FRACTURE MECHANICS MODELS FOR SENSITIVITY ANALYSIS OF HYDRAULIC FRACTURING

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Keywords: hydraulic fracturing, Monte Carlo, fracture mechanics

ARSTRACT

Most hydrofrac data interpretation for stress derivation only use singular pressure values of characteristic pressurisation phases such as breakdown, fracture reopening or shut-in values. This paper presents an algorithm for automatic sensitivity analysis of hydraulic fracturing parameters on tlic basis of fracture mechanics. The algorithm can be separated into two parts. The first part consists of calculating synthetic pressure records for random modcl assumptions of the underground. The second part compares the nieasured pressure records with the synthetic pressure records by a least square fit. The result of this calculations are presented in the form of intervals for horizontal and vertical stresses, rock mass density, fracture toughness, fracture height and initial fracture length, given by tlic model parameters with the best fit of synthetic and measured pressure records.

1. INTRODUCTION

In this paper, a combination of a fracture mechanics model for hydraulic fracturing strcss determination and an inversion algorithm hased on the monte-carlo principle is presented. The numerical hydrofrac-model takes into account the specific experimental conditions of in-situ stress measurements (small injection rates, low permeability of the crystalline rock, small diameter boreholes and short test interval-length).
The basic idea of the monte-carlo principle (Press, 1968) is to reconstruct a measurement by testing a model of the underground with random modcl parameters on a computer. The most important advantages of this method are:

- this method works for under-determined (more model parameters than measurements) as well as for over detemiined (more measurements than model parameters) problems
- the result of the inversion process shows absolute and not absolute minimums of the error distribution
- no singularities during the determination of the new model parameters
- the performance of tlic algorithm is high. so that is is possible to combine it with a data aquisition program
- the memory requirement of the whole program is very small

2. FRACTURE MECHANICS MODEL OF HYDROFHACTURING

Theoretical pressure records are calculated based on fracture mechanics presented by Winter (1983), Rummel (1978) and Rummel and Hansen (1989). The algorithm assumes a symmetrical double crack of length a and width w in a two dimensional plate containing a borehole with radius R, as shown in figure 1.

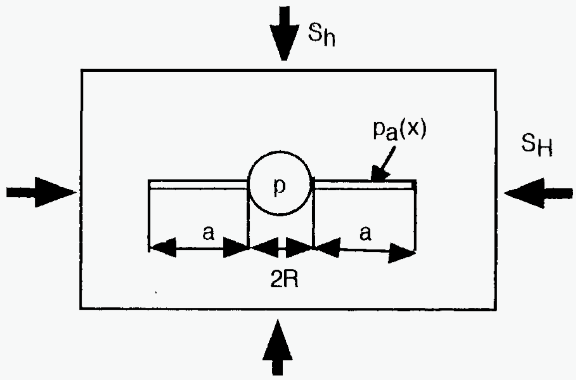


Figure 1: Geometry of the fracture mechanics model (two dimensional).

The plate is subjected to tlic far field stresses S_H and S_h and to the borehole pressure p. The pressure profile within the crack is given by $p_a(x)$. The orientation of S_H is parallel to the propagation of tlic crack. Assuming these conditions, the critical pressure for crack growth p_c is given by:

$$p_c=(K_{IC}/R^{1/2}+f*S_H+g*S_h)/(h_0+h_a) \quad (1)$$

with K_{IC}: fracture toughness

f, g, h_0, h_a: normalized stress intensity functions, depending on the crack length.

The pressure distribution $p_a(x)$ within the crack is calculated by:

$$p_a(x)=\begin{cases} c_1*p*(1-((x-R)/a)*exp(-c_2*R/a), & \text{for } R\le x\le R+a \\ c_1*p*(1+((x+R)/a)*exp(-c_2*R/a), & \text{for } -R-a\le x\le -R \end{cases} \quad (2)$$

with c_1: pressure drop at the fracture inlet; c_2 parameter for modelling the change of pressure gradient in the propagating fracture. Figure 2 shows an example of the pressure distribution within the fracture as the fracture increases in length.

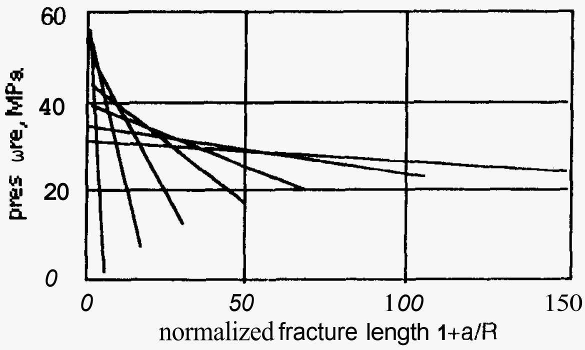


Figure 2: Typical pressure distribution within the fracture.

During fluid injection, elastic energy is stored in the pressurised fluid in the pressure lines. in the test interval and in the fracture itself. The stored energy will partially be released during unstable fracture growth. Episodic dynamic frac growth will crease whenever the potential energy supply is balanced by the energy demand to create a new fracture surface. If the pressure of the total system reaches the critical pressure p_c, the frac-length and the system volume increases, tluid penetrates into the newly created frac space and the pressure will increase. Frac growth will come to a halt if the potential energy of the total system is balanced by tlie energy demand to create new fracture surface.

Fluid loss induced hy fluid penetration lrom the fracture into the rock matrix of the crystalline rock formations is described by Darcy's law:

$$v_y=(k/\eta)*(p_a(x)-p_0)/L(x,t) \qquad (3)$$

where η is the fluid viscosity, k the rock permeability, $p_a(x)$ the fluid pressure distribution within the fracture, p_0 the formation pressure, $L(x,t)$ the penetration depth at each fracture clement at each moment and v_y is the flow velocity into the rock.

3. INVERSION ALGORIHTM

The principle of the Monte-Carlo-Method is to simulate the reality by using a stochastical model of the underground, which is tested by random model parameters. If the difference between the measured and theoretical pressure curves is greater than the threshold the underground assumptions are ignored, otherwise the model parameters are stored. The result of this inversion algorithm is a scatter field of model parameters, given by the ten hest fitting model assumptions. The structure of the inversion procedure is given in the flowchart in figure 3. The input values are: test depth, borehole diameter, diameter of the hydraulic hose, rock permeability, range of rock density, initial fluid penetration depth, range of the fracture toughness, range of the horizontal stresses S_H and S_h, range of intrinsic fracture length, range of tlie fracture height and maximum estimation error between measured and theoretical frac-cycles. In the nest step the measured pressure - time values are stored in a vectorfield, to decrease the performance of the program. After initialising the random number generator the inversion of the measured date started by the calculation of theoretical pressure records for random underground parameters in between the parameter ranges. The following parameters are selected by random numbers: rock density, fracture toughness, horizontal stresses S_H and S_h, intrinsic fracture length aiid fracture height. If the difference between synthetic and measured pressure records is less than a free defined error value according to the L_2-norm (Least-square-norm) the model-assumptions are stored. The inversion algorithm ends when 500 fitting pressure records were estimated. The result of the inversion is a scatter field, estimated by the ten best fitting models.

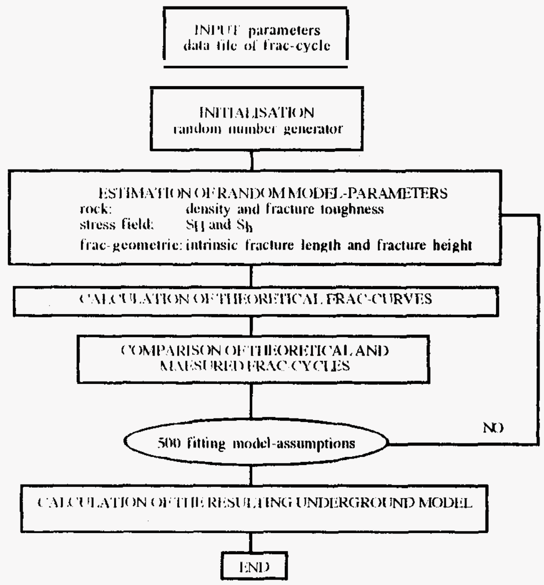
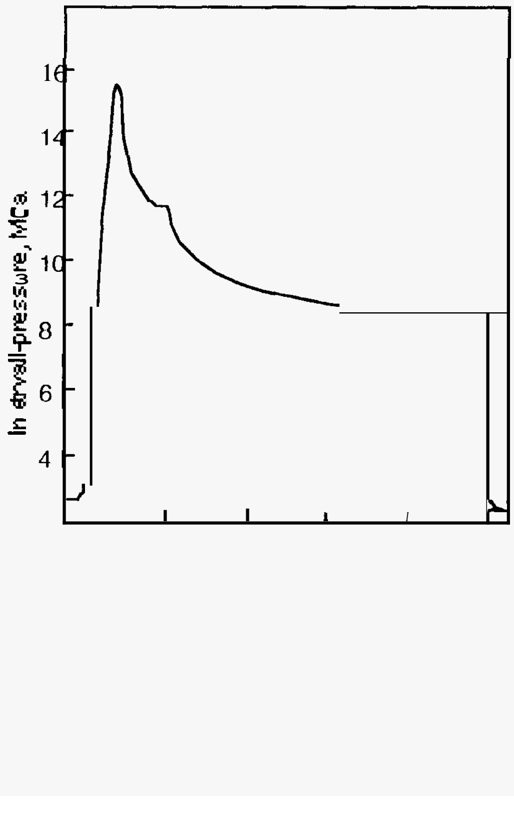


Figure 3: Flow-chart of the Monte-Carlo-Inversion.

4. INVERSION RESULTS OF IN-SITU STRESSMEASUREMENTS

In tlie following the result of the inversion of two different frac cycles are presented. The first frac-cycle used for the inversion was taken from a hydraulic fracturing experiment at a depth of 262 m. The direction of the induced vertical frac is parallel to the direction of S_H. The frac-cycle is given in figure 4.



by the

P_{SI}-method presented by Baumgärtner (1987) on the basis of 12 hydraulic fracturing and 12 impression packer tests. The result of the in-situ stress determination was:

S_H, MPa = 15.1
S_h, MPa = 10.0
S_v, MPa = 7.1

The results of laboratory core testing yield a mean core density of 2.73 g/cm^3 and a mean fracture toughness of 1.5 MN/m$^{3/2}$. For the inversion of the frac-cycle at 202 m depth tlie following input parameters were used:

depth, m	= 202
borehole radius, mm	= 73
hydraulic hose radius, mm	= 5
permeability, 10^{-6} µdarcy	= 1
density, g/cm^3	= 2.5 - 3.0
fracture tounghness, MN/m$^{3/2}$	= 1.0 - 2.0
major horizontal stress, MPa	= 10 - 20
minor horizontal stress, MPa	= 5 - 15
intrinsic fracture length, mm	= 1 - 50
fracture height, m	= 0.5 - 1.0

The result of the inversion can be summarised as follows:

density, g/cm^3	= 2.75
fracture tounghness, MN/m$^{3/2}$	= 1.3
major horizontal stress, MPa	= 18.7
minor horizontal stress, MPa	= 11.1
vertical stress, MPa	= 7.1
intrinsic fracture length, mm	= 17
fracture height, m	= 0.86

Figure 5 shows the result of the inversion in the form of inversion parameter vs. average plots. This type of presentation makes it possible to analyse the sensitivity of each parameter. Sharp minimums show a greater influence of tlie corrosponding parameter on the pressure record than diffuse minimums. The error of tlie corrospondinp parameter is smaller, if the minimum is sharp. The plots of figure 5 show miniums which are quite near by tlie estimated values, determined by tlie P_{SI}-method or measured in the laboratory, although the input parameter ranges are chosen quite wide.
The second frac-cycle used for the inversion was taken from a hydraulic fracturing experiment at a depth of 110 m. The direction of the induced vertical frac is parallel to the direction of S_H. The frac-cycle is given in figure 6.

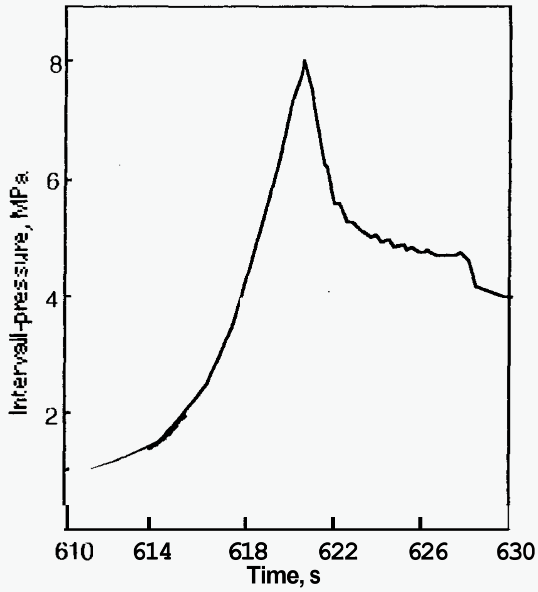


Figure 6 Frac-cycle in a depth of 110 m

The stress regime around this borehole was estimated by the classical Hubbert arid Willis concept (6) on the basis of 12 hydraulic fracturing and 12 impression packer tests. The result of the in-situ stress determination was:

S_H, MPa = 5.1
S_h, MPa = 3.1
S_v, MPa = 2.9

The results of laboratory core testing yield a mean cure density of 2.73 g/cm^3. The following input parameters were used to estimate the model parameters:

depth, m	= 110
borehole radius, mm	= 48
hydraulic hose radius, mm	= 5
permeability, 10^{-6} µdarcy	= 1
density, g/cm^3	= 2.5 - 2.8
fracture tounghness, MN/m$^{3/2}$	= 1.0 - 2.0
major horizontal stress, MPa	= 1 - 10
minor horizontal stress, MPa	= 1 - 10
intrinsic fracture length, mm	= 1 - 50
fracture height, m	= 0.5 - 1.0

The estimated model parameters are:

density, g/cm^3	= 2.65
fracture tounghness, MN/m$^{3/2}$	= 1.60
major horizontal stress, MPa	= 4.77
minor horizontal stress, MPa	= 2.86
intrinsic fracture length, mm	= 16
fracture height, m	= 0,68

Figure 7 gives the model parameter-error distribution of each parameter. These plots can be summarised as follows. The minimums are quite wide, so the corrosponding error is larger than in tlie first example. Although tlie error is larger tlie mean result of the inversion fits quite well the laboratory and classical analysis of the characteristic pressure values

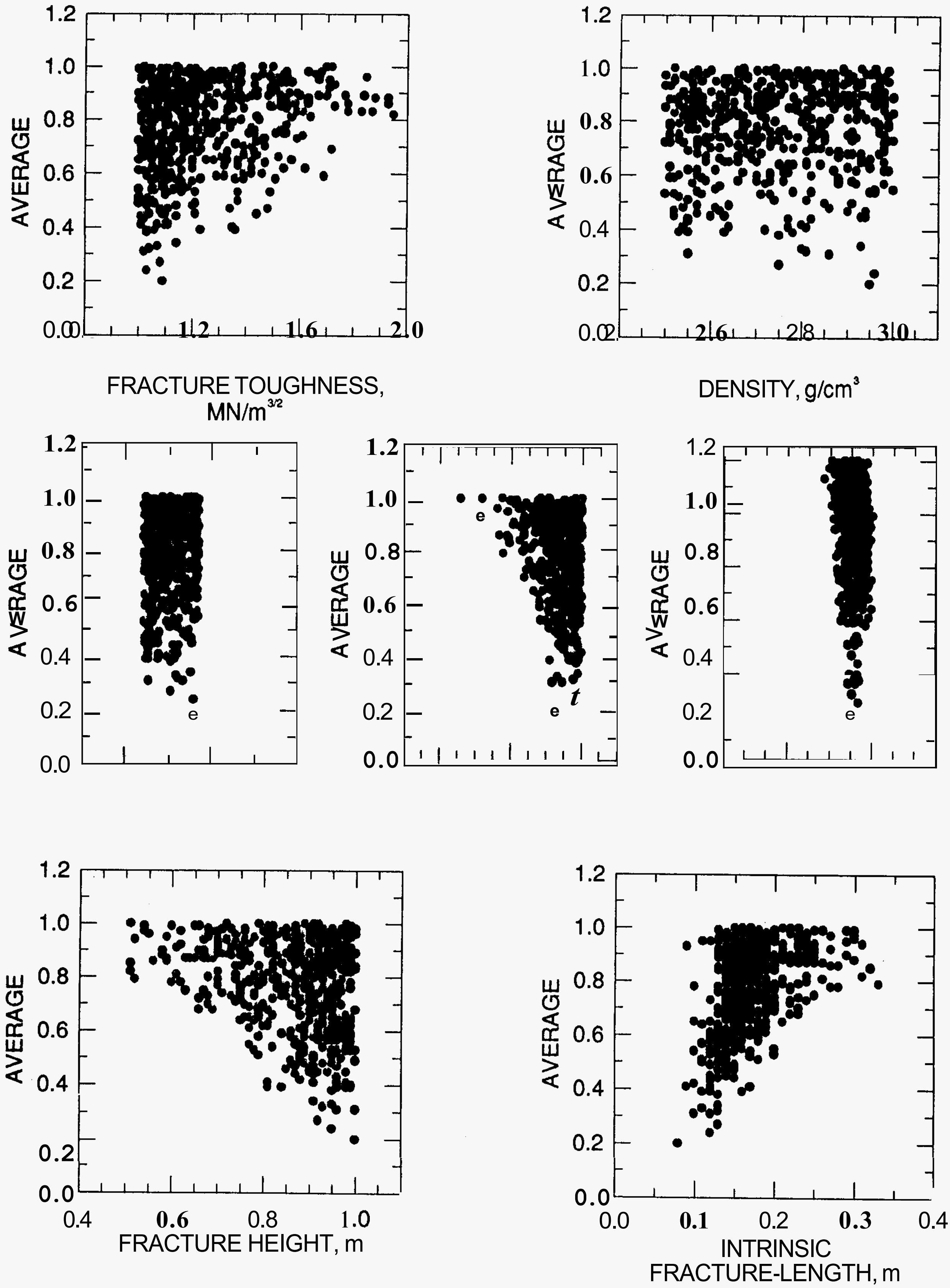


Figure 5: Results of the inversion of the Frac-cycle in a depth of 262 m.

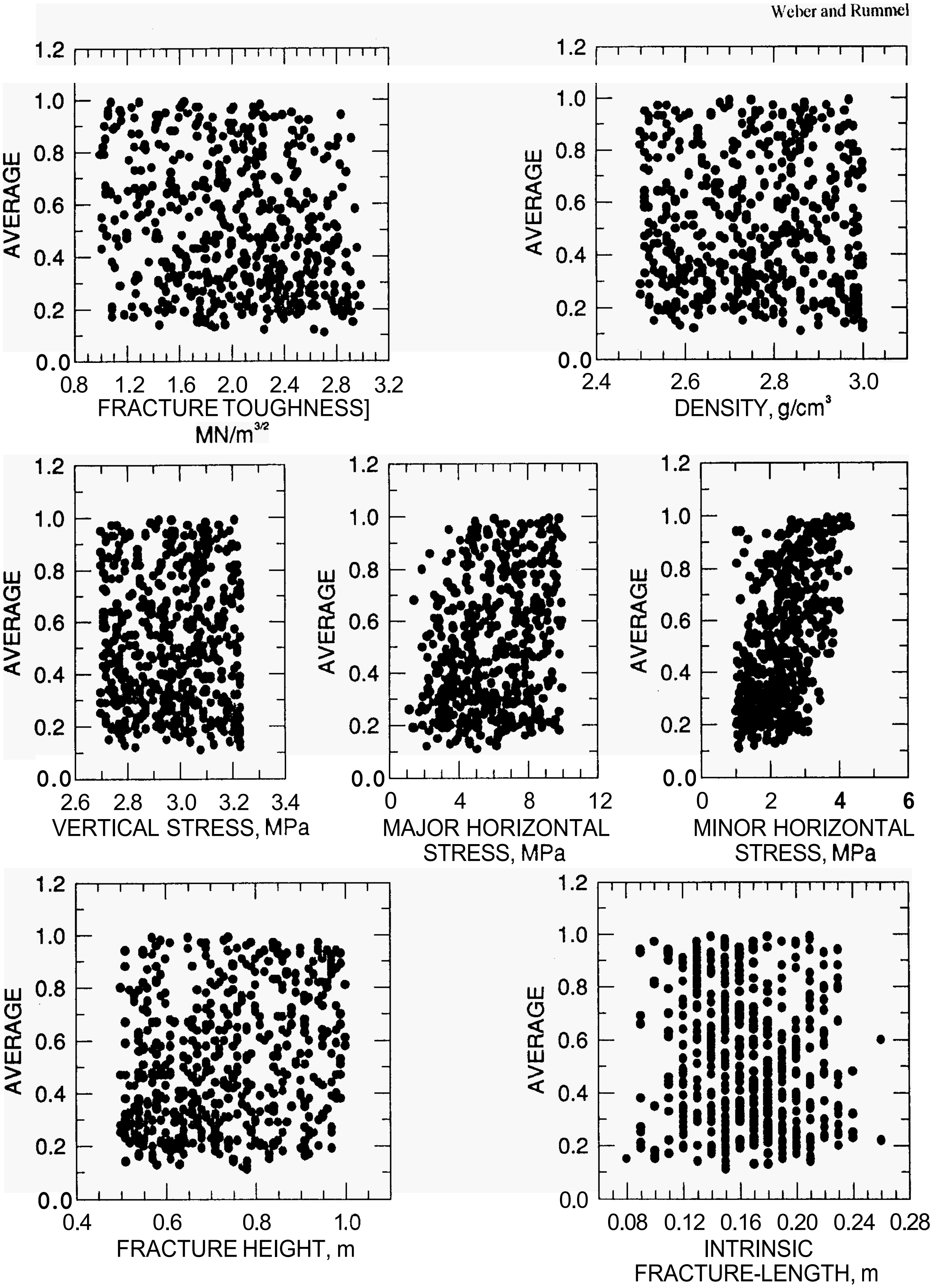


Figure 7: Results of the inversion of the Frac-cycle in a depth of 110 m

5. CONCLUSIONS

All parameters (density, fracture tounghness, major horizontal stress, minor horizontal stress.intrinsic fracture length and fracture height) estimated by the inversion of a single hydrofrac-cycle yield acceptable values. The density, fracture toughness and intrinsic fracture values are fitting the laboratory results measured on representative core material of the borehole. The minor horizontal stressfield components S_h reproduce the values estimated on the basis of the shut-in values. The mean major horizontal stress S_H calculated by the inversion is larger than the values recieved by the shut-in and retmc-pressure values. The fracture height is in the range of heights seen on the impression packers.
In the future this algorithm will be combined with a data aquisition program to estimate the characteristic parameters during the hydrofracturing experiments.

6. ACKNOWLEDGEMENT

This work is founded by the Germean Ministry of Research and Technology (BMFT) under contract no 03266901B.

7. LITERATURE

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