Non-linear Modelling of a Geothermal Pipe Network System - A computer approach to network analysis

Huang Yicun and D.H. **Freeston**

Geothermal Institute, University of Auckland, Private Bag 92019, Auckland, New Zealand

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ABSTRACT -A computer model has been developed for simulating and analyzing the performance of a geothermal pipe network system. A non-linear numerical method is used for solving the mass and energy balance equation set of the model. A typical system includes: a number of production and reinjection wells, a steam turbine, separation plant, a reinjection pump station, booster reinjection pump where necessary, and all the pipelines and fittings which handle steam, water and two-phase steam/water mixtures. By utilizing the computer code developed, the performance of a network system, i.e., the pressure and mass flow rate in each pipeline, can be predicted for operational conditions. This paper provides a methodology for solving the effect of the interactions between the different production and reinjection wells and the reinjection pumps in a complex system of pipes. The computer code of this model is PC-based and the program can be used as a design tool or as a simulator.

1. INTRODUCTION

Geothermal supply and reinjection pipe networks are a necessary part of a geothermal utilization system, particularly in a geothermal power plant. The operation and control of such a system depend on the characteristic of the pipe network as well as the production rate, reinjection rate, reinjection pump characteristic, turbine inlet pressure,etc. For a certain operational condition, the prediction of the performance of a network system are of great importance for both the design and the operation of the system. In many geothermal power plants around the world, these predictions are dependent upon the engineer's experience rather than a more effective and reliable tool.

A computer simulator for pipe network performance analysis can offer people an effective way of dealing with the above problem. A numerical simulation of a steam pipeline network in Larderello geothermal field, Italy, made use of a simulator VAPSAT1 (Marconcini and Neri, 1979). This work was based on the calculation for localiwd head losses caused by variations in diameter or sudden changes in direction of the fluid. The pressure-mass flow rate curve at the manifold was calculated based on the back-pressure curves of all the wells supplying the network. Another computer approach of pipe network analysis which has a wider range of application, although designed for an oil-field production and distribution system is that of Sachdeva, 1990. This method relies on modifications of Newton-Raphson's techniques to give solutions for pressure and flow rate residuals at each node of a pipe line system. The practical use of the approach is largely limited because of the requirement of the writing of a rather cumbersome computer code every time.

This paper presents recent development work on a numerical model which uses a non-linear method to simulate a geothermal pipeline network system. The constraints which control the mass and pressure balance at each node and loop of a pipe network are considered to be the characteristics of the production and reinjection wells, the reinjection pump characteristic and turbine inlet pressure etc. The two-phase distribution pipeline network and the steam and water pipeline network are described with different models respectively. In order to handle a more complex system with a high and intermediate pressure steam turbine, a two stage separation process is considered in the computer model developed. Different numbers of reinjection pumps linked in parallel and any booster pump if needed, can be arranged by this simulator. Since the non-linear model solves a set of equations describing a network system simultaneously, it has good flexibility, making it particularly applicable to looped networks which are much more difficult to solve than a tree-like network. Several pipe network systems have been tested with this simulator, the results converged satisfactorily. Using this computer code, the user can change the connection and the characteristics of each individual demand in a network and chose a desired turbine pressure, the simulator will then calculate the balanced mass flow rate and the pressure distribution along the defined pipe network.

2. PHYSICAL AND MATHEMATICAL MODEL

All the pipelines, wells, separators, pumps and turbines and the way they are linked together compose the basic elements of a pipeline network. Physical and mathematical models were used to describe the inter-relationship between each elements of a network and the process of the pressure and mass flow distribution.

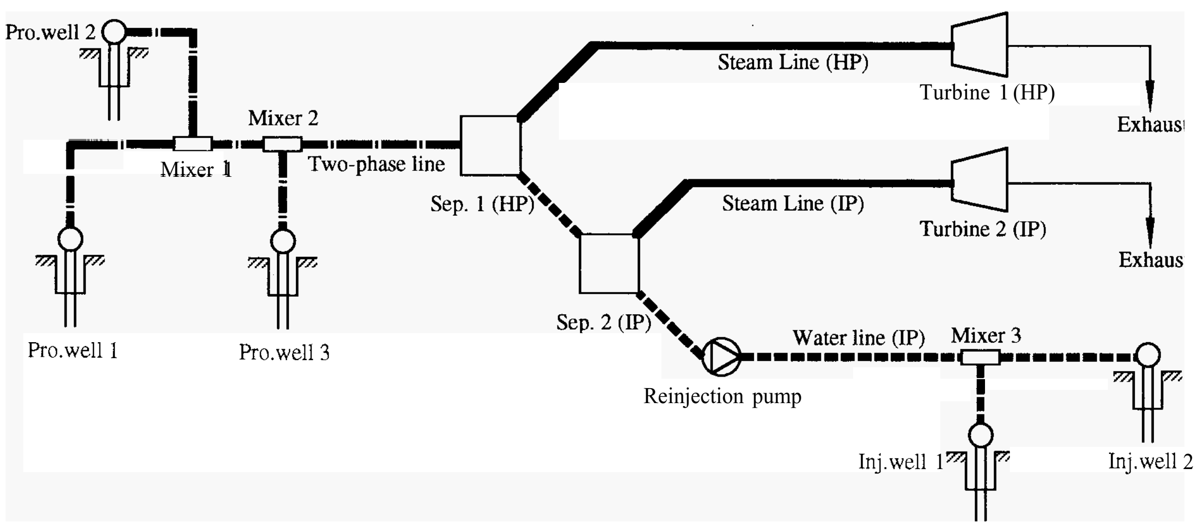


Fig. 1 Simplified model system flow diagram

2.1 Outline of a Pipe Network

A simplified model of a typical system is shown in Fig. 1, which consists of the main elements existing in a pipe network for a geothermal power plant. Two-phase pipelines are used for the model system from production wells to separator 1, as shown in some developed geothermal supply systems. Alternatively, these two-phase lines can be either dry steam lines or water lines, as necessary. The two-stage flashing/separator system is modelled here though a single stage system can be run with the same computer code. In fact, the high pressure separated water coming out from separator 1 flashes and produces the steam at intermediate pressure in separator 2 which then supplies the separated water at intermediate pressure for reinjection. Since the reinjection pump is connected to the supply system at the downstream end and is also connected with reinjection system at the upstream end, the operational parameters of reinjection pumps are affected by both of the systems simultaneously.

2.2 Two Phase lines

Several methods have been developed for the calculation of two-phase flow. Among them, the Lockhart-Martinelli Method is the most frequently used. Since it is found to predict higher values than observations, another method - the Harrison and Freeston method - has been used instead, which showed good accuracy in an experimental two-phase line in Wairakei, NZ (Freeston et al., 1983). Some of the assumptions of the method are:

(1) Annularflow;
(2) Horizontal or small angle inclined transmission pipes.

The pressure drop on a length of pipe AL is given by:

$$\Delta P=\left(\frac{\lambda\Delta L}{D}\frac{K_1}{2\rho_f(1-K_2)}+\rho_m g\Delta Z\right)\cdot 10^{-5} \quad (1)$$

where ρ_m and ρ_f are the two-phase and liquid densities, K_1 and K_2 are functions of liquid, λ is Moody diagram friction factor and L and Z are pipe length and elevation respectively.

2.3 Steam and Water Pipelines

A saturated "pure" steam without any gas has been assumed. For a superheated steam line, the pressure drop should be very close to the calculated pressure drop using saturated steam properties although the enthalpy calculation might be quite different. Single phase steam pressure drop is calculated using the standard formulation as follow:

$$\Delta P=\frac{\lambda\Delta L}{D}\frac{\rho_g V_s^2}{2} \quad (2)$$

where ρ_g and V_s are steam density and velocity.

The pressure drop on fittings is calculated by:

$$\Delta P_f=\frac{8m_t^2}{\rho_g P^2 D^4}\frac{K}{10^5} \quad (3)$$

where K is the pressure loss factor for the fitting.

Pressure drop calculations for water pipe are the same as shown in equations (2) and (3) using the properties of saturated water.

2.4 Mixer

The calculation for the mixing process is based on the mass and energy balance equations. Because of the balance of the two input flows, the pressure in the two inlet pipes should be the same. If the pressure drop in the mixer is small enough, the output flow is the sum of the two input flows under the same mixing pressure individually. Then the following equations can be written:

$$P_o=P_{i1}=P_{i2}$$

$$m_o=m_{i1}+m_{i2}$$

$$h_o=\frac{h_{i1}m_{i1}+h_{i2}m_{i2}}{m_o}$$

where subscript "i" indicates the input flow and "o" the output flow.

2.5 Separator

Two-stage separation is designed for a wider range of applications. The process is considered to be similar to that of mixer but in the opposite direction. Under the separation pressure, the input flow is separated into two different steam and water flows according to:

$$P_i=P_{og}=P_{of}$$

$$m_i=m_{og}+m_{of}$$

$$x_{i1}=\frac{h_i-h_{of}}{h_{og}-h_{of}}$$

$$m_{og}=m_i x_i$$

$$m_{of}=m_i(1-x_{i1})$$

where subscript "og" indicates output steam,"of" indicates output water, and x_{i1} is steam quality of input flow for one-stage separator or for the first stage of a two-stage separator. For the second stage of a two stage separator, the input flow is the separated water that has come out from the first stage separation. This separated water has a process of adiabatic flashing between the two stages while the pressure changes from high pressure in the first stage to intermediate pressure in the second stage. This makes only a small change in the calculation of steam quality of input flow for second stage separation:

$$x_{i2}=\frac{h_{i1}-h_{of}}{h_{og}-h_{of}}$$

where h_{i1} is input flow enthalpy which has the same value with the separated water enthalpy at high pressure in stage one; h_{of} and h_{og} are enthalpies of separated water and steam at intermediate pressure in stage two.

2.6 Solution of the Model.

At the beginning of the modelling developing process, a graphic method of transfering the pressure vs. mass flow curves from wells to the pipeline networks gave a satisfactory solution on a two-phase pipeline network from production wells to separation plant. It was then extended to try to solve a larger system including the separation plant and the reinjection network system. For a pipeline network like Fig. 1, the graphic solution transfered the characteristic curve of the wells along the pipeline sequentially, the simulation started at the production and reinjection wells separately and proceeded toward the inlet and outlet of reinjection pump. At each mixer or separator, mass conservation applies. For a reinjection system, the water head offered by reinjection pumps should overcome all the pressure losses along the system and match with both production and reinjection well curves at the two ends of the system, ie.:

$$\Delta P_{pump}=P_{inj}-P_{pro}+\Delta P_{pro\text{-}inj} \quad (4)$$

where ΔP_{pum} is water head offered by reinjection pump, P_{inj} and P_{pro} are well Read pressures of the production and reinjection wells. $\Delta P_{pro\text{-}inj}$ is the pressure loss along the system from production well to reinjection well. (Huang and Freeston, 1989).

With the more complicated networks which contains more then one separation plant, the graphic method was found to be unadequate. It was impossible for this method to handle any network with enclosed loops or with more than one separation plant. To solve the above problem, a new mathematical model and numerical solution were developed as follows.

3. NUMERICAL SOLUTION

Computer modelling of geothermal pipe networks for both design and simulation purposes has been reported in the technical literature. Most of these are linear computing models, which can work well on a tree-like linked simple pipe network. However with more complex practical systems, it is easy to find that both loops of pipelines and a series of non-linear well characteristics have to be considered. These two factors contribute to the difficulty of the modelling work. Loops in a pipe network can normally be solved using a number of established methods (Stephenson, 1989). However if the pipe network is connected to a number of wells with parabolic like characteristic curves rathei than linear curves, even the calculation of a simple network can be a cumbersome process. Convergence of the solution is often very slow because of the trial and error methods used.

The non-linear simulation model presented in this paper solves the set of equations describing a network simultaneously. It has good flexibility, making it particularly applicable to looped networks.

A conceptual model which can represent the real pipe network is the first requirement for setting up a non-linear equation set. An effective numerical method is necessary for an accurate solution with quick convergence.

3.1 Conceptual Model

A conceptual model should reflect all the interrelations between each part of the network and should be concise and easy to use. Fig. 2 shows a typical conceptual model of a network where $P_1(m_1)$, $P_2(m_2)$ and $P_3(m_3)$ indicate the characteristic curves of the three steam wells, P_T indicates the required steam pressure at the manifold, m_1, m_2 , m_3 , m_4 and m_5 are the stable-state mass flows in the network, and the nodes are numbered as 1, 2, 3, 4, **5** and 6. Dummy lines, represented by the dashed lines are used between each input and output point of the system. With the help of these dummy lines, the network is linked by a number of enclosed loops on which the pressure balance rule of a loop can be applied. The expected flow directions are marked on each pipeline. The following conventions are then applied to establish an equation set for the network.

- At a balanced node the input flow is positive, output flow is negative.
- In a balanced loop, an arbitrary calculation direction of the loop is assumed. If a well output has a same direction as the calculation direction the well head pressure $P_i(m_i)$ is positive, otherwise it is negative; the opposite rule is applied for output pressure P_T in the steam manifold.
- If a pipe flow m_i has a same direction as the calculation direction, the frictional pressure drop is negative, otherwise it is positive.
- In the dummy pipe, the mass flow rate is defined as zero.

3.2 Establishing the Equation Set

In the conceptual model of Fig. 1, well head pressure can be expressed as:

$$P_i(m_i) = A_i + B_i m_i + C_i m_i^2 \qquad (5)$$

where A, B and C are regression coefficients of the well characteristic curve and m is mass flow of the well.

Frictional pressure loss along the pipelines are given as:

$$F_i(m_i) = K_i m_i^2 \qquad (6)$$

where K is the loss factor on a section of a pipeline.

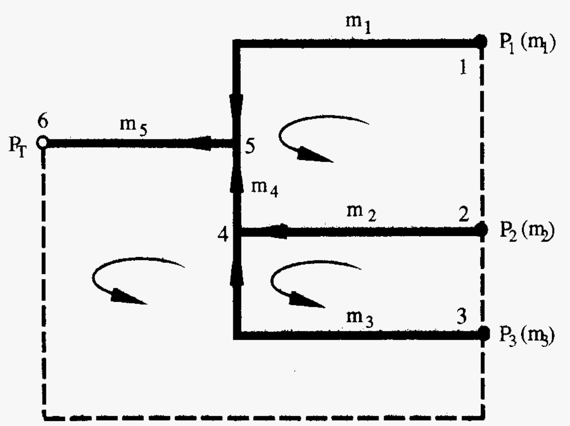


Fig. **2** A typical conceptual model of a network

A system constraint is the requirement of a fixed output pressure P_T at node *6*, so the mass and pressure balance equation set can be written as:

$$\begin{aligned}
&m_2 + m_3 - m_4 = 0 \\
&m_1 + m_4 - m_5 = 0 \\
&P_1(m_1) - F_1(m_1) + F_4(m_4) + F_2(m_2) - P_2(m_2) = 0 \qquad (7) \\
&P_2(m_2) - F_2(m_2) + F_3(m_3) - P_3(m_3) = 0 \\
&P_3(m_3) - F_3(m_3) - F_4(m_4) - F_5(m_5) - P_T = 0
\end{aligned}$$

From equation *(5)* and *(6)* it is obvious that the equation set (7) is a non-linear set and should have **a simultaneous solution** for m_1, m_2, m_3, m_4 and m_5, if they exist.

3.3 Numerical Approach

For a nonlinear equation set such as

$$f_i(x_1, x_2, \cdots, x_n) = 0 \qquad i = 1, 2, \cdots, n \qquad (8)$$

We can define an objective function

$$F(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} f_i^2 (x_1, x_2, \cdots, x_n) \qquad (9)$$

When we have $F(x_1^*, x_2^*, \cdots, x_n^*) < \varepsilon$ (10)

then $x_1^*, x_2^*, \cdots, x_n^*$ are roots of the nonlinear equation set (7).

4. SIMULATION PRACTICE AND SENSITIVITY ANALYSIS

In order to test and validate the performance of the simulator, well data from a geothermal field have been used in a series of simulation exercises. The operation of the program and the interrelations between parameters have been investigated (Huang, 1992). Different types of system and connections were simulated and compared with the available measurements.

4.1 Simulation of a Steam Pipe Network

Among a number of simulation exercises, this code was applied to one of the longest steam pipe networks operating at Larderello, Italy and the results compared to published data. The conceptual model of the network and the simulation results are presented (Huang and Freeston, 1992).

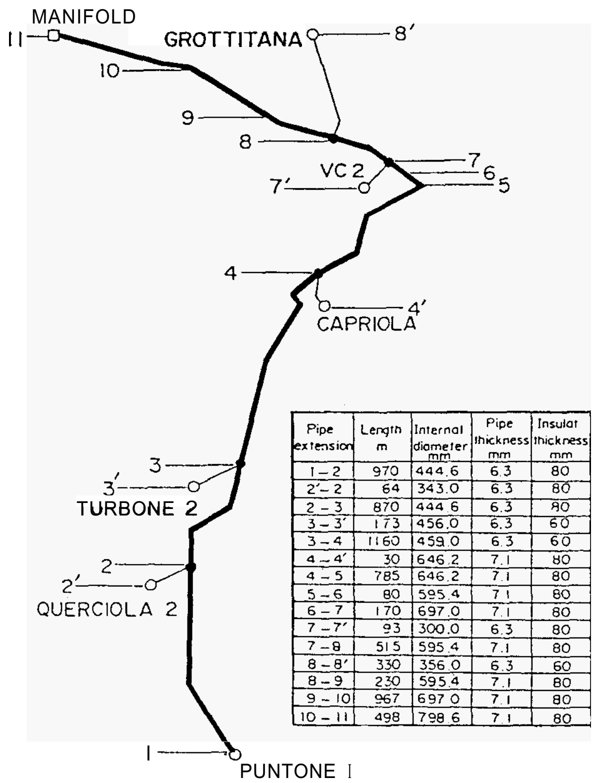
4.1.1 Network layout and conceptual model

This network carries fluid from well Puntone **1**, Querciola 2, Capriola, Grottitana and VC2 to the Serrazzano power plant. Fig. *3* illustrates the layout of the pipe system. Several condensate dischargers are placed along the line. The characteristic curves of the different wells were calculated from published data. The loss factor, K, is based on the geometry of each pipeline and the corresponding steam state. A conceptual model for the network is shown in Fig.4.

4.1.2 Simulation results

A non-linear numerical simulation of the Larderello pipeline network was successfully performed with three different sets of input data in TEST **4**, TEST **5** and TEST 6. The iterative procedure takes about **5** CPU minutes to converge on a IBM **PC** *386* computer. The results were printed as the mass flow rate from m1 to m11. The required manifold pressure was taken from published data (Marconcini and Neri, 1979). In TEST **5,** the K value for pipelines between well Capriola and VC 2 has been modified by an increase shown as follows

	$K_{4\text{-}5} + K_{5\text{-}6} + K_{6\text{-}7}$	$K_{7\text{-}8}$
TEST 4	0.00024	0.00020
TEST **5**	**0.00037**	**0.00030**



Pipe extension	Length m	Internal diameter mm	Pipe thickness mm	Insulat thickness mm
1 – 2	970	444.6	6.3	80
2' – 2	64	343.0	6.3	80
2 – 3	870	444.6	6.3	80
3 – 3'	173	456.0	6.3	60
3 – 4	1160	459.0	6.3	60
4 – 4'	30	646.2	7.1	80
4 – 5	785	646.2	7.1	80
5 – 6	80	595.4	7.1	80
6 – 7	170	697.0	7.1	80
7 – 7'	93	300.0	6.3	80
7 – 8	515	595.4	7.1	80
8 – 8'	330	356.0	6.3	60
8 – 9	230	595.4	7.1	80
9 – 10	967	697.0	7.1	80
10 – 11	498	798.6	7.1	80

Fig. 4 Conceptual model of the steam pipe network in Laraderello, Italy.

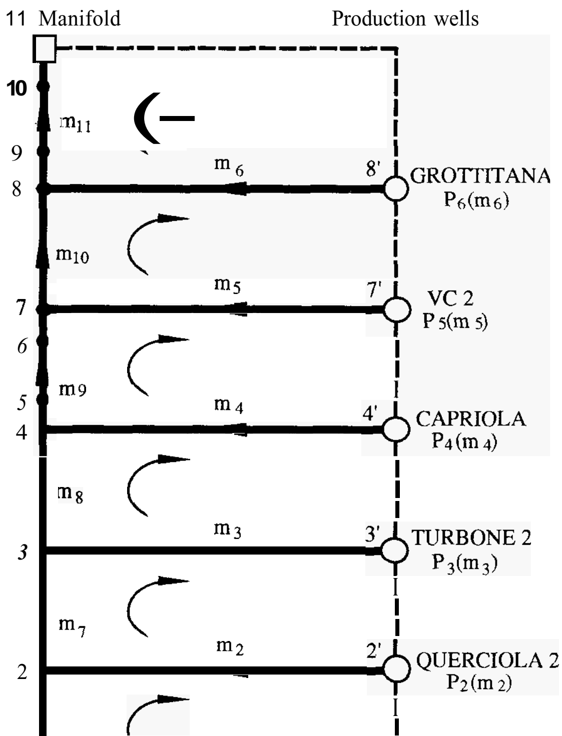


Fig. 4 Conceptual model layout of the steam pipe network in Larderello, Italy

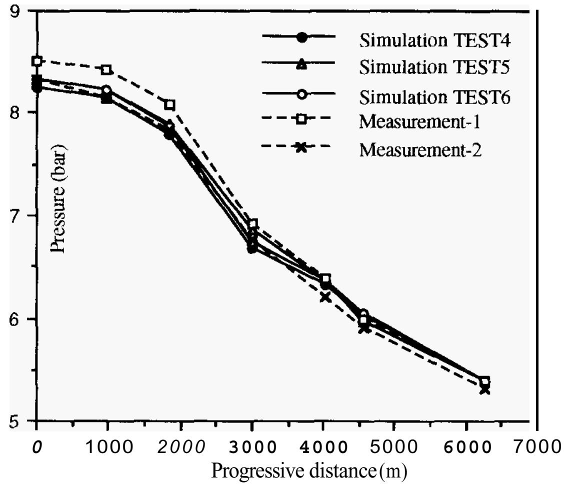


Fig. 5 Simulation results compared with measurements

In TEST 6, a constant Fanning friction factor f=0.0033, which is an average value for geothermal piping (Huang, 1992), was used in the calculation of all the K values throughout the network. The mass flow rate for each well from the results is used in the well characteristic curves to give the operating wellhead pressures. Pressure profiles are then plotted over the measured data along the pipeline network, as shown in Fig. 5. The resultant curves representing TEST 4, TEST 5 and TEST 6 fall within a close range of the measured data irrespective of the difference in input data. Considering the simplification made, the result is good enough to show an agreement between the numerical simulation and the measurements.

4.1.3 Discussion

One of the sensitive parameters in the simulation is the loss factor K. Improperly used K values in the system may lead to a non-convergent simulation. Fortunately. the K value found in practice is dependent on the change of Fanning friction factor f, which happens to fall in a narrow range. Different tests have shown that the simulation results for pressure drop are not too sensitive to Fanning friction factor f.

In Fig.5, the comparison between the simulation and measurement is illustrated. The narrow gap between the two measurements may indicate measurement errors. Most of the simulation results fall within the range of measurement error, which indicate a good agreement with measurement. All three simulations, TEST 4, TEST 5 and TEST 6 have a slightly flatter pressure profile along the pipeline than the measured data. This is because localized frictional loss has been neglected for the simulation at this stage. For the results for TEST 4, the pressure profile has a similar slope to the measurements except for the pipelines from node 4 to 8. This might be due to some additional localized frictional loss. In TEST 5, the K value for all pipelines has been modified by a small increase. The simulation result shows that the slope of the pressure profile between node 4 and 8 is closer to the measured one, demonstrating the sensitivity to the K value.

TEST 6 is based on a constant Fanning friction factor f of 0.0033. The objective of this test was to investigate the sensitivity to f. It is interesting to note that the resultant pressure profile of this test is very close to that of TEST 4. This is an indication that the f value, if within a reasonable range, is not a sensitive parameter in this numerical simulation. Among the variables involved in evaluating K, pipe diameter is the most sensitive one for estimating pressure drop. Fortunately it is one of the most well specified parameters.

Of the main parameters the production well characteristic curves are of special importance. Since each well may have its own characteristic curve, any pressure change along the pipeline can cause a corresponding change of the working point of the well along its production curve. As a result, the well production rate is changed which then leads to consequent changes in pressure drop on the network. The influence on the other well operating points of changing a well characteristic curve is very complex. It is a function of their curve shape, the frictional characteristic of each pipeline and the mass and energy balance of the whole network. In some cases, it can cause an increase in the total system output while in some other cases, an opposite effect occurs. With the help of the simulation, well head control of the working point can be used as a systematic control to a pipe network, especially after there has been a change to the network system.

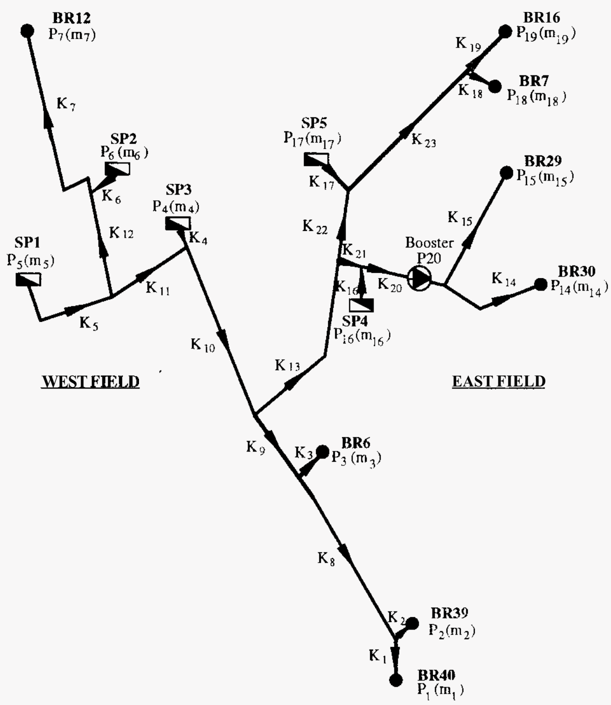


Fig.6 Layout diagram of the reinjection pipe network at Ohaaki Power Station

4.2 Simulation of a Reinjection Pipe Network

Another example is the simulation of the reinjection pipe network in Ohaaki Power Station, New Zealand. Investigation on simulation sensitivity to surface roughness was made based on normally used engineering data.

4.2.1 Network layout and conceptual model

The layout diagram of the reinjection pipe network at Ohaaki Power Station is shown in Fig. 6. There are 5 separation plants and 8 reinjection wells working as the sources and sinks of the system. At each separation plant a reinjection pump has been taken into account with a fixed head lift purely to simplify the model. A booster pump is located on the down-stream side of the separation plant SP4. which gives a fixed head lift of 10bar. The production curves of mass flow rate versus pressure at each separation plant are given for the simulation. The injectivity, ie. reinjection mass flow versus wellhead pressure, lor the reinjection wells are given from well test data. The computer simulation calculates for a stable balanced mass flow rate in each pipeline. Fig.7 shows the conceptual model of the reinjection pipe network in Ohaaki Power Station. As illustrated in the model, there are twenty-three unknown mass flow rates to he solved.

4.2.2 Simulation results

A series of simulations each with different values of pipe surface roughness was produced in this study. First, a constant value of Fanning friction factor, F=0.0033 was used for all pipes in the system. The solution of the mass flow rates among the 23 pipelines are shown to be close to the measurement data. An error analysis for the simulation result compared to that measured shows that the relative error of the numerical solution for mass flow rates are between 4.2% and -8.1%.

Secondly. the calculated Fanning friction factors were used in the simulation. A constant pipe surface roughness, ie. ε=0.0000457 m, was used for all the pipelines in the evaluation of Fanning friction factors. With all the calculated values off, the numerical solution for the network gives a small change in magnitude. The comparison between the measurement and the simulation results is similar to that for the constant Fanning friction factor. The error analysis on the simulation result shows only about 1% improvement in the relative error against the measurement. All the relative errors are between 4.2% and -7.18.

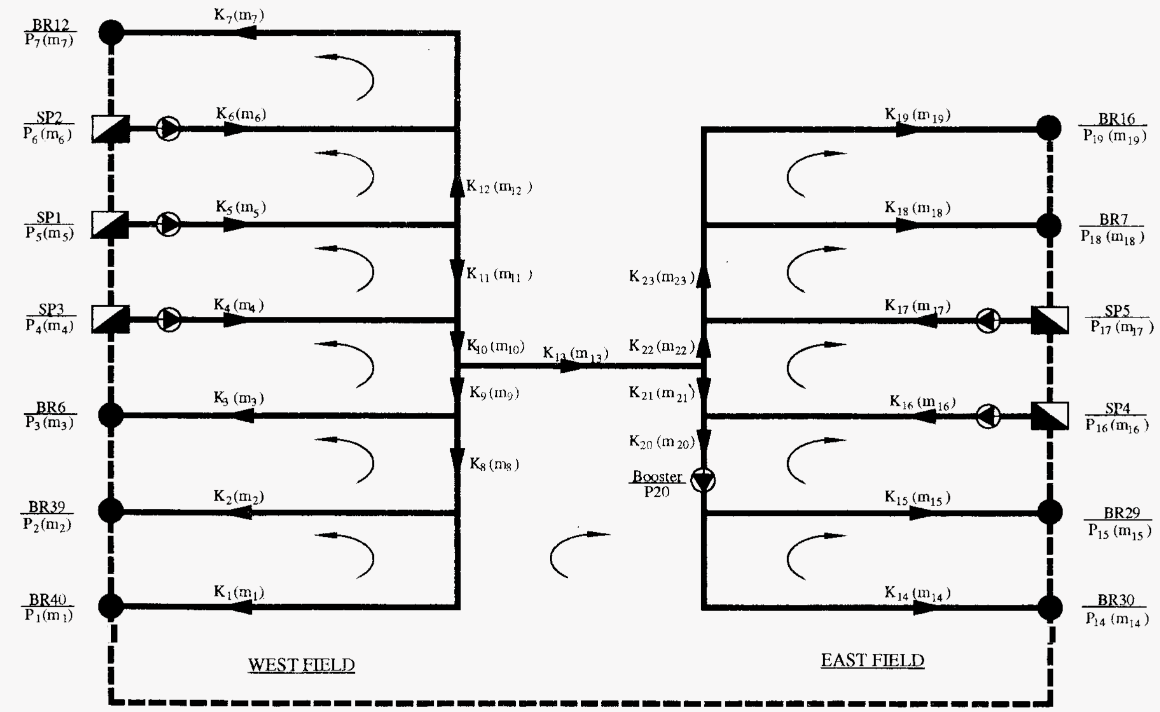


Fig.7 Conceptual model of the reinjection pipe network in Ohaaki

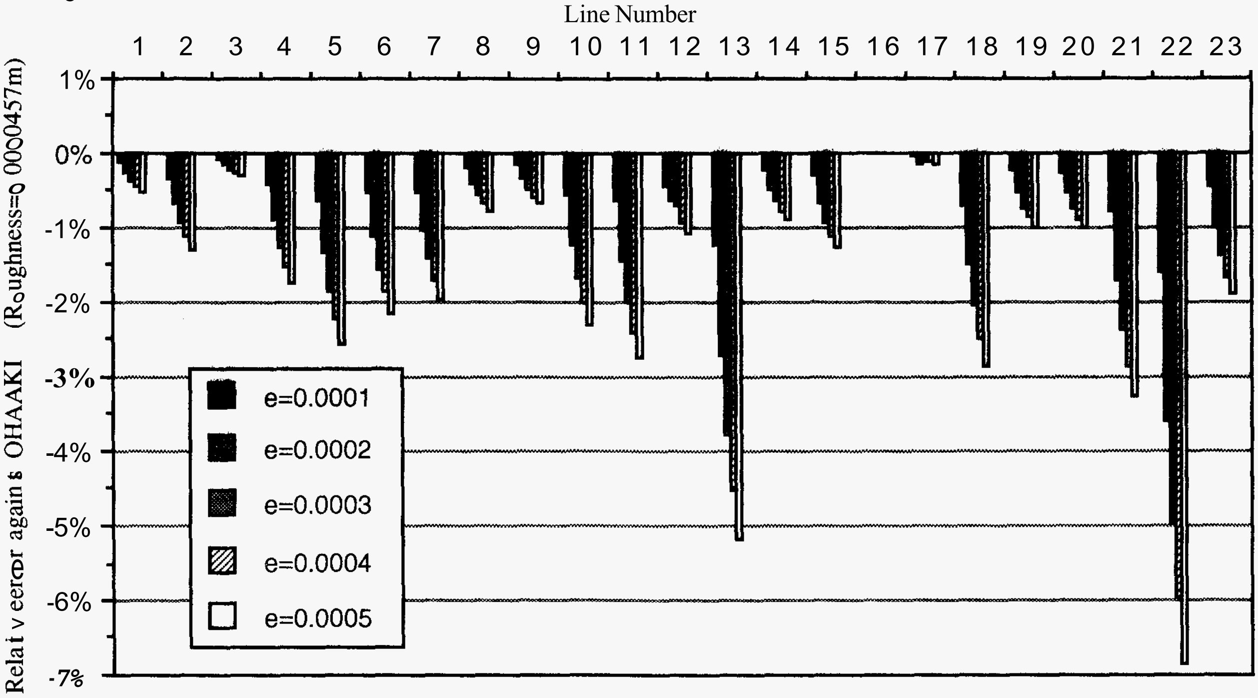


Fig. 8 Error analysis on the simulation results for different pipe surface roughness , e in meter

It is suggested that differcnt pipe surface roughness values would influence the performance of an pipe network. Comparisons from the simulation for pipes with with different surface roughness are presented. Fig.8 shows the error analysis on the simulation results using pipe roughness value of 0.0001, 0.0002, 0.0003, 0.0004 and 0.0005 m respectively. These results are compared with the one with roughness= 0.0000457. The maximum relative error or rather, the maximum relative decrease in mass flow rate among the 23 pipelines when the roughness is increased to 0.0002m is 3.6%, which becomes **6.8%** for a roughness of 0.0005m.

4.2.3 Discussion

The simulation results of the above reinjection network system for differcnt values of roughness demonstrate that in the range of 0.0000457 - 0.0005m, mass flows and pressures in the network pipes do not change significantly.

The comparison between the results for constant and calculated Fanning friction factors has also confirmed that the simulation is not roughness sensitive. So that a small variation of the pipe surface roughness with time should not cause any significant effect on the network performance. It also proves that the unified Fanning friction factor of 0.0033 is a properly chosen one for the numerical simplicity and engineering accuracy of this simulation.

The maximum relative change of mass flow rate, with an increasing roughness up to 0.0005 m, is -6.8% found in Fig.9. The top value of 0.0005 m would be a typical roughness for concrete which may also be used for the aged geothermal pipeline with a layer of scale or deposition, The maximum -6.8% change in mass flow gives the magnitude of the possible drop of the mass flow rate which is important data for network performance prediction.

5. Conclusion

(1) A non-linear model for the numerical simulation of a geothermal pipe network has been developed. **A** mass flow and pressure balance at each node and loop in the network is used to establish a non-linear equation set. Numerical methods have been employed for the modelling. The simulator developed by this work offers a viable methodology for analyzing the performance of a geothermal pipe network system.

(2) With the simulator presented, the simulation of a pipeline network can be applied not only to a tree-like system but also to a looped one. More then one separation plant each with a two stage separation process and a reinjection pump station can be managed. This simulator allows any change of conditions in the network system and changes to the operation condition to be made at the discretion of the user. Designed with a user friendly PC-based operating process, the simulator is attractive for engineers who need a design tool in pipe network design. It can also benefit the user of a pipe network system potentially from the view point of prediction of the performance of a network system.

(3) According to the simulations completed 'on some published geothermal pipe network data, convergency is satisfactory and the results are close to measurements. These results have shown that they are not sensitive to the Fanning friction factor f for the pipe roughness range tested. This may indicate the possibility of using a constant or linear (against pipe roughness ε/D) f value for a simplified calculation of pressure drop.

6. ACKNOWLEDGEMENT

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