NUMERICAL SIMULATION OF STEAM *AND* WATER PRODUCING WELL TESTING

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ABSTRACT

The main method of **well** and high temperature **hydrothermal reservoirs testing** at **different stages** of **their exploration** is production flow test of **a well producing** *steam or steam* and water mixture **with different vapor** saturation The data **obtained** are used to find the correlation between mass production rate and well head pressure that **represents** very important characteristics of well productive capacity. **However** the use **of** these **data** to **obtain** the characteristics of the productive **zones** and the processes **generated** in them is still not enough investigated problem. We have **developed** the **methods** of numerical simulation **that makes it possible** to **take** into consideration the **different** conditions of fluid flow in the productive **zones and** in the wellbore and thus to **obtain the** calculated **relationships between** mass production rate of **the** fluid as **well as its** enthalpy and the **pressure** at the **well** head. **As the example** driven **shows** the use of numerical simulation **provides obtaining** additional **information about** the **process** parameters that leads to higher **effectiveness of exploration works.**

1. INTRODUCTION

The **main purpose** of *standard* field **investigations** *carried* **out** at the testing of *steam* and water **producing** wells and high temperature hydrothermal **reservoirs** is to **get** the following **data:**

— pressure and **enthalpy** of *steam* and water **mixture** at **the well outlet** p_h;

— mass **production** rate and regime of fluid **discharge at different well head** pressure;

— mass **concentration** of water and *steam* in the fluid prod——

— chemical and *gaseous* composition of water and *steam.*

Considering these **data** as **a** very significant **information** about the productive capacity of **the well** tested **it** should be **noted** that **they** do not **provide** obtaining the **number** of *other* parameters **important** for hydrothermal **reservoir** investigation (phase state of **the** fluid in the productive zone, its fluid conductivity, etc.). Attempts to use traditional methods of **hydrodynamic** investigations **by plotting** $p - \ln\tau$ **and** $p - \tau/r^2$ diagrams **proved** to **be inefficient at** high **temperature hydrothermal reservoirs because of** the **effect** of **non-**isothermal filtration and fluid phase state changing in the process of its filtration in the productive zone *(Kiryukhin et al, 1991).*

Nevertheless, the existing methods of well testing **that provide the** *obtaining* $Q = f(p_h)$ **and** $e = f(p_h)$ **diagrams make it** possible **to** identify **the** conditions of *steam* and water producing **well testing** in more detail and thus to **increase** the informativeness of **testing** results. The identification of **the well** testing conditions can be made **on** the **base** of numerical simulation combined with **stochastic** variation of the process parameters, which **characterize the** fluid flow conditions in the productive **zone** and in the **wellbore.** Results **of the** combined simulation compared **with** the **observed** data of the process **development** **($Q = f(p_h)$ and $e = f(p_h)$ diagrams) allow to** determine **with some** degree of insured probablity to **what** particular type of fluid flow conditions these data can be **related. To realize** this **approach it is** *necessary* to **develop** the **mathematical** model of well **and productive zone compatible performance** that **can be** used as the base for the numerical simulation of the **well testing processes.**

2. METHODS OF NUMERICAL SIMULATION OF WELL AND PRODUCTIVE ZONE COMPATIBLE PERFORMANCE

Fluid in the productive zone of the high temperature hydrothermal **reservoir can be** in **one-phase (superheated** water **or steam) or in two-phase state** (SWM) **depending on** the **geothermal conditions** and **well** depth. In the **process** of production flow **testing** initially **one-**phase state of the fluid *can* be changed into two-phase one if the pressure in the wellbore where it **intersects** the productive zone drops to the **saturation** pressure **corresponding** to the **temperature** in that **interval.** Considering possible phase transfer of the fluid **the** *general* **set** of equations for the non-isothermal fluid filtration in **the** productive **zone we represent** by the equations of mass conservation

$$\partial(m\rho)/\partial\tau = q_m - div\,(V' + V''), \qquad (1)$$

motion

(2)

$$V'' = -k\rho'' R''(\nabla p - \rho'' g)/\mu'', \qquad (3)$$

and *energy* **conservation**

$$\partial E/\partial\tau = \nabla(\lambda\nabla t) - div\,(V'e' + V''e'') + q_e, \qquad (4)$$

where $\rho = \rho'(1-S) + \rho'' S$

$$E = m\rho e + (1-m)\rho_0 e_0;$$
$$e = \eta e'' + (1-\eta)e'$$
$$e_0 = c_0 t$$
$$\eta = V''/(V' + V'')$$
$$1 = m[\lambda'(1-s) + \lambda'' s] + (1-m)\lambda_0$$

(Piscacheva, 1991).

Set of equations **(1)** - *(4)* is **related** to non-isothermal **one-** *or* two-phase filtration in **a** porous or **fractured** medium where **rock matrix** and fluid are considered to be in **local thermal** equilibrium.

In response to the problems of **well and productive** me compatibility simulation equations **(1) - (4) were** transferred to **the** cylindrical coordinates r, z with the substitution of new **variable** $x = r^2$ and **represented as follows**

$$\beta^* \frac{\partial p}{\partial\tau} = 4\frac{\partial}{\partial x}\left(xT\frac{\partial p}{\partial x}\right) + $$
$$+ \frac{\partial}{\partial z}\left(T\frac{\partial p}{\partial z}\right) - g\frac{\partial}{\partial z}(T'\rho' + T''\rho'') + q_m, \qquad (5)$$

$$\alpha^* \frac{\partial e}{\partial\tau} = 4\frac{\partial}{\partial x}\left(x\lambda\frac{\partial t}{\partial x}\right) + $$
$$+ \frac{\partial}{\partial z}\left(\lambda\frac{\partial t}{\partial z}\right) - 4xV_x\frac{\partial e}{\partial x} - V_z\frac{\partial e}{\partial z} + q_e, \qquad (6)$$

where $\beta^* = m\partial\rho/\partial p + (1-m)\rho\beta_s$

$$\alpha^* = m\rho + (1-m)c_s\partial t/\partial e;$$
$$T' = k\rho' R'/\mu';$$
$$T'' = k\rho'' R''/\mu''$$
$$T = T' + T'.$$

Set of equations *(5)* - (6) is solved numerically **with** the use of fully implicit **finite difference** method that **approximates** the **original** equations to the following **schemes** *(Samarsky, 1977)*

$$C\frac{y^{n+1/2} - y^n}{\Delta\tau} = 4\Lambda_1 y^{n+1/2} + \varphi_1, \qquad (7)$$

$$C\frac{y^{n+1}-y^{n+\frac{1}{2}}}{\Delta\tau}=\Lambda_j y^{n+1}+\varphi_2\,, \qquad (8)$$

where y is pressure or energy value, $\Delta\tau$ is time step. Calculational ana is discretizied into N horizontal steps of h_i and M vertical steps of h_j size. For the general case of non-uniform discretization the average steps will be

$\hbar_i = (h_i + h_{i+1})/2\,,\ i = 1,2...(N-1),$

$\hbar_j = (h_j + h_{j+1})/2\,,\ j = 1,2...(M-1).$

When the filtration equation is solved operator $\Lambda_\alpha y^t$ is referred to

$$\Lambda_\alpha y^t = a_{\alpha+1}\frac{p^t_{\alpha+1}-p^t_\alpha}{h_{\alpha+1}} - a_\alpha\frac{p^t_\alpha - p^t_{\alpha-1}}{h_\alpha}\,,\ \alpha = i,j\,, \qquad (9)$$

It is assumed that $t = n+\frac{1}{2}$, if $\alpha = i$, and $t = n+1$, if $\alpha = j$. The coefficients φ, C and a_α are calculated as follows:

$\varphi_1 = q_m/2\,; \qquad \varphi_2 = q_m/2 -$

$- g(a'_{j+1}\rho'_{j+\frac{1}{2}} + a''_{j+1}\rho''_{j+\frac{1}{2}} - a'_j\rho'_{j-\frac{1}{2}} - a''_j\rho''_{j-\frac{1}{2}})/\hbar_j\,;$

$C = \beta^*\,; \quad a'_i = x_{i-\frac{1}{2}}T'_i T'_{i-1}\ln(r_i/r_{i-1})/$

$/[T'_{i-1}\ln(r_i/r_{i-\frac{1}{2}}) + T'_i\ln(r_{i-\frac{1}{2}}/r_{i-1})]\hbar_i\,;$

$a'_j = 2T'_j T'_{j-1}/(T'_j + T'_{j-1})\hbar_j\,; \qquad a_\alpha = a'_\alpha + a''_\alpha\,.$

Coefficients a'_{i+1}, a'_{j+1}, etc. are determined the same way.

When the equation of energy is solved operator $\Lambda_\alpha y^t$ is referred to

$$\Lambda_\alpha y^t = r^+_\alpha\frac{e^t_{\alpha+1}-e^t_\alpha}{h_{\alpha+1}} + r^-_\alpha\frac{e^t_\alpha - e^t_{\alpha-1}}{h_\alpha}\,,\ \alpha = i,\ j \qquad (10)$$

The coefficients are determined by the following expressions:

$C = a'$;

$\varphi_1 = q_e/2 + [x_{i+1/2}\lambda_{i+1/2}(t_{i+1}-t_i)/h_{i+1}$

$\qquad - x_{i-1/2}\lambda_{i-1/2}(t_i - t_{i-1})/h_i]/\hbar_i\,;$

$\varphi_2 = q_e/2 + [\lambda_{j+1/2}(t_{j+1}-t_j)/h_{j+1}$

$\qquad - \lambda_{j-1/2}(t_j - t_{j-1})/h_j]/\hbar_j\,;$

$r_i = -x_i V_i \quad ; \quad r_j = -V_j \quad ; \quad r^+_\alpha = (r_\alpha + |r_\alpha|)/2 \quad ;$

$r^-_\alpha = (r_\alpha - |r_\alpha|)/2\,.$

Values of y at $i=1,N$ and $j=1,M$ are to be found according to the given border conditions.

Mathematical model of two-phase flow in a wellbore can be represented by the equations of motion

$$-\partial p/\partial Z = \psi\rho''g + (1-\psi)\rho'g + 2\tau_k/r_w\,, \qquad (11)$$

and energy conservation

$$\partial(\rho e)/\partial\tau = q_e - \partial(Ve)/\partial z + 2\tau_k\upsilon/r_w\,, \qquad (12)$$

where $\upsilon = \psi\upsilon'' + (1-\psi)\upsilon'$

$V = \psi\rho''\upsilon'' + (1-\psi)\rho'\upsilon'$

(*Mamaev*, et *al*, 1978, *Kutateladze*, 1973, *Berglas and Sju*, 1970, *Averjev*, *1960*, *Droznin*, 1980, *Zabarny, et al*, 1992). The main difficullties in the solution of equations (11) - (12) are o —— with the variety of possible types of two-phase fluid flow in the wellbore influencing the hydraulic resistance to flow and heat exchange between two-phase flow and surrounding rocks.

Summarizing the results of previous *studies* of two-phase flow in the pipes and wellbores we have obtained the relations that provide the determination of the shear stress value τ_k for different patterns of two-phase fluid flow in the wellbore (pellet or annular type) as well as the criteria for the type of flow pattern identification (*Piscacheva, 1991*). This made it possible to find the relationship between the pressure *gradient* in the wellbore and mass production rate of the fluid

$$(\partial p/\partial z)_f = 2\tau_k/r_w = FQ^2\,, \qquad (13)$$

where the coefficient F is dependent *on* the pattern type of two-phase fluid flow. As far as the intensity of the heat *exchange* is concerned it is found that it *can* be *calculated* as follows

$$g_w = 2K_\tau[t(z) - t]/r_w\,, \qquad (14)$$

where K_τ is transient heat exchange coefficient determined by the expression (*Pudovkin*, et *al*, 19..)

$$K_\tau = k_t/[1 + Bi\ \ln(1 + \gamma\sqrt{Fo})]\,, \qquad (15)$$

$t(z)$ is initial temperature of rocks at the depth; t is temperature of the two-phase fluid at the same depth;

$Bi = a_0 r_w/\lambda_0$

$Fo = a_0\tau/r_w^2$

$a_0 = \lambda_0/(\rho_0 c_0)$

γ - coefficient approximating the time dependence of K_τ (if $Bi < 30$ $\gamma = 2$, if $Bi > 30$ $\gamma = \pi$);

$k_t = 1/(1/\alpha_0 + \delta_c/\lambda_c + \delta_t/\lambda_t)\,,$

Set of equations (11) - (12) is solved by finite difference method using the explicit schemes. The equation of motion (11) is solved first accounting to the pressure value found at the wellbore from the solution of equations *(5)* - (6) for the fluid flow in the productive zone or to the accepted pressure at the well head The pressure distribution along the wellbore obtained is used then to solve the equation of energy (12). *After* that all coefficients in the equations (11) and (12) arc recalculated according to the obtained values of pressure and enthalpy and *set* of equations (11) - (12) is solved again. Phase state of the fluid in the wellbore is determined by the value of its specific enthalpy. If $e(z) > e'_s$ (e'_s is the specific enthalpy of saturated water) it means that at the depth z phase transfer takes place. In *order* to provide the compatible performance of a well and productive zone *set* of equations *(5)* - (6) and that of equations (11) - (12) must be simultaneously satisfied that requires a proper selection of the solution methods depending on the spatial discretization scheme used and well location in the area of calculation.

If the radial grid with distributed nodes is used and well is located at the *center* of the calculational area (for instance, circular), the discrete equation *(5)* in cylindrical coordinates r , z can be *represented* as the following balance relation for the block containing the well

$$Q_k = q_{kr+1/2} + q_{k+1/2} - q_{k-1/2} + q_k\,, \qquad (16)$$

where Q_k is mass fluid discharge from the block located in the layer "k"; $q_{kr+1/2}$ is mass fluid flux in radial flow across the circumference located at the middle between the first and second nodes of the grid; $q_{k+1/2}$ is the same for the ascending flow of fluid across the bottom of the block; $q_{k-1/2}$ is the same for the fluid flow across the upper cross section of the block; q_k is mass of fluid extracted from the block volume due to *compressibility* of the productive zone and vaporization of water.

Discrete equation of fluid flow in the wellbore regarding to the relationship (13) *can* be represented as follows

$$p^w_{k+1} =$$

$$= p^w_k + \rho_{k+1/2}\,g(z_{k+1} - z_k) + F_{k+1/2}(z_{k+1} - z_k)Q^2_{k+1/2}\,, \qquad (17)$$

where $Q_{k+1/2}$ is mass fluid flow rate across the wellbore between the nodes z_k and z_{k+1}; $\rho_{k+1/2}$,$F_{k+1/2}$ are average values of fluid density and of the coefficient F characterizing the hydraulic resistance in the same interval of the wellbore. The value of $Q_{k+1/2}$ in the *cased* interval of the wellbore is to be *taken equal* to the total mass production of fluid Q_T , while in the non-cased (e. g. productive) interval consisting in general of several grid blocks this value must be determined from the following equation of mass balance for the *interval* of the wellbore between the adjacent grid blocks

$$Q_k - Q_{k+1/2} - Q_{k-1/2} = 0\,, \qquad (18)$$

where $Q_{k-1/2}$ is mass flow rate in the wellbore between the nodes z_k and z_{k-1}.

The compatibility condition for the well and productive zone performance will be satisfied in the considered case *if* the pressure in a grid block containing the well is *equal* to the pressure in the wellbore, e. g.

$$p_k = p^w_k \qquad (19).$$

Considering the equations (16) - (18) it is *easy* to note that it is impossible to find a general solution which will satisfy the condition (19) using *any* direct method because of non-linear dependence of mass flow rates $Q_{k+1/2}$ and $Q_{k-1/2}$ on the pressure drop in the wellbore $\Delta p^w_{k+1/2}$ and $\Delta p^w_{k-1/2}$. It means that the model proposed by K. Aziz and A. Settari *(Aziz and Settari, 1982)* for the well and productive zone compatibility can not be used in the case of steam and water producing wells and the solution of the equations (16) - (18) has to be found by the iteration procedure.

In the case of well displacement from the center of the calculational area as well as when the block-centered grid is used for spatial discretization the compatibility condition (19) becomes incorrect. In these cases the pressure in a grid block p_{kr} must satisfy to the mass balance equation similar to the equation (16)

$$Q_k = q_{kr+1/2} - q_{kr-1/2} + q_{k+1/2} - q_{k-1/2} + q_k, \qquad (20)$$

which differs from (16) only by presence of radial mass flow flux $q_{kr-1/2}$ from inner contour of the grid block to the center of the calculational area. The pressure p_{kr} can be considered as the pressure at the circular gallery with the radius r_e that is equal to the well displacement from the center of the calculational area It is clear that the pressure in the wellbore p^w_k must be less than the pressure in the grid block p_{kr} in order to provide fluid extraction from the productive mne. Approximate value of the pressure p^w_k can be found at the assumption that the fluid discharge from a grid block Q_k corresponding to the equation (20) is equal to the mass production rate of a well located at the center of a circular reservoir which side surface area is equal to that of the grid block and regards to the circumference with the radius r_e, where the pressure is equal to the pressure in a grid block p_{kr} (Fig. 1). Taking into consideration the

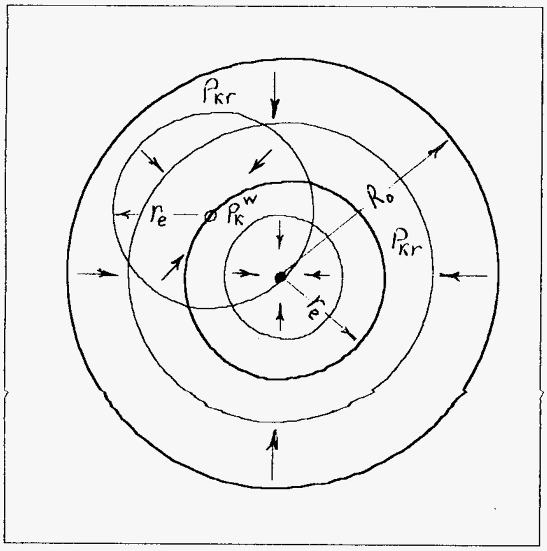

Fig. 1. Schematization of flow to excentricaly located well.

possible change of pressure and vapor saturation in a grid block in time fluid discharge rate from the fictitious reservoir can be expressed as an implicit function of these parameters *(Aziz and Settari, 1982)*,

$$Q^{n+1}_k = C_k(p^{n+1}_{kr} - p^w_k) + (C'_k)_s(p^n_{kr} - p^w_k)(S^{n+1}_k - S^n_k), \qquad (21)$$

where C_k is productivity factor of a well producing in the fictitious reservoir; $(C'_k)_s = \partial C_k / \partial S_k$; p^n_{kr}, S^n_k, p^{n+1}_{kr}, S^{n+1}_k - values of pressure and vapor saturation at the time steps n and $n+1$ respectively.

The solution of the equations (17), (18), (20) and (21) also requires the use of iteration methods. The solution of energy equation (6) for the fluid flow in the productive zone and that for the flow in the wellbore (12) has to be done after obtaining the pressure values in grid blocks p_{kr} and in the wellbore p^w_k as well as the mass fluid rate distribution Q_k and $Q_{k+1/2}$ and $Q_{k-1/2}$

3. RESULTS

To probe the developed model it was applied to the interpretation of field test data obtained from the wells K-8 and E-1 drilled at the northern part of Pauzhetka geothermal field. The wells intercepted the psephite tuff aquifer with superheated water at the depth from 40 - 90 to 270 - 350 m *(Sugrobov and Kraevoy, 1966)*.

The most simple scheme of the well and productive zone interaction was used for the calculations: radial steady state one-phase flow in a homogeneous bed to the well having ideal conditions for fluid entry. That made. numerical simulation of the filtration problems in the productive zone unnecessary.

Initial pressure in the productive zone was assumed to be equal to hydrostatic one with regard to the depth of its bedding while the pressure in the vicinity of the well was supposed to be equal to the pressure at the wellbore that has been found from the solution of equation (18) taking into account the values of the mass production rate Q_T and well head pressure PI, accepted. As the results the pressure at the well bottom (e. g. pressure drop relative to the initial value). the depth of vaporizing front in the wellbore and regime of fluid flow in it (pellet or annular) were determined for each combination of Q_T and PI, .

Since under considered conditions the dependence of Q_T on p_h is determined by fluid transmissibility kh and its temperature t_f these two parameters were subjected to stochastic variation within some range of their accepted values made by Monte-Carlo method that was accompanied by numerical simulation of the fluid flow. Results of simulation are presented in Fig. 2.

Testing of the developed numerical program for the simulation of two-phase non-isothermal filtration in the productive zone was done by comparison the results obtained with known numerical and quasi-analytical solutions of the problems concerned with the radial flow in a homogeneous formation at the different values of initial vapor saturation and constant rate of fluid discharge per 1 m of effective thickness of the reservoir (Fig. 3). As it seen from these data the results of all calculations are in a very good agreement that proves the adequateness of the developed model.

4. DISCUSSION OF THE RESULTS

Since the $Q - p_h$ diagram obtained as the result of numerical simulation (Fig. 2) is influenced by two parameters characterizing the fluid flow in the productive zone (e. g. fluid transmissibility kh and its temperature t_f) they can be considered as the additional conditions of the well and productive zone compatibility besides the condition (19) prescribed before. Comparing the field well test data with the results of numerical simulation of the well and productive zone compatible performance combined with the stochastic simulation of their compatibility conditions makes it possible to estimate the value of these parameters.

To get a definite result of such estimation it is important to run the test at each step of the well head pressure long enough to provide quasi-stationary fluid flow in the productive zone and heat exchange in the wellbore.

5. CONCLUSIONS

The results of the fulfilled studies show that numerical simulation of the well and productive zone compatible performance combined with stochastic simulation of their compatibility conditions is capable to serve as an effective mean to increase the informativeness of all testing works at high temperature hydrothermal reservoirs.

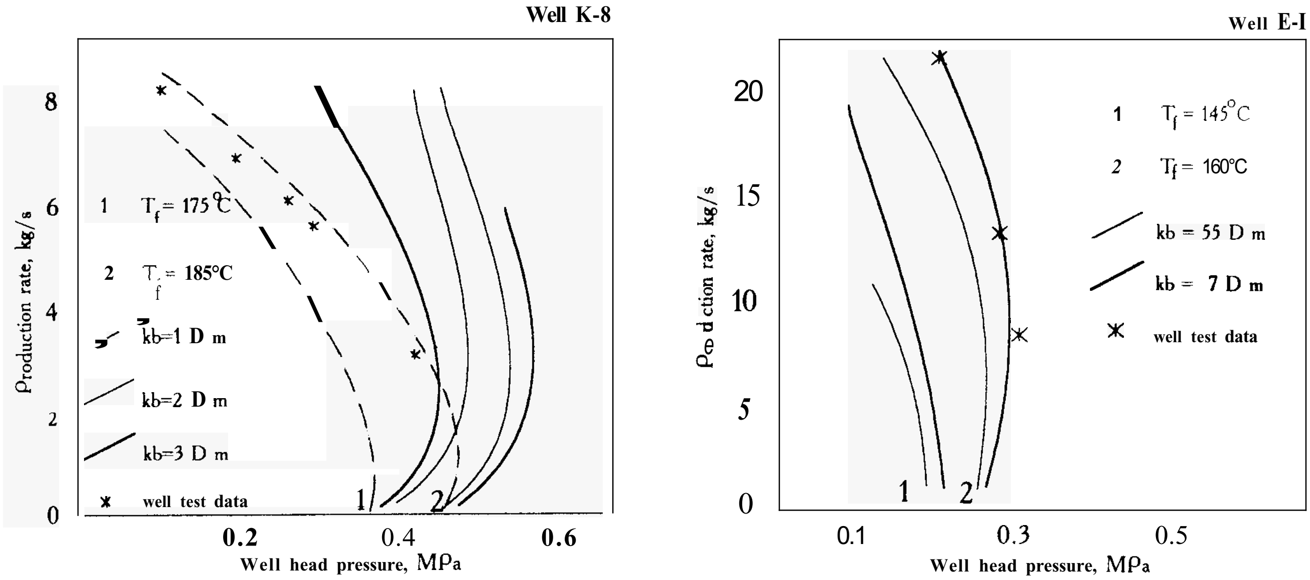

Fig. 2. Calculated production rate vs well head pressure.

a) initial vapour saturation $\sigma_o = 0$

b) initial vapour saturation $\sigma_o = 0.35$

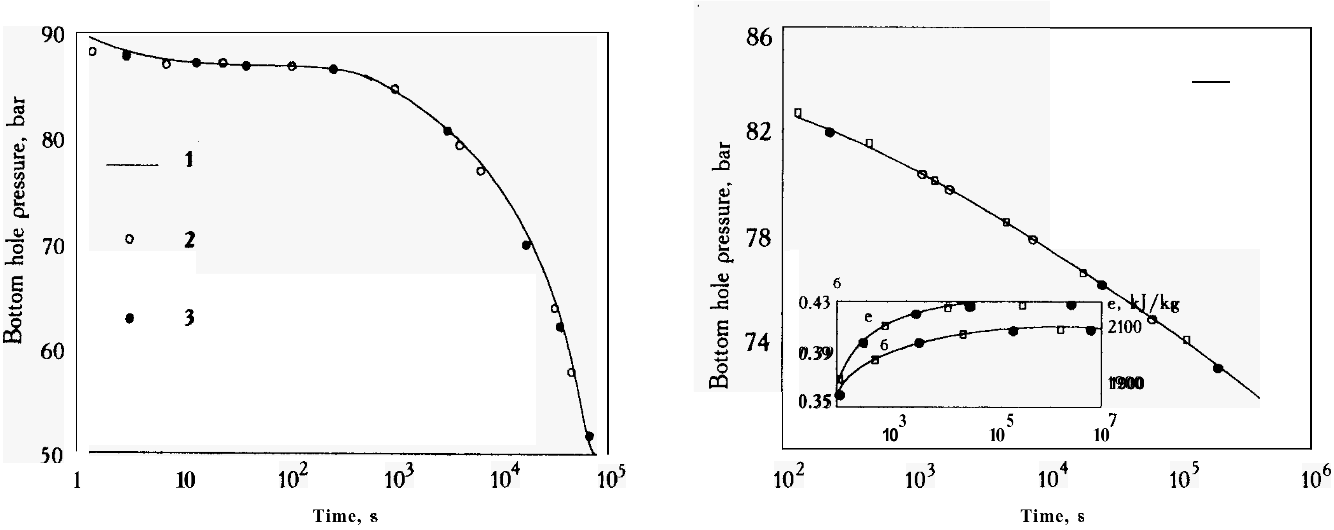

Fig. 3. Comparison of the simulation results (3) with other solutions of the test problem : by TOUGH Program (1a), Garg (2) ,O'Sullivan (4) , Grant (1b) *(Grant and Sorry, 1979).*

6. NOMENCLATURE

a_α , C	-coefficients in finite difference equations
a_0	-thermal conductivity
b	-effective thickness of the productive zone
Bi	-Biot criterion
c_0	-specific heat capacity
c_k	-well productivity factor
e	-enthalpy of the fluid
E	-specific volume enthalpy of the productive zone
Fo	-Fourier number
g	-gravity acceleration
h	-spatial step size
i, j	-grid node number
k	-absolute permeability
k_t	-heat transfer coefficient
K_τ	-transient heat exchange coefficient
m	-porosity
n	-time-step number
p	-pressure
q	-mass fluid flux
q_m, q_e	-sources of mass and energy
q_w	-heat exchange intensity between the fluid flow in the wellbore and surrounding rock
Q	-mass fluid flow rate

r	-radial coordinate
$r_{i,j}$	-grid function of mass fluid flux
R', R''	-relative liquid and vapor phase permeabilities
S	-vapor saturation of the fluid
t	-temperature
T	-fluid mobility coefficients
T', T''	-fluid phase mobility coefficients
υ	-fluid flow flux
V	-mass fluid flow flux
V', V''	-liquid and vapor phase mass fluxes
x, z	-horisontal and vertical coordinates
α_0	-heat exchange coefficient
β_s	-rock compressibility
δ'	-fluid and rock matrix compressibility
δ_c, δ_t	-thickness of the cement and casing pipe
a	-productive zone heat conductivity coefficient
λ', λ''	-liquid and vapor phase heat conductivity coefficients
μ', μ''	-fluid phase dynamic viscosities
η	-mass flux vapor saturation of the fluid
ρ	-fluid density
ρ', ρ''	-liquid and vapor phase densities
ρ_0	-rock density
τ	-time
τ_k	-shear stress at the flow - wellbore wall contact
φ_1, φ_2	-source terms in finite difference equations
ψ	-real vapor saturation of the fluid flow in the wellbore

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