3/fv$, respectively, and f is the ratio of the width of Layer 3 to Layer 2.

From (4), $\dot{A}_2 = \theta\, \dot{A}_1$, and so from (7) and (8),

$$\frac{w}{v} = \frac{z_3 t}{z_3 + (1+\theta) z_1 - z_2} \quad (11)$$

and for θ about 2, the ratio of w to v is fixed (at about 2.3 Ma), since all the terms on the right hand side of (11) are known at present. Note that (11) is independent of $\dot{A}_3$, because in this rifting model, there is no relationship between volumes below z_2 and those above z_2. Equation (11) then represents a fundamental constraint on the rifting parameters, and consequently on the flows possible in such rifting models.

Choosing $w = 30$ km, then from (11), $w_0 = 9$ km, and $v = 13$ mm/a. Choosing θ as 2, and using present layer depths, gives λ as 0.4 from (4), and so for these values and this rifting model, Layer 2 is about 40% full of rhyolite. Similarly, the Layer areas increase as $\dot{A}_1 = 1.5\times10^{-6}\ \mathrm{m^2/s}$, $\dot{A}_2 = 3\times10^{-6}\ \mathrm{m^2/s}$, $\dot{A}_3 = 10\times10^{-6}\ \mathrm{m^2/s}$ for $f = 1.6$. Thus volume is being created much faster in Layer 3 than it can be removed above, and so the displaced greywacke initially occupying Layer 3 must be moved out of the TVZ region, probably mostly under the North Island Shear Belt, which must move to the east in response to this flow of incoming rock. Thus uplift of the Axial Ranges of the NISB, erosion and or displacement is needed in this model. "Major uplift and dissection west, south and east of the TVZ between c. 1 Ma and c. 0.34 Ma" is reported by Wilson et al. (p.21, 1995).

Further calculations show, provided $\theta = 2$, that this transient rifting model has the same extension rate (13 mm/a) and initial width (9 km) as the transient spreading model discussed previously by Weir (1998a). Consequently, the two models have identical thermal histories, and so previous difficulties in obtaining 4200 MW from the TVZ are also present in this rifting model. Additionally, present GPS measurements do not support an extension rate of 13 mm/a, but rather only about 8 ± 4 mm/a (Darby and Willaims, 1991). The main difference between the rifting and spreading models then is the composition of Layer 3. The spreading model requires that about 56% of Layer 3 is greywacke, but in the rifting model, no greywacke is present in Layer 3.

3. DISCUSSION AND CONCLUSIONS

A three layer rifting model between the surface and a depth of 25 km was formulated. Material erupts onto the surface, forming Layer 1, to balance the higher density material in Layer 3. The thermal output was equivalent to an earlier spreading model, and consequently the predicted heat output of the model contained a component from intrusions above 8 km which tended to produce a constant heat flow with time of about 1000 MW. Intrusives below 8 km produced an increasing conductive heat flow with time (because of the increasing surface area), which now could be about 2000 MW. Additional to this may be another 1000 MW from andesitic water.

Rising material which stops at a depth above about 8 km is cooled by circulating meteoric water to release its heat and chemicals into rising geothermal plumes. Material which stops below about 8 km depth can contribute some heat by conduction to the circulating groundwater above about 8 km depth. Additionally, andesitic water to the east will enter the geothermal plumes, adding chemicals and heat there.

The key difference between this rifting model and an earlier spreading model is the composition of the deep Layer 3, which is devoid of greywacke in a rifting model, but contains a significant fraction of greywacke in a spreading model. A fundamental constraint in the rifting model on the nature of the extension was derived in (11), which couples the present average width of the TVZ to its present average extension rate, if the model is correct.

A model valid throughout the 1.6 Ma of the TVZ was not found. No heat model for the basic heat source driving the TVZ was presented. However, if the mechanisms operating in the TVZ are those of rifting, then this simple model showed that, to be consistent with the heat flow and seismic records, required spreading rates (13 mm/a) larger than are being currently measured (8 mm/a). The rifting model suggested in this paper was associated with the eastward motion of the NISB. Clearly, more research is needed to clarify these issues.

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Symbol	Meaning	Assumed Value
f	mantle to TVZ width ratio	1.6
t	time	
v	extension rate	13 mm/a
w	average TVZ width	30 km
w_0	initial average TVZ width	9 km
z	depth	
z_1	lower infill surface depth	
z_2	depth of upper Layer 3 surface	
$z_{1\infty}$	asymptotic thickness of Layer 1	4 km
$z_{2\infty}$	asymptotic thickness of Layer 2	7 km
$z_{3\infty}$	asymptotic thickness of Layer 3	14 km
z_3	depth of mantle	25 km
$\dot{A}_1$	Rate of increase of Layer 1	$1.5\ 10^{-6}\,m^2/s$
$\dot{A}_2$	Rate of increase of Layer 2	$3\ 10^{-6}\,m^2/s$
$\dot{A}_3$	Rate of increase of Layer 3	$10\ 10^{-6}\,m^2/s$
R	TVZ eruption rate per unit length	$1.9\times10^{-6}\,m^2/s$
ε	Fraction of greywacke in rhyolite	0
λ	Fraction of rhyolite in Layer 2	0.4
ρ_1	Layer 1 density	2170 kg/m^3
ρ_2	Layer 2 density	2670 kg/m^3
ρ_3	Layer 3 density	2795 kg/m^3
θ	Ratio of rhyolite in Layer 2 wrt Layer 1	

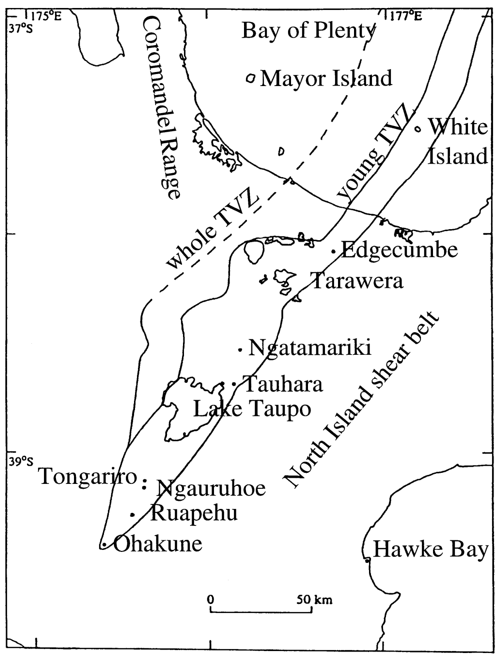

Figure 1. Location of the TVZ

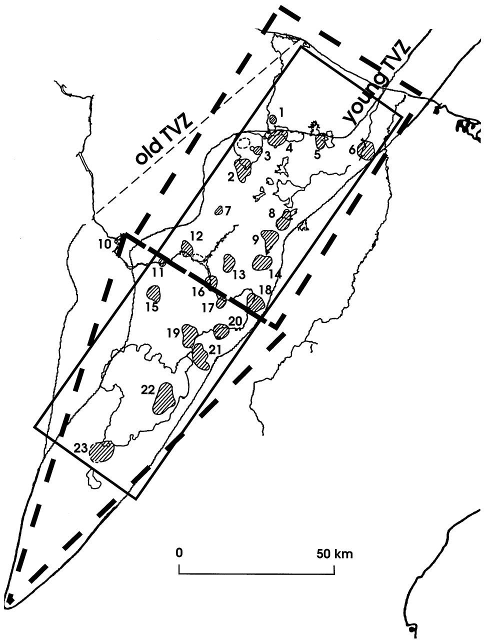

Figure 2. Area estimates for the TVZ. The shaded and labelled areas are the geothermal fields: 1. Taheke, 2. Rotorua, 3. East Rotorua, 4. Rotoiti, 5. Rotoma, 6. Kawerau, 7. Horohoro, 8. Waimangu, 9. Waiotapu-Waikite, 10. Mangakino, 11. Ongaroto, 12. Atiamuri, 13. Te Kopia, 14. Reporoa, 15. Mokai, 16. Orakeikorako, 17. Ngatamariki, 18. Ohaaki, 19. Wairakei, 20. Rotokawa, 21. Tauhara, 22. Lake Taupo, 23. Tokaanu.

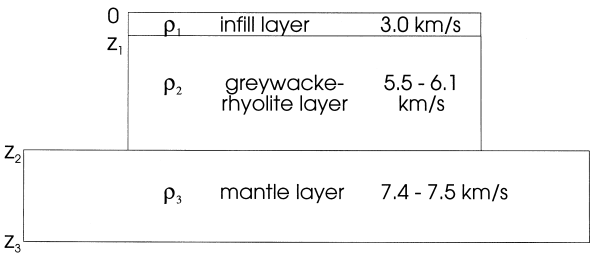

Fig 3. Assumed depth structure in the TVZ