

HYDROFRACTURE STRESS MEASUREMENTS UNDER CONDITIONS OF HIGH TEMPERATURE

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ABSTRACT

The effect of thermal stress on the so-called shut-in pressure is investigated theoretically and experimentally for in-situ stress measurements. It is considered here that a pre-existing high temperature around a borehole is disturbed due to circulation of cold water before a packer system is set at a depth in the borehole. The results show that a value of the in-situ compressive stress normal to the crack plane can be estimated from variation of the shut-in pressure with the period of circulating time as follows. At first, the shut-in pressure is plotted as a function of the period of circulating time to the power of 0.68. The plot fits to a straight line. The line is extrapolated to the time when the period of circulating time is zero. Then, a value of pressure determined by the extrapolation gives the value of the in-situ compressive stress normal to the crack plane.

1. INTRODUCTION

In-situ stresses are key parameters in the design of a hot-dry-rock (HDR) operation for extraction and the economic use of geothermal energy. In-situ stresses control the operation pressure to activate pre-existing joint systems, determine the underground fluid flow paths, or influence borehole stability in uncased borehole sections (Klee and Rummel, 1993).

Hydraulic fracturing is the method commonly used for the in-situ stress measurement at great depths (e.g., Mizuta et al., 1987; Baumgärtner and Rummel, 1989; Ikeda and Tsukahara, 1989; Cornet, 1992; Haimson et al., 1993; Hayashi et al., 1989, 1993). With this method, a portion of borehole which is free from natural fissures is sealed off and is pressurized with a hydraulic fluid such as water. At the so-called breakdown pressure, the rock bursts and tensile cracks are developed at the borehole wall. The cracks are extended by continued pumping. When the pump is shut off without venting the hydraulic line, the so-called shut-in pressure is recorded. This is interpreted as the pressure necessary to keep the crack open. Finally, the in-situ stresses are computed from the breakdown and shut-in pressures based on relations among the in-situ stresses and those pressures which are derived theoretically in advance.

In the procedure of hydraulic fracturing, the test interval of borehole is sealed off with a straddle packer system. Because the system

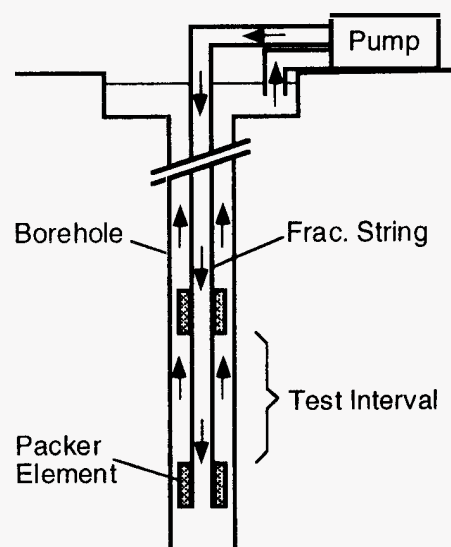


Figure 1. Setup for circulation of cold water.

consists of two inflatable packer elements which are basically made of rubber, a maximum temperature of the system is usually so low ($< 150^{\circ}\text{C}$) that the system cannot function under a high temperature environment in a geothermal field. Hence, if the hydraulic fracturing is conducted in the geothermal field, the borehole wall must be cooled somehow, e.g., circulation of cold water in the borehole as shown in Figure 1 before the straddle packer system is set at the depth of test interval. In such a case, due to the cooling of the borehole wall, a pre-existing high temperature around the borehole is disturbed, so that the disturbance induces thermal stresses in the region surrounding the borehole. Therefore, these thermal stresses must be taken into account in the fracturing theory, i.e., the relations among the in-situ stresses and the observed pressure data, for the in-situ stress measurement by hydraulic fracturing. Stephan and Voight (1982) have studied the effect of the thermal stress on the breakdown pressure. The results show that an important effect of thermal stress is imposed on the results of in-situ stress measurement. Then, they have presented a modified hydraulic fracturing theory with the breakdown pressure. However, the effect of thermal stresses on another pressure data, i.e., the shut-in pressure, has not been investigated so far.

In this regard, in the present paper, the effect of thermal stress on the shut-in pressure was investigated theoretically and experimentally for

the case that the pre-existing high temperature around the borehole was disturbed due to the circulation of cold water. To this end, the induced thermal stresses in the region surrounding the borehole were investigated theoretically, and we discuss the variation of the shut-in pressure due to the thermal stresses. Furthermore, the variation of the shut-in pressure with the period of circulating time was investigated through laboratory hydraulic fracturing experiments.

2. THERMAL STRESSES AROUND BOREHOLE

Let us consider a cylindrical borehole in an infinite homogeneous elastic medium. The initial temperature of medium is v_0 everywhere, and its temperature at the borehole wall is changed to V , at the time t is zero. The temperature change induces radial variation of temperature field in the medium surrounding the borehole. The variation of temperature field induces thermal stresses around the borehole. Here, it is assumed that existence of cracks created by hydraulic fracturing does not affect the distribution of thermal stress. Then, the thermal stresses are given by (Nowacki, 1962):

$$\left. \begin{aligned} \sigma_r &= \frac{1}{r^2} \int_0^r v(s, T) s ds \\ \sigma_\theta &= -\frac{1}{r^2} \int_0^r v(s, T) s ds + v(r, T) \\ \sigma_z &= v(r, T) \end{aligned} \right\}$$

where tensile stresses are taken to be positive. The notations σ_r , σ_θ and σ_z are non-dimensional notations of the stress components σ_r , σ_θ and σ_z referred to a cylindrical coordinate system (r, θ, z) (Figure 2), and \bar{r} , \bar{t} and \bar{v} are non-dimensional notations of r , t and the radially varying temperature v respectively. They are defined as follows:

$$\left. \begin{aligned} \sigma_r &= -\frac{1-\nu}{\alpha E \Delta v} \sigma_r, \quad \sigma_\theta = -\frac{1-\nu}{\alpha E \Delta v} \sigma_\theta, \quad \sigma_z = -\frac{1-\nu}{\alpha E \Delta v} \sigma_z, \\ \bar{r} &= \frac{r}{a}, \quad \bar{t} = \frac{v}{\Delta v}, \quad \bar{T} = \frac{\kappa t}{a^2} \end{aligned} \right\}$$

where a is the borehole radius, ν , E , α and κ are Poisson's ratio, Young's modulus, coefficient of thermal expansion and thermal diffusivity of the medium respectively, and Δv is $\Delta v = V - v_0$. Therefore, the value of Δv is negative for the case that the borehole is cooled, i.e., $V < v_0$, by means of the circulation of cold water as described in the introduction. On the other hand, \bar{v} is given by (Carslaw and Jaeger, 1959):

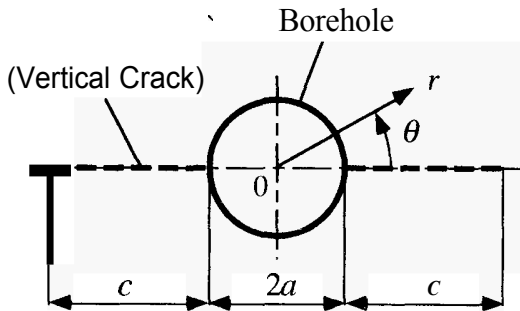


Figure 2. Vertical cracks and a cylindrical coordinate system. It is assumed here that the existence of cracks does not affect the distribution of thermal stress.

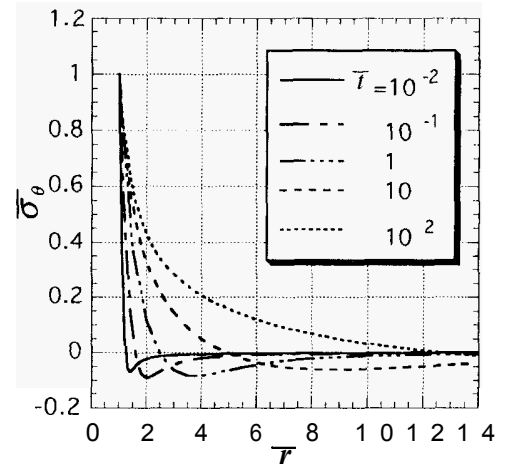


Figure 3. Distribution of thermal stress around a borehole

$$\bar{v}(\bar{r}, \bar{T}) = 1 + \frac{2}{\pi} \int_0^\infty e^{-u^2 \bar{T}} \frac{J_0(u \bar{r}) Y_0(u) - Y_0(u \bar{r}) J_0(u)}{J_0^2(u) + Y_0^2(u)} \frac{du}{u} \quad (3)$$

where J_0 and Y_0 are the Bessel functions of the first and second kinds of order zero, respectively.

Figure 3 shows distribution of the thermal circumferential stress σ_θ , which is obtained from Eqs.(1)-(3). As can be seen from the figure, σ_θ has a maximum value at the borehole wall and decreases with radial distance from the borehole wall. Its maximum value is independent of time, however, the slope of the stress distribution decreases gradually with time. When it is assumed that $\kappa = 3 \times 10^{-7}$ in 2 /sec and $a = 50$ mm, a value of $\bar{T} = 10$ corresponds to about 23 hours.

On the other hand, the shut-in pressure is interpreted as the pressure necessary to keep the crack open, therefore, the shut-in pressure is considered to indicate a net compressive stress across a crack plane which is induced by hydraulic fracturing. Usually, the net compressive stress is almost equal to the in-situ compressive stress, i.e., a far-field compressive stress, normal to the crack plane and so the shut-in pressure can be used as an indicator of the in-situ compressive stress. However, as shown in Figure 3, if the pre-existing high temperature around the borehole is disturbed, the thermal stress is induced in the region surrounding the borehole due to the disturbance of temperature. Taking account of the results shown in Figure 3 and the fact that the shut-in pressure indicates the net compressive stress across the crack plane, we can consider as follows. When the shut-in pressure is obtained after the cooling of borehole, the value of shut-in pressure will be generally smaller than that of compressive stress normal to the crack plane due to the effect of thermal stress. The difference between the values of the shut-in pressure and the in-situ compressive stress normal to the crack plane will increase with increasing period of the cooling time. But, if the period of cooling time is relatively short, the value of shut-in pressure will be almost equal to the value of the in-situ compressive stress normal to the crack plane. The reason is that the distribution of thermal stress is localized in the region fairly close to the borehole wall in such a case. Next, let us discuss these concepts based on the results of laboratory hydraulic fracturing experiments.

3. LABORATORY HYDRAULIC FRACTURING EXPERIMENTS AND DISCUSSIONS

3.1 Experimental Procedure

Kofu andesite was used as a rock sample. Specimen size was $0.15 \times 0.15 \times 0.15 \text{ m}^3$ and the borehole with diameter of $2a = 15 \text{ mm}$ was drilled into the specimen. Three specimens were used. We will refer to them as specimens A, B and C. In order to simulate the in-situ stresses, the specimen was loaded under biaxial compressive stresses, i.e., the two horizontal compressive stresses S_1 and S_2 ($S_1 \geq S_2$), by using flat jacks. The borehole was pressurized by using a specially designed double packer jig analogous to a straddle packer system (Figure 4) and by using a hydraulic booster connected to a motor driven pump, where water was used as a fracturing fluid. The borehole was vertical and the induced cracks were also vertical, so that the shut-in pressure should indicate the applied compressive stress S_2 , if there is no effect of thermal stress. For the case of experiment under a high temperature environment, the specimen was heated by a servo-controlled electric resistance furnace and the temperature at the side wall of specimen was held constant throughout the experiment. The borehole wall was cooled by circulation of cold water with a constant temperature as shown in Figure 4, where the temperature at the borehole wall of the specimen, V , is assumed to be equal to that of circulated water. By using such a setup, the experiments were conducted through the following procedure:

1. Apply the compressive stresses, i.e., $S_1 = 15$, $S_2 = 10 \text{ MPa}$, to the specimen. Set the packer jig with $b = 40 \text{ mm}$ at a middle depth of the borehole, where b is the axial length of pressurized interval of the jig, and pressurize the interval with injection of fluid until the vertical cracks are created from the borehole wall.
2. Extend b to 60 mm so that the interval contained the cracks and then raise the temperature of specimen to $v = v_0$.

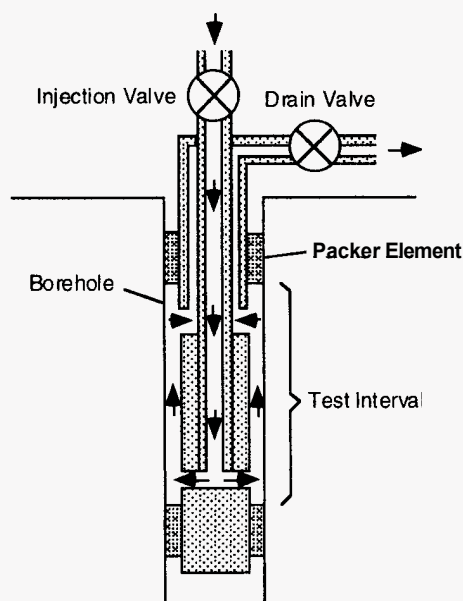


Figure 4. Schematic illustration of a packer jig and its setup for the circulation of water.

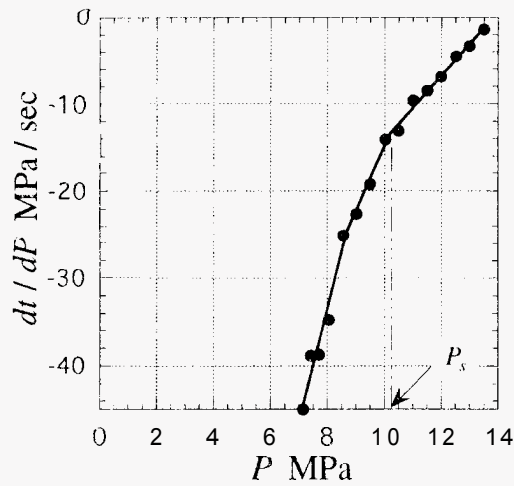
3. Inject water into the borehole with a constant injection flow rate of 12000 cc/h ($= 3.3 \times 10^{-6} \text{ m}^3/\text{sec}$) for a period of time, t , with the drain valve held open (Figure 4).
4. Close the drain valve and start immediately to pressurize the borehole with a constant injection flow rate, i.e., 100 cc/h ($= 2.8 \times 10^{-6} \text{ m}^3/\text{sec}$), in order to reopen the cracks.
5. When the crack reopening is observed from the borehole pressure vs time record, close the injection valve, i.e., shut-in, and monitor the decay of borehole pressure after the shut-in.
6. Detect the shut-in pressure, P_s , from the pressure decay.

In the present study, the shut-in pressure is detected based on the method presented by Hayashi and Haimson (1991). They have suggested that the pressure decay curve is trilinear in the plot of the inverse of pressure decrease rate, dt/dP , vs the borehole pressure P and the pressure at the intersection of the first and second lines corresponds to the shut-in pressure. The temperature of either 60 or 80°C was applied to the specimen for this experiment under a high temperature environment. It should be noted that the temperatures of 60 and 80°C correspond respectively to those of 233 and 275°C in the case of an actual field test (see the appendix). Also, in some cases, the experiments were conducted under the conditions that the specimen was heated but the borehole was not cooled by the circulation of water, i.e., $t = 0$. Then, the inner tube in the packer jig (Figure 4) was replaced by a thermocouple and the temperature in the borehole was monitored during the pressurization for the reopening of the cracks, where the pressurization was conducted through the drain line in this case. However, the temperature was almost constant, i.e., $V \approx v_0$ (the difference is less than 2°C), in spite of the fact that water at 20°C was injected continuously into the borehole in the specimen with either $v_0 = 60$ or 80°C . The reason for this is considered to be that the mass of injected fluid is very small compared to the mass of fluid stored initially in the pressurized interval and tubing.

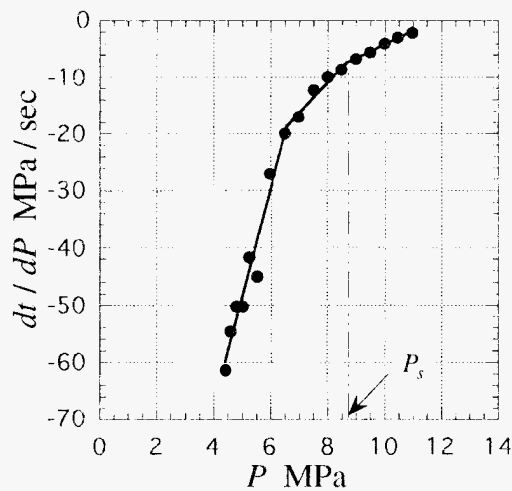
3.2 Results of Experiments

Figures 5 (a) and (b) show examples of the plot of dt/dP vs P which are replotted from the pressure decay curves after the shut-in. In these figures, arrows point to the shut-in pressures which are determined by the method of Hayashi and Haimson (1991). Figure 5 (a) shows the result of an experiment which was conducted under usual conditions, i.e., the temperature of the specimen, v_0 , was 60°C but the borehole was not cooled by the circulation of water, i.e., $t = 0$. The results show that the value of shut-in pressure is almost equal to the value of applied compressive stress normal to the crack plane, i.e., $S_2 = 10 \text{ MPa}$. The reason is that, if there is no difference between the temperature of fluid in the borehole and that of the specimen, a thermal stress is not induced.

On the other hand, Figure 5 (b) shows the result of an experiment conducted under the conditions that $v_0 = 60^\circ \text{C}$, $V = 20^\circ \text{C}$ and the borehole was cooled by the circulation of water for a period of time, i.e., $t = 1300 \text{ sec}$. As can be seen, the value of shut-in pressure is clearly lower than that of S_2 in this case. Thus, the relation between P_s and t was investigated experimentally. Examples of the results are summarized in Figure 6. In the figure,



(a)



(b)

Figure 5. Variation of dt/dP with P ; (a) for the case of $v_0 = 60$ °C, $V = 20$ °C and $t = 0$, (b) for the case of $v_0 = 60$ °C, $V = 20$ °C and $t = 1300$ sec.

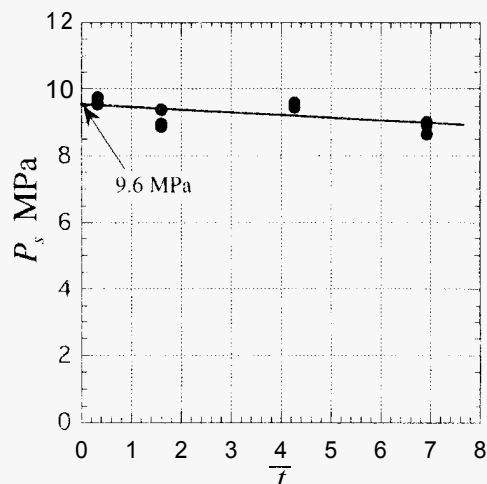


Figure 6. Variation of P_s with \bar{T} for the case of $v_0 = 60$ °C, $V = 20$ °C.

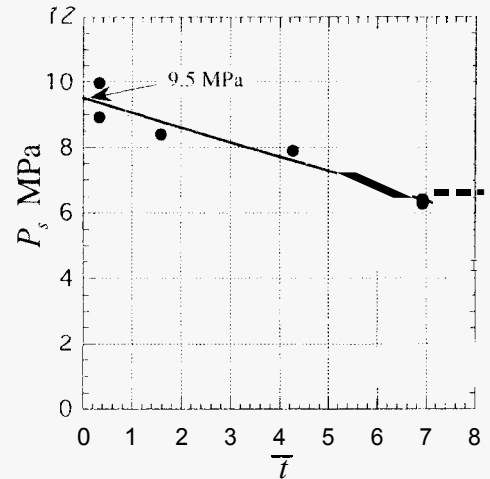


Figure 7. Variation of P_s with \bar{T} for the case of $v_0 = 80$ °C, $V = 20$ °C.

Table 1. Physical properties of rocks

physical properties	rock	
	Kofu andesite	Kowasegawa tuff
ν	0.22	0.37
E MPa	59	20
α (°C) ⁻¹	7×10^{-6}	8×10^{-6}
κ m ² /sec	3×10^{-7}	---

P_s is plotted as a function of \bar{T} in order for comparison with the results shown in Figure 3. For instance, the time 1300 sec of t corresponds to the non-dimensional time 6.9 of \bar{T} , where the thermal constants of Kofu andesite are shown in Table 1. Also, the experimental results for the case of $v_0 = 80$ °C, $V = 20$ °C are shown in Figure 7. As shown in Figures 6 and 7, if \bar{T} is short, P_s is almost equal to S_2 , however, P_s decreases with increasing \bar{T} . Furthermore, the slope of the plot in Figure 7 is steeper than that in Figure 6.

4 DISCUSSION AND CONCLUSIONS

Such results as those shown in Figures 6 and 7 must be caused by the fact that the shut-in pressure is affected by the thermal stress around the borehole and its effect becomes larger with increasing \bar{T} and $|\Delta v|$. Now let us discuss in more detail the relation between the shut-in pressure and the thermal stress.

At first, the amount of decrease in P_s due to the thermal stress is denoted by f , i.e., the relation between P_s and the in-situ stress S_2 is given by

$$P_s = S_2 - f \quad (4)$$

Furthermore, it is assumed that the value of f is approximately equal to the net value of the circumferential thermal stress σ_θ (Eq.(3)) given by

$$f = -\frac{1-\nu}{\alpha E \Delta v} \bar{T} = -\frac{1-\nu}{\alpha E \Delta v} \frac{1}{c} \int_1^{\bar{r}+1} \sigma_\theta d\bar{r} \quad (5)$$

where

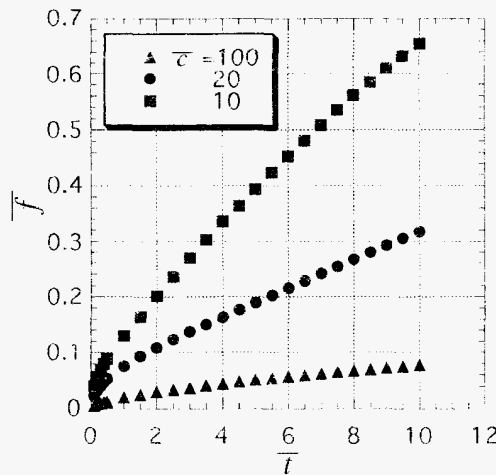


Figure 8. Variation of \bar{f} with \bar{T} for several values of crack length, $\bar{\tau}$.

$$\bar{\tau} = \frac{c}{a} \quad (6)$$

A two dimensional crack is considered here and c is the length of the crack (Figure 2). Figure 8 shows the relation between \bar{f} and \bar{T} based on Eq.(5) for several values of $\bar{\tau}$. The results show that the value of \bar{f} increases with \bar{T} as can be expected from the experimental results shown in Figures 6 and 7. Furthermore, the results suggest that the relation between \bar{f} and \bar{T} can be expressed approximately by $\bar{f} = \beta \bar{T}^n$, where β and n are constants independent of t . Therefore, we tried to find a value of n from the several plots of \bar{f} vs \bar{T}^n by visual inspection. The result shows that the value 0.68 for n is appropriate in the range of $\bar{\tau}$ between 10 and 1000. This result means that the relation between P_s and \bar{T} is approximately given by

$$P_s \approx S_2 - \beta' \bar{T}^{0.68} \quad (7)$$

where

$$\beta' = \left(-\frac{1-\nu}{\alpha E \Delta v} \right) \beta \quad (8)$$

Moreover, it is found out from Eq.(7) that the in-situ compressive stress normal to the crack plane, S_2 , can be determined from the variation of P_s with \bar{T} as follows. At first, we plot the variation of P_s with \bar{T} on the plot of P_s vs $\bar{T}^{0.68}$. The plot is fitted by a straight line. We extrapolate the line to $\bar{T} = 0$ to get the value of P_s which corresponds to the value of S_2 .

By these considerations, the plots of P_s vs \bar{T} shown in Figures 6 and 7 were replotted in the form of a P_s vs $\bar{T}^{0.68}$ plot. The results are shown in Figures 9 and 10 corresponding to Figures 6 and 7 respectively. These figures show that each data plot can be fitted by a straight line (Where the line was obtained based on the least square method). The values of S_2 estimated by the present method are summarized in Table 2. The results show that each estimated value of S_2 is sufficiently close to the applied value of S_2 . Also, the values of S_2 which were estimated through the similar procedure but directly from the plot of P_s vs \bar{T} , i.e., $n = 1$, are shown in Table 2. As can be seen from the table, the values of S_2 which were estimated from the plot of P_s vs \bar{T} are also close to the applied value of S_2 , although the results are seemed to be less reliable compared with the results based on the plot of P_s vs $\bar{T}^{0.68}$.

Incidentally, Eq.(7) can be modified as follows:

$$P_s \approx S_2 - \beta'' t^{0.68} \quad (9)$$

where

$$\beta'' = \left(\frac{\kappa}{a^2} \right)^{0.68} \beta' \quad (10)$$

The constant β'' is independent of t . Therefore, the value of S_2 can be estimated from the plot of P_s vs $t^{0.68}$ through a procedure similar to that based on the plot of P_s vs $\bar{T}^{0.68}$.

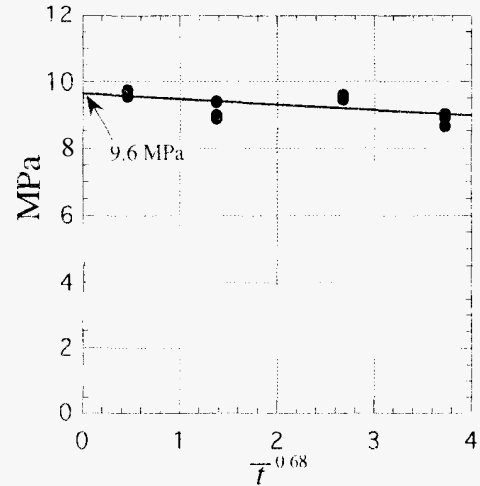


Figure 9. Variation of P_s with $\bar{T}^{0.68}$ for the case of $v_0 = 60^\circ\text{C}$, $V = 20^\circ\text{C}$.

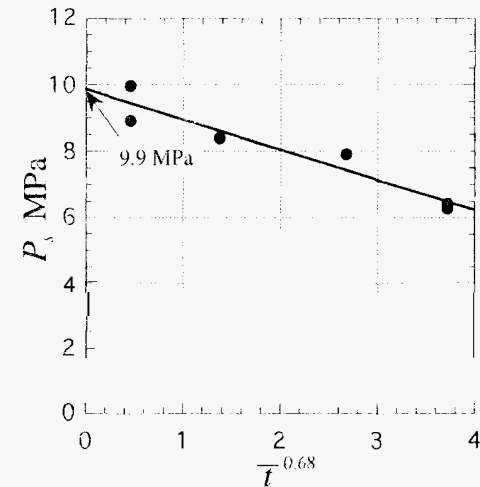


Figure 10. Variation of P_s with $\bar{T}^{0.68}$ for the case of $v_0 = 80^\circ\text{C}$, $V = 20^\circ\text{C}$.

Table 2. Comparison among the applied value of S_2 ($= 10$ MPa) and its values estimated by the present method.

v_0 °C	specimen	estimated S_2 MPa	
		P_s vs $\bar{T}^{0.68}$	P_s vs \bar{T}
60	A	9.7	9.6
	B	9.6	9.6
80	B	9.9	9.5
	C	9.2	8.9

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APPENDIX - SCALING OF RESULTS TO FIELD TEST CONDITIONS

Let us consider the relation between the shut-in pressures which are obtained in two cases referred as case L and case F (Table AI). It is assumed that, in the two cases, the geometry of borehole and crack,

material constants of rock and the temperature difference $\Delta v (= V - v_{ol})$ are different but the ratio c/a and the in-situ stresses, i.e., the stresses corresponding to S_1 and S_2 for the present laboratory experiment, are same. Besides, the parameters in the cases L and F are represented by the subscripts L and F respectively, e.g., E , is Young's modulus of rock in the case L for instance. Furthermore, it is assumed that the thermal stress is approximately given by Eq.(1). Then, taking account of Eq.(1) and the manner of stress distribution induced by the in-situ stresses, it can be easily understood that the stress state at (τ_L, θ_L) in the case L is same as the stress state at (τ_F, θ_F) in the case F, when $\tau_L = \tau_F$, $\theta_L = \theta_F$, $T_L = T_F$ and a following condition is satisfied.

$$\frac{\alpha_L E_L}{1 - \nu_L} \Delta v_L = \frac{\alpha_F E_F}{1 - \nu_F} \Delta v_F \quad (A1)$$

It means that, when T is same in the two cases and Eq.(A1) is satisfied, the shut-in pressures observed in the two cases are same, because the stress distribution surrounding the borehole and crack are then similar in the two cases. Thus, from Eq.(A1), we can evaluate Δv_L (or Δv_F) corresponding to Δv (or Δv_{ol}) which satisfies the condition that the shut-in pressures observed in the cases L and F are same. As an example, it is considered here that the cases L and F correspond to the case of present laboratory experiment and to the case that the hydraulic fracturing is conducted in Kowasegawa tuff (see Niitsuma et al., 1989) in an actual geothermal field. In the case L, the experiment was conducted by using Kofu andesite and the temperature differences Δv_L were -40 and -60 °C, i.e., $V_L = 20$ (°C) and $v_{ol} = 60$ and 80 (°C). On the other hand, in the case F, the temperature of rock at the borehole wall, V_F , is assumed to be cooled down to 150 °C. From these conditions and Eq.(A1), the temperature differences in the case F, Δv_F , are evaluated as -83 and -125 °C correspondingly to $\Delta v_L = -40$ and -60 (°C) respectively, where the material constants of Kofu andesite and Kowasegawa tuff are summarized in Table I. Hence, the initial rock temperatures in the case F are evaluated as $v_{oF} (= V_F - \Delta v_F) = 233$ and 275 (°C) correspondingly to $v_{oL} = 60$ and 80 (°C) in the case L respectively.

Table AI Relation between case L and case F.

	case L	case F
geometry	a_L, c_L	c_F
	$c_L / a_L = c_F / a_F$	
material constants	α_L, E_L, ν_L	α_F, E_F, ν_F
	(independent)	
temperature	$\Delta v_L (= V_L - v_{oL})$	$\Delta v_F (= V_F - v_{oF})$
	(see the text (Eq.(A1)))	
in-situ stresses	S_{1L}, S_{2L}	S_{1F}, S_{2F}
	$S_{1F} = S_{1L}, S_{2L} = S_{2F}$	