INJECTION INTO VAPOUR-SATURATED GEOTHERMAL RESERVOIRS

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ABSTRACT

We present a model of the rate at which water migrates under gravity through a vapour-saturated geothermal reservoir from an injection well. Our model accounts for the vaporisation of a fraction of the liquid as it is heated by the hot permeable rock. We show that injection from a central well at constant flux leads to a radial migration of liquid into the reservoir at a rate scaling as $t^{1/2}$, with the depth of the liquid layer decreasing with radius.

INTRODUCTION

The injection of fluid into vapour-saturated geothermal reservoirs is an important and increasingly necessary process as fluid reserves in such reservoirs become depleted (Enedy 1989; Pruess and Enedy 1993). Recent work on the process of injection has considered the detailed dynamics of the vaporising liquid-vapour interface, assuming that liquid migrates through the reservoir with a uniform velocity from a line or planar source (Pruess et *al.* 1987; Woods and Fitzgerald 1993). To date the role of gravity upon liquid injection into vapour-dominated systems has only been considered through the application of numerical models because of the complex three-dimensional nature of the problem (Pruess and Enedy 1993; Pruess 1991a,b). The results of Pruess (1991a,b) predict that as liquid is injected from a well into a vapour-filled reservoir, a plume of liquid sinks to the base of the permeable zone of the reservoir. Thereafter, the liquid spreads out as a gravity current. In this paper we focus attention upon the motion of the liquid as it spreads out radially along the base of the reservoir and present a simple analytical means of calculating the evolution of the current.

THE MODEL

Gravitational forces cause a spreading liquid-vapour interface to become two-dimensional and may have a dominant control upon the flux of liquid supplied to a reservoir as well as the mass fraction of this liquid which may vaporise (Pruess 1991b). In this work, we present a model which describes how liquid injected from a central source into a hot permeable rock spreads out under gravity along the base of a superheated geothermal reservoir, vaporising a fraction of the liquid en route (figure 1).

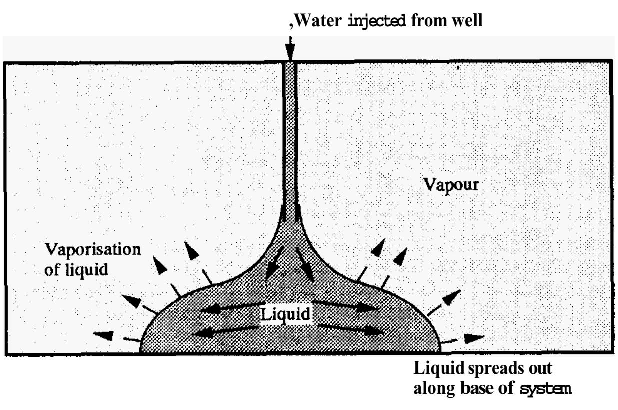

Figure 1 Schematic diagram of the radial spreading of water after it has descended to the base of the permeable zone of a vapour-dominated reservoir.

Our present work builds upon the result of Woods and Fitzgerald (1993) that if liquid is injected sufficiently slowly then the fraction of liquid which vaporises is a maximum. This maximum occurs when the interfacial pressure equals the far-field pressure of the reservoir. It is attained when the interface migrates at a rate slower than the vapour diffusion speed which is approximately $(KP_\infty/\phi\mu_v t)^{1/2}$ where K is the permeability, ϕ the porosity, P_∞ the far-field pressure, μ_v the viscosity of the vapour and t is time. In our model we assume this to be the case and we subsequently check for the consistency of our results

The maximum mass fraction of liquid which vaporises, F, is given by the relation

$$F = \frac{1}{1+\dfrac{\phi\rho_w\left(h_{v\infty}-C_{pw}T_{sat}(P_\infty)\right)}{(1-\phi)\rho_r C_{pr}\left(T_\infty-T_{sat}(P_\infty)\right)}} \quad (1)$$

where h_v represents the enthalpy of the vapour in the far-field, C_{pw} the specific heat capacity of water, C_{pr} the specific heat capacity of the rock, $T_{sat}(P_\infty)$ the saturation temperature of the far-field pressure, ρ_w the density of water, ρ_r the density of the rock and T_∞ the reservoir temperature. Figure 2 shows how the fraction vaporising F varies with reservoir pressure for a given constant reservoir temperature. As the reservoir pressure increases towards $P_{sat}(T_\infty)$, the fraction vaporising tends towards zero.

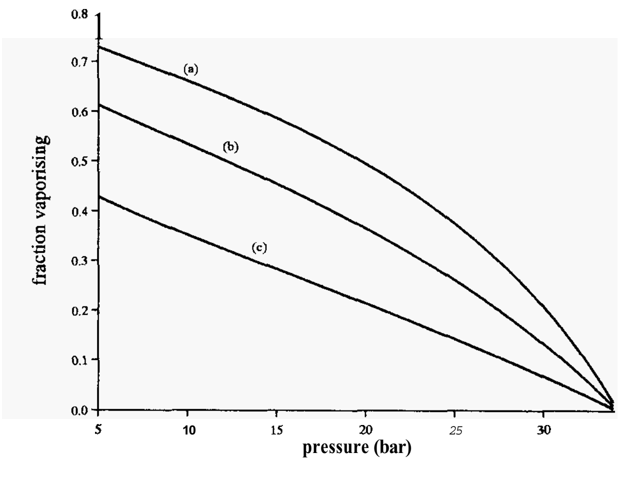

Figure 2 The fraction of liquid that can vaporise at a migrating liquid-vapour interface as a function of reservoir pressure. In this figure the reservoir is at 240°C. Curves (a), (b) and (c) indicate reservoirs of porosity 3, 5 and 10% respectively.

We assume that the liquid migrates through the reservoir from an injection well at position $r=0$ with the depth of the liquid at any position expressed as $h(r,t)$. The liquid is assumed to spread radially and axisymmetrically from the source. If the depth of the layer remains sufficiently small then the vertical pressure gradient is approximately hydrostatic and the motion is purely radial (Bear 1972; Batchelor 1967). The radial velocity u of liquid of viscosity μ_w satisfies Darcy's Law

$$u = \frac{-K}{\mu_w}\frac{\partial P}{\partial r} \quad (2)$$

where the pressure at a height y above the base of the reservoir is

$$P(r,y) = P_\infty + \rho_w g(h-y) \quad (3)$$

and g is the acceleration due to gravity. Hence,

$$u = \frac{-K}{\mu_w}\rho_w g\frac{\partial h}{\partial r} \quad (4)$$

Since a fraction (1-F) of the liquid does not vaporise, conservation of mass requires that locally the liquid current deepens

$$r\frac{\partial h}{\partial t}=\left(\frac{\rho_w g K(1-F)}{\phi\mu_w}\right)\frac{\partial}{\partial r}\left(rh\frac{\partial h}{\partial r}\right) \quad (5)$$

The model is completed by imposing global conservation of mass. This relates the total mass of water injected Q with the mass of water in the reservoir

$$Q(1-F)=\rho_w 2\pi\phi\int_0^{L(t)} rh\, dr \quad (6)$$

where $L(t)$ is the radial extent of the current at time t. The model is valid so long as $\partial h/\partial t>0$ and that $(\partial h/\partial t) \ll \sqrt{(\alpha/t)}$ where $\alpha=KP_\infty/\phi\mu_v$ (Woods and Fitzgerald 1993). Equations (5) and *(6)* govern the rate of propagation of the liquid into the reservoir. In practice, F, the mass fraction vaporising is a function of the reservoir pressure which changes as the liquid invades the reservoir and supplies new vapour (figure 1). In a large or highly superheated reservoir, F may only vary slowly as the current propagates along the base of the reservoir and supplies new vapour to the system (figure 2). In contrast, in a smaller reservoir, F will indeed evolve and thereby alter the propagation rate of the current. For simplicity, we restrict attention to the case in which F is approximately constant. In this case equations (5) and (6) admit similarity solutions which provide a useful reference with which more complex flows may be compared.

In the case of a constant flux q at the base of the injection well, we seek a solution of the form

$$h(r,t)=h_o f(\eta) \qquad L(t)=\lambda(\beta t)^{1/2} \qquad \eta=r/(\beta t)^{1/2} \quad (7)$$

where $h_o=\sqrt{(q\mu_w/(\rho_w{}^2 gK2\pi))}$, $\beta=((1-F)/\phi)\sqrt{(qgK/(2\pi\mu_w))}$ and λ is a dimensionless constant. In a typical vapour-dominated geothermal reservoir, reservoir and injection parameters may have the following values, q**=30kg/s**, $K=10^{-13}m^2$, $\rho_w=1000kg/m^3$, $g=9.81m/s^2$, $F=0.4$, $\phi=0.05$ and $\mu_w=1.3\times10^{-4}kg/sm$, in which case we find that $h_o=25m$ and $\beta=2.3\times10^{-3}m^2/s$. In this solution the rate of propagation of the liquid $\sqrt{(\beta/t)}$ is much smaller than that of the vapour $\sqrt{(\alpha/t)}$ if the applied flux

$$q \ll \frac{2\pi\mu_w P_\infty\phi}{g(1-F)^2\mu_v}=q_m \quad (8)$$

In this case the fraction vaporising F indeed attains the maximum value. Substituting (7) into equation (5) we find that the liquid-vapour interface has the shape given by the equation

$$\frac{-\eta^2}{2}\frac{df}{d\eta}=\frac{d}{d\eta}\left(\eta f\frac{df}{d\eta}\right) \quad (9)$$

with boundary conditions

$$-\eta f\frac{df}{d\eta}=1 \text{ at } \eta=0 \quad (10a)$$

and

$$f(\lambda)=0 \quad (10b)$$

where λ is the dimensionless length of the current. A further constraint is imposed by the global conservation of mass within the current *(6)* which is now given by

$$\int_0^\lambda \eta f(\eta)d\eta=1 \quad (11)$$

The eigenvalue λ may be found by solving equation (9) subject to boundary conditions (loa) and (11) We find $\lambda=1.5$ and show the shape of the current in figure 3. Although in reality the source of the liquid is of finite radius, as the flow leaves the source it rapidly converges towards the similarity solution shown in figure 3. Note however that as a consequence of the finite radius source, the flow near the source may differ in detail to that of the similarity solution .

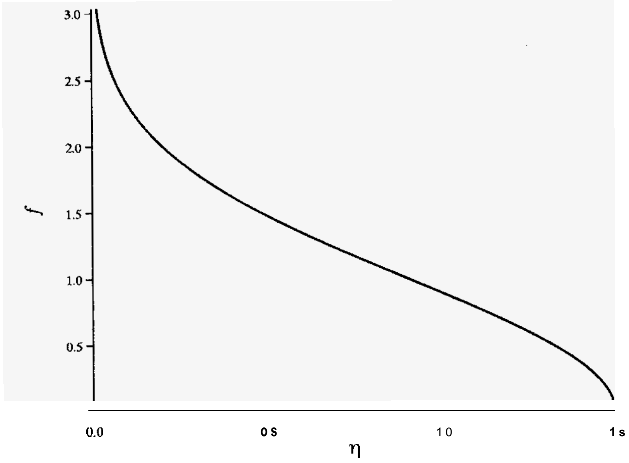

Figure 3 The height of the liquid zone above the base of the reservoir f as a function of similarity variable η.

In an example where liquid is injected at a rate of 30 kg/s into a reservoir whose properties are as listed after equation (7) except where stated otherwise, the position of the liquid front **L(t)** is given in Table 1.

Table 1 Distance of liquid front from source as function of time

	Time				
	1 hour	1 day	1 week	1 year	10 years
Distance (m) for $K=10^{-13}m^2$	**4.3**	21	56	**400**	1280
Distance (m) for $K=10^{-15}$	0.43	2.1	5.6	**40**	128

The mass of vapour added to the reservoir after a time t is Fqt. Assuming the density of the liquid far exceeds that of the vapour, it follows that the increase in pressure of a closed reservoir of volume V is

$$\frac{dP}{dt}=\left(\frac{F}{\phi V}(R\theta_\infty)\right)q \quad (12)$$

where θ_∞ is the reservoir temperature in degrees Kelvin and R is the gas constant for vapour (Elder 1981). According to this simple theory the mass fraction vaporising F, as given by equation (1), changes at a rate

$$\frac{dF}{dt}=F^2\phi\rho_w\frac{[C_{pw}(T_\infty-T_{sat}(P_\infty))-(h_{v\infty}-C_{pw}T_{sat}(P_\infty))]}{(1-\phi)\rho_r C_{pr}(T_\infty-T_{sat}(P_\infty))^2}\left(\frac{dT}{dP}\right)_c\frac{dP}{dt} \quad (13)$$

where $(dT/dP)_c$ is the slope of the Clausius Clapeyron saturation curve. We expect the theory presented herein applies when the mass of vapour added to the reservoir is much smaller than that initially contained within the system. For an initial vapour flux qF_o, we therefore expect the model to be valid for times $t \le t^*$ where

$$t^*=\left(\frac{P_{o\infty}V\phi}{qR\theta_\infty F_o}\right) \quad (14)$$

and where subscript $_o$ refers to an initial value (figure 4).

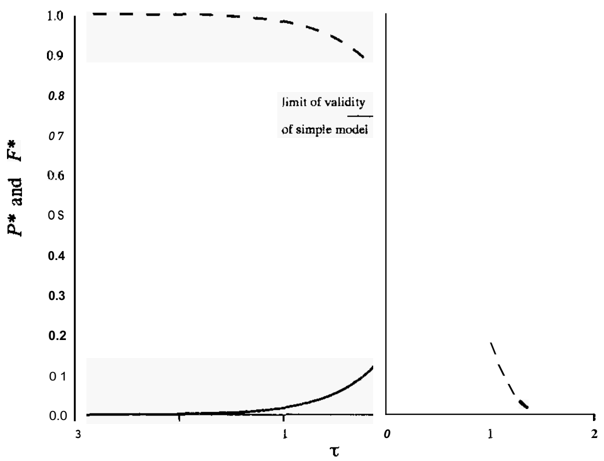

Figure 4 The reservoir pressure $P^*=(P_\infty-P_{o\infty})/(P_{sat}(T_\infty)-P_{o\infty})$ (solid line) and fraction vaporising $F^*=F/F_o$ (dashed line) as a function of time $\tau=t/t^*$ as liquid is injected at a constant rate into a reservoir of porosity 10% and temperature 240°C.

CONCLUSIONS

We have presented a simple new theory to predict the rate of propagation of liquid from an injection well along the base of a geothermal reservoir. Our model shows that if water is supplied at a constant rate then the liquid spreads radially from the well according to

$$L(t)=\lambda\sqrt{\beta t} \qquad (15)$$

Our theory assumes that the fraction of liquid which vaporises remains constant. This is valid as long as $q \ll q_1$ (8) so that diffusion of pressure signals within the vapour is more rapid than the rate of propagation of the current. The theory is valid for times of order $t \le t^*$ (14) for the region in which the liquid is spreading laterally. On longer time scales, the reservoir vapour pressure increases so much that the fraction of liquid which vaporises begins to decrease and the theory needs modifying.

This novel method of calculating the two-dimensional spreading of a vaporising liquid-vapour interface in a hot superheated rock may be extended to other injection conditions and we are presently developing a method to apply this modelling technique for times $t \ge t^*$. This modelling technique provides a simple method of evaluating the potential impact of injected fluid upon the spatial and temporal distribution of liquid and vapour in vapour-dominated reservoirs. Also, this model provides a powerful and complimentary method of understanding liquid injection following the numerical studies of Pruess (1991a,b).

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REFERENCES

Bear, J. 1972 ***Dynamics of Fluids in Porous Media.*** Dover.

Batchelor, G.K. 1967 ***An Introduction to Fluid Dynamics.*** Cambridge.

Elder, J. 1981 ***Geothermal Systems.*** Academic.

Enedy, K.L. 1989 The role of decline curve analysis at the Geysers. ***Trans. Geoth. Res. Counc.*** **13,** 383-392.

Pruess, K. Calore, C., Celati, R. and Wu, Y.S. 1987 An analytical solution for heat transfer at a boiling front moving through a porous medium. ***Int. J. Heat Mass Transfer*** **30**(12), 2595-2602.

Pruess, K. 1991a Grid orientation and capillary pressure effects in the simulation of water injection into depleted vapour zones. ***Geothermics*** **20,** 257-277.

Pruess, K. 1991b Grid orientation effects in the simulation of cold water injection into depleted vapour zones. ***Proc. Stanford Geothermal Workshop*** **16.**

Pruess, K. and Enedy, S. 1993 Numerical modelling of injection experiments at the Geysers. ***Proc. Stanford Geothermal Workshop*** **18.**

Woods, A.W. and Fitzgerald, S.D. 1993 The vaporization of a liquid front moving through a hot porous rock. ***J. Fluid Mech.*** **251,** 563-579.