Estimation of Hcat Discharge Rate at the Permeable Surface of a Geothermal Reservoir

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ABSTRACT

This paper describes a method for estimating the heat discharge rate at the permeable surface of a geothermal reservoir. This method is based on the so–called, film theory, in which a liquid film is assumed to be a thin layer of water on the ground surface. This assumption yields a new boundary condition, which is applicable to a numerical analysis of mass and heat transfer in the reser– voir. Using the numerical solutions, we can estimate the mass and the heat discharge rates and their distributions on the surface in detail. The boundary condition pro– posed applies to the analysis of already existing observed data on the Takenoyu geothermal area (Kumamoto, Japan), whose special feature is the non–existence of cap rock. The calculatcd values correspond quite well with the observed data on the mass and the heat discharge rates on the surface, their distributions, and the tempera– ture distribution in the reservoir.

1.INTRODUCTION

Simultaneous mass and heat discharges, **i.e.** hot springs, fumaroles, or hot pools, are observed on the ground surface of geothermal fields (Yuhara et ai., 1983, Chida and Niibori, 1986, Hochstein, 1988, Hochstein et al., 1990, Lippmann et al.,1991, Weir et al., 1992), where the mass transfer process is significantly important in order to comprehend geothermal reservoirs. This matter demands a more suitable boundary condition at the ground surface than the so–called, Dirichlet type B.C. used by, e.g. Ingebritsen and Sorey, 1988, Niibori and Chida, 1989, Mongelli, 1994, Lai et al., 1994, Oldenburg et al., 1994 and others.

Chida and Niibori (1986) and Niibori et al. (1990) have discussed such a boundary condition to analyze the heat discharge rate at the permeable surface of a geothermal reservoir. This paper describes its application through numerical analysis of observed data, for the natural state **of** Takenoyu geothermal field. The data, reported by Yuhara et al.(1983), summarize the mass and heat dis– charge rates on the ground surface, their distribution and the temperature distribution in the reservoir. This method gives realistic estimates of how much surface heat dis– charge is due to fluid flow, and of heat transfer coeffi– cients.

2.BOUNDARY CONDITION CONSIDERING HEAT DISCHARGE WITH MASS TRANSFER

Figure 1 shows an illustration of a heat discharge process with mass and heat transfer. Ignoring the x(horizontal)–direction flow at the ground surface, the heat balance at the surface is

$$(c_p\rho)_f w\theta_{1+0} + h(\theta_{1+0} - \theta_a) = (c_p\rho)_f w\theta_{1-0} + q_{z1} \quad , \qquad (1)$$

where c_p is the specific heat (J/kg) in a constant pressure, ρ, the density (kg/m^3), w , the flow velocity (m/s) in the z (vertical) direction, 8, the temperature (K), h, the heat transfer coefficient (W/m^2 K) to the atmosphere layer, and the subscripts f and a are fluid and atmosphere, respectively. q_{z1} is the conductive heat **flux** (W/m^2) at the permeable surface, defined by

$$q_{z1} = -\lambda_e \frac{\partial\theta}{\partial z}\bigg|_{z=z1} \quad , \qquad (2)$$

where λ_e is the effect thermal conductivity(W/m K). In Equation (1), θ_{1+0} and θ_{1-0} are $\lim_{\delta\to 0}\theta_{z1+\delta}$ and $\lim_{\delta\to 0}\theta_{z1-\delta}$, respectively. The two values of θ at $z=z_1$ are due to exist– ence of mass discharge, i.e. $w\neq 0$. (When $\theta_{1+0}=\theta_{1-0}$, the boundary condition is equivalent to the Newton's low of cooling (Bird et al.,1960).) The derivative in Equation (2) should be evaluated at $\lim_{\delta\to 0+}(z_1-\delta)$, so that it is defined. The temperature rapidly changes between θ_{1+0} and θ_{1-0} in a liquid film supposed on the border of the reservoir. In a numerical calculation, however, it is impossible to esti– mate θ_{1+0} and θ_{1-0}, because the values are apparently discontinuous in the mathematical description. Therefore, in order to analyze the heat discharge, it is necessary to improve upon this model.

Figure 2 illustrates the concept of the proposed improved boundary condition, where we assume a thin water layer between the atmosphere layer and the reservoir. The water layer has the two sub–layers, i.e. a free convection zone and a thermal boundary layer.

When the thickness of the water layer, $(z_{sur}-z_1)$, is small compared to the atmosphere layer or the reservoir, the heat balance when $w>0$ is

$$(c_p\rho)_f w\theta_{sur} + h(\theta_{sur} - \theta_a) = (c_p\rho)_f w\theta_1 + q_{z1} \quad , \qquad (3)$$

where h is the heat transfer coefficient (W/m^2 K) to the atmosphere, and q_{z1}, defined in Equation (2), also satis– fies:

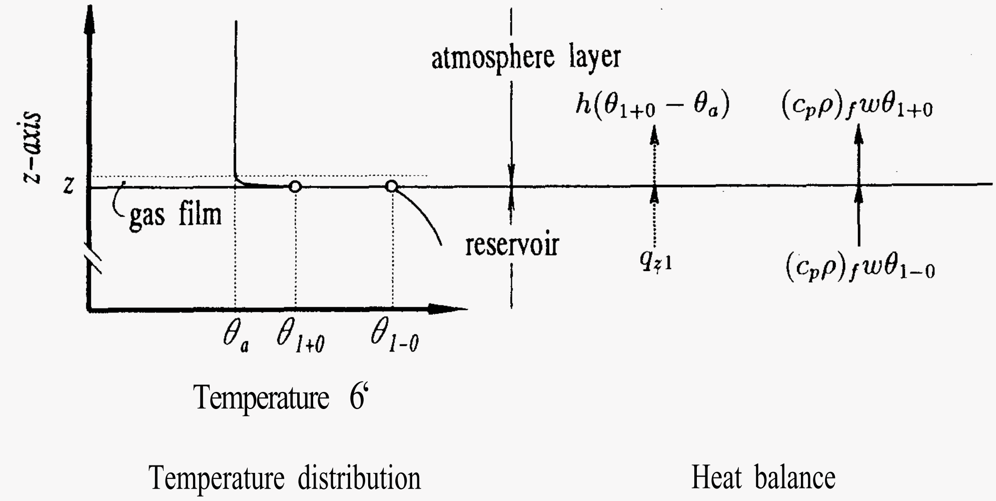

Figure 1 Illustration of heat and mass discharge process on a permeable ground surface. (z_1:the location of the surface, z–axis:vertical(depth) direction.)

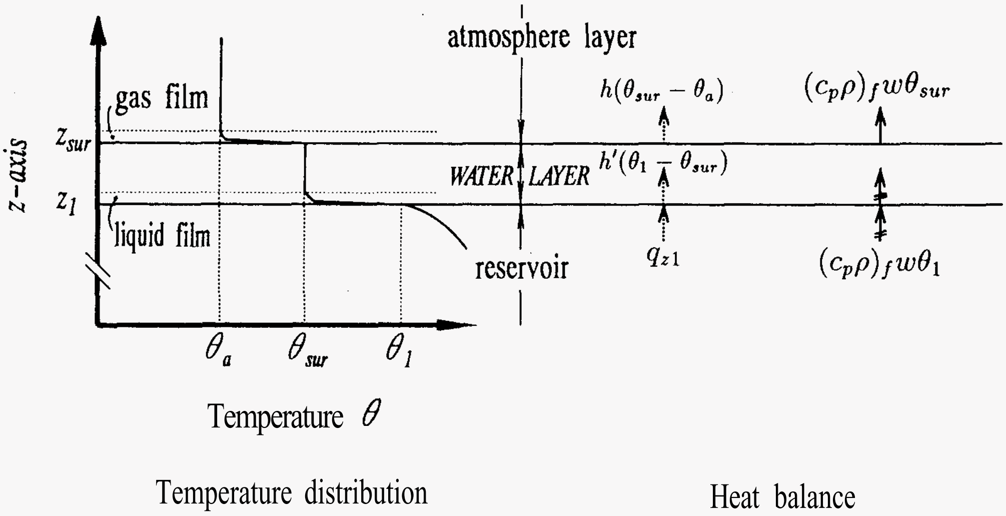

Figure 2 Concept of the permeable boundary ($w>0$, w:fluid flow velocity in z–direction. the thickness of water layer, i.e. ($z_{sur}-z_1$), is assumed to be thin, relative to the atmosphere layer or depth of reservoir, z_1, in the heat balance equation (3).)

$$q_{z1}=h'(\theta_1-\theta_{sur}) \quad , \qquad (4)$$

where h'is the heat transfer coefficient (W/m^2 K) to the water layer. Substituting Equation (4) to Equation (3) yields

$$h'(\theta_1-\theta_{sur})-h(\theta_{sur}-\theta_a)=(c_p\rho)_f w(\theta_{sur}-\theta_1) \quad . \qquad (5)$$

Further, we define the following dimensionless variables:

$$\Theta_1=\frac{\theta_1-\theta_a}{\theta_h-\theta_a} \; , \quad \Theta_{sur}=\frac{\theta_{sur}-\theta_a}{\theta_h-\theta_a} \; , \quad W=\frac{z_1 w\rho}{\mu}P_r \; ,$$

$$P_r=\frac{c_p\mu}{\lambda_e} \; , \quad H=\frac{h}{h'} \; , \quad N_u=\frac{h'z_1}{\lambda_e} \; , \quad Z=\frac{z}{z_1} \; ,$$

where μ is the viscosity (Pa s) and the subscript of 0, h, denotes heat source. Then, Equation (5) gives

$$\Theta_{sur}=\frac{1+W/N_u}{1+H+W/N_u}\Theta_1 \quad . \qquad (6)$$

Therefore, Θ_{sur} can be derived from Θ_1. Further, using the dimensionless variables, Equations (2) and (4) yield

$$-\frac{\partial\Theta}{\partial Z}\bigg|_{Z=1-0}=N_u(\Theta_1-\Theta_{sur}) \quad . \qquad (7)$$

According to Equation (7), the temperature gradient at the ground surface is decided by ($\Theta_1-\Theta_{sur}$) obtained from Equation (6). In other words, Equations (6) and (7) avoid the discontinuity of temperature in Equation (1).

When $w<0$, the heat balance at the ground surface is

$$(c_p\rho)_f w\theta_a+h(\theta_{sur}-\theta_a)=(c_p\rho)_f w\theta_1+q_{z1} \quad , \qquad (8)$$

where the temperature of inflow water to the reservoir is assumed to equal the atmosphere temperature θ_a. (Note that the first term of the left side is different from that of Equation (3) for $w>0$.) The relation between Θ_1 and Θ_{sur} are

$$\Theta_{sur} = \frac{1 + W/N_u}{1 + H}\Theta_1 \qquad (9)$$

When W=0, Equation (6) coincides with Equation (9).

3.EVALUATION OF HEAT DISCHARGE

Now consider a hydrothermal system with permeable ground surface. Figure 3 shows its schematic diagram, where X, and Y_1 are defined by x_1/z_1 and y_1/z_1, respectively. x, y_1 and z_1 determine its size (m) in space. The total heat discharge rate at the ground surface, Q,, is divided into two kinds of heat rates:

$$Q_1 = Q_{1f} + Q_{1h} \quad , \qquad (10)$$

where Q_{1f} and Q_{1h} are the heat discharge rates (W) due to fluid flow (convection) and thermal conduction, respectively. These are

$$Q_{1f} = (c_p\rho)_f \int_{-y_1}^{y_1}\int_{-x_1}^{x_1}(w\theta)_{z=z_1}dxdy \quad , \qquad (11)$$

$$(12)$$

Substituting Equations (11) and (12) into Equation (10), we get the following dimensionless formula:

$$\mathbf{Q}_1 = \mathbf{Q}_{1f} + \mathbf{Q}_{1h} \quad , \qquad (13)$$

where

$$\mathbf{Q}_1 = \frac{Q_1}{z_1\lambda_e(\theta_h - \theta_a)} \quad , \qquad (14)$$

$$\mathbf{Q}_{1f} = \int_{-Y_1}^{Y_1}\int_{-X_1}^{X_1}(W\Theta)_{Z=1}dXdY \quad , \qquad (15)$$

$$\mathbf{Q}_{1h} = N_u\int_{-Y_1}^{Y_1}\int_{-X_1}^{X_1}(\Theta_1 - \Theta_{sur})_{Z=1}dXdY \quad . \qquad (16)$$

Thus, the ratio of discharge by fluid flow to total heat discharge rate, f_{Qf}, is

$$f_{Qf} = \frac{\mathbf{Q}_{1f}}{\mathbf{Q}_1} \quad . \qquad (17)$$

4.APPLICATION AND DISCUSSION

4.1 Fundamental Equations

To apply this boundary condition proposed to a numerical analysis, let us consider in its natural state, a two-dimensional hydrothermal reservoir of single-phase liquid. Figure **4** illustrates the boundary conditions. At the permeable surface we apply Equations **(6),(7)** and **(9).** Further, we assume a heat source of $\Theta=1$ at $Z=0$, $0\leq X\leq X_h$, where X_h denotes its spread, as shown in Figure **4.** The other boundaries are the insulated and no flow boundary. In the pseudo steady state of such a reservoir, the momentum and heat balances (Cheng and Lau, **1974,** Faust and Mercer, **1979)** are respectively described by

$$\frac{\partial}{\partial Z}\left(\frac{1}{K_x}\frac{\partial\psi}{\partial Z}\right) + \frac{\partial}{\partial X}\left(\frac{1}{K_z}\frac{\partial\psi}{\partial X}\right) = -R_a\frac{\partial\Theta}{\partial X} \quad , \qquad (18)$$

$$0 = \frac{\partial^2\Theta}{\partial X^2} + \frac{\partial^2\Theta}{\partial Z^2} - U\frac{\partial\Theta}{\partial X} - W\frac{\partial\Theta}{\partial Z} \quad , \qquad (19)$$

where ψ is the dimensionless, stream function:

$$U = \frac{\partial\psi}{\partial Z}, \qquad W = -\frac{\partial\psi}{\partial X} \quad .$$

K_x, K_z, U and R_a are the dimensionless variables:

$$K_x = \frac{k_x}{k^*} \quad , \quad K_z = \frac{k_z}{k^*} \quad , \quad U = \frac{z_1 u \rho_f}{\mu}P_r \quad ,$$

$$R_a = \frac{z_1 k^* \rho_f^2 g\beta(\theta_h - \theta_a)}{\mu^2}P_r \quad ,$$

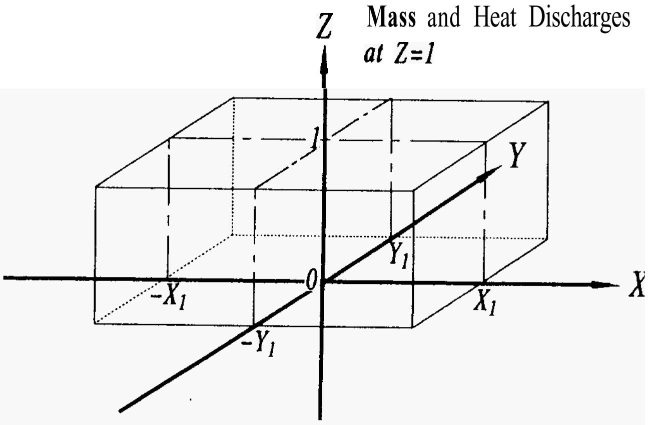

Figure 3 The schematic diagram of a reservoir to be analyzed.

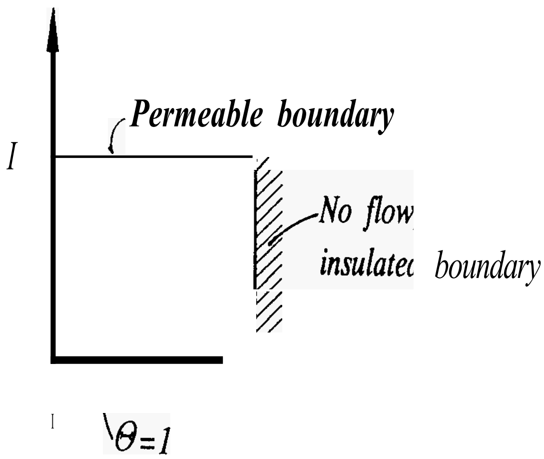

Figure **4** Illustration of the boundary conditions where u is the fluid flow velocity (m/s) in x−direction, k is the permeability (m^2) in each direction, g, the gravity acceleration (m/s^2), β, the heat expansion coefficient (1/K) and k^* is the characteristic permeability (m^2).

Equations (18) and (19) with the boundary conditions are solved by the finite difference method. Then, we need to determine some dimensionless parameters. Those are values of R_a, N_u, H, X_1, X_L, K_x and K_z.

4.2 Application and Discussion

Takenoyu geothermal field is located in Kumamoto prefecture, Japan (Yuhara et al., 1983). A special feature is the non−existence of a cap rock. Yuhara et al. (1983) reported the field data of heat discharge rate on the ground surface, also the temperature distributions obtained from some wells. The mass and heat rates discharged by fluid flow were evaluated by measuring the flow rate and temperature of hot water at each of fumaroles, hot springs and hot pools. The total number was more than one hundred. Also, the heat rate discharged by heat conduction was calculated by using some data of the thermal conductivity and temperature gradient at measurement points on the ground surface. Further, Yuhara et al.(1983) have proposed some data required to analyze the reservoir, as shown in Table 1. These valucs are applicable to calculate the dimensionless parameters in the mathematical model. However, there is little data on the permeability distribution. Thus, this paper assumes a homogeneous medium, that is, $K_x=K_z=1$.

Table 1: Takenoyu field data (Yuhara et al.,1983)

Parameter	Symbol	Value
characteristic effective permeability, m/s	$k^*\rho g/\mu$	2.0×10^{-7}
breadth of the reservoir, m	$2y_1$	1400
volumetric expansion coefficient, 1/K	β	8.4×10^{-4}
heat resource temperature, K	θ_h	523
average atmosphere temperature, K	θ_a	285
horizontal size, m	$2x_1$	2500
effective heat conductivity, W/(m K)	λ_e	2.1
heat capacity, J/(m^3 K)	$(\rho c_p)_f$	4.2×10^6

Figure 5 shows comparisons of the calculated temperature with the observed values in four wells, i.e. (a)TY−1, (b)K−6, (c)B−1 and (d)GSR−3, whose locations are $X=0.09$, $X=0.06$, $X=0.0$, and $X=0.11$, respectively. The calculated results and the observed data are good agreements when the following parameters arc used in the calculation: $R_a=80$, $H=3$, $N_u=67$, $X_1=1.25$, and $X_h=1.0$. These were estimated from the values of parameters in Table 1, assuming the other unknown parameters, i.e. $z_1=1$km, $x_h=1$km, and $h=0.42$ W/(m^2 K). The assumptions are on the basis of several trials to fit the observed temperature data of every one hundred meter in depth (represented by square symbols in Figure 5). The estimated value $h=0.42$ W/(m^2 K) is relatively small, compared with the range of values in Bird et al.,1960, pp.393. At this stage, all values of the dimensionless parameters are fixed.

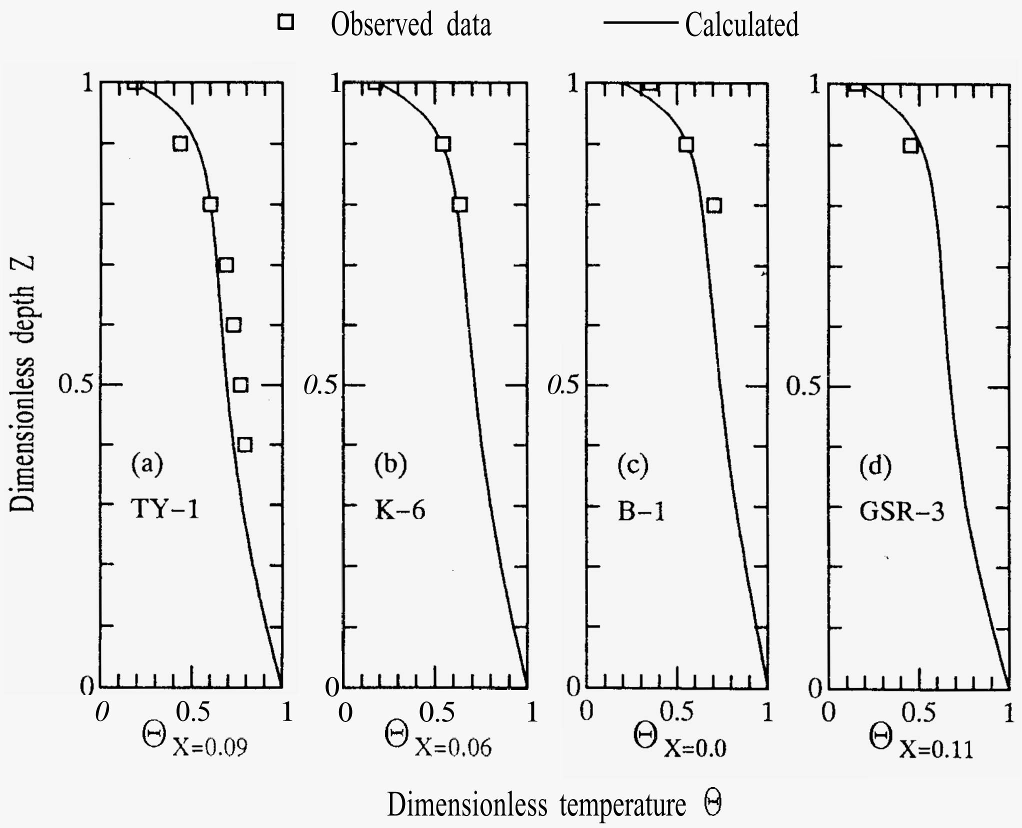

Figure 5 The calculated temperatures and the observed data (Yuhara et al.,1983) for each well ((a)TY−1,(b)K−6,(c)B−1 and (d)GSR−3) in Takenoyu geothermal field.

Table *2* shows the calculated results and the observed data of the mass and heat discharge rates. The calculated values correspond well with the observed data. Figure **6** shows the calculated, dimensionless flow velocity, i.e. *W*, at *Z=1*. The region where W>0 is equivalent to the mass discharge area. Yuhara et al. (1983) have reported that the mass discharge area is *600*m by *1400*m on the basis of the observed data. On the other hand, the calculated result was *850*m by *1400*m. Thus, its difference from the observed data is 250m in width, as shown by the hatched region in Figure **6.** The hatched region is only 8%of the whole region where *W>0*.

Table 2: Comparison of calculated results(this work) with field data (Yuhara et al.,1983)

	Calculated	Observed data
mass discharge	*10.2*kg/s	*13.0*kg/s
heat discharge	5.0×10^6 W	6.3×10^6 W

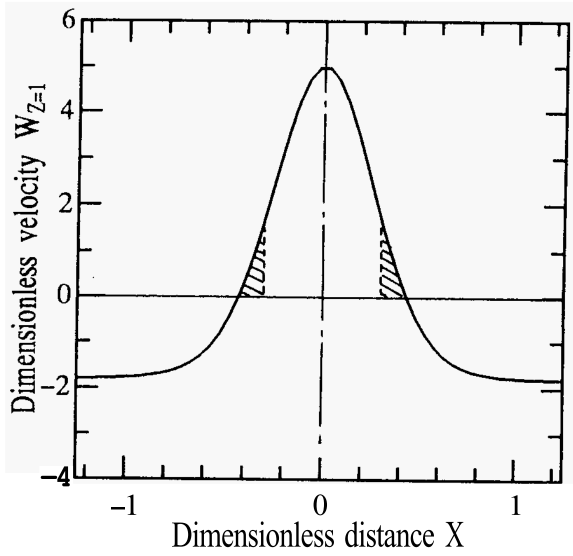

Figure **6** Distribution of the dimensionless velocity in *Z*-direction, *W*, at the ground surface at *Z=1*. ($X=x/z_1$, z_1 is *1000*m. The breadth in y-direction, $2y_1$, is assumed to be *1400*m in Table 1.)

Yuhara et al.(1983) have concluded that ***0.92*** is the heat discharge ratio by fluid flow, i.e. f_{Qf} defined by Equation (17) on the basis of the observed data for the Takenoyu geothermal field. On the contrary, the value estimated from the calculation in Figures 5 and **6** and Table *2* is 0.32. Nevertheless the value of ***0.32*** calculated here **is** more realistic than would be obtained using the Dirichlet type B.C., which always gives $f_{Qf}=0$. **A** calculation that used a Dirichlet type B.C. cannot estimate heat and mass discharge rates.

Figure 7 shows some calculated temperature distributions at *X=0* and their f_{Qf}-values, where f_{Qf} denotes the heat discharge rate by fluid flow against total heat discharge rate (see Equation (17)) and the values of the dimensionless parameters except *H* are equal to those for the Takenoyu geothermal field, respectively. In Figure 7, the f_{Qf}-value is found to be larger, as the H-value increases, while the absolute value of the temperature gradient at *Z=1* is smaller. In addition, strong sensitivity of *H* on the temperature distribution appears only in the zone relatively close to *Z=1*. This fact suggests that we need more detail of the temperature distribution observed in the domain close to the ground surface, in order to evaluate f_{Qf} with higher accuracy.

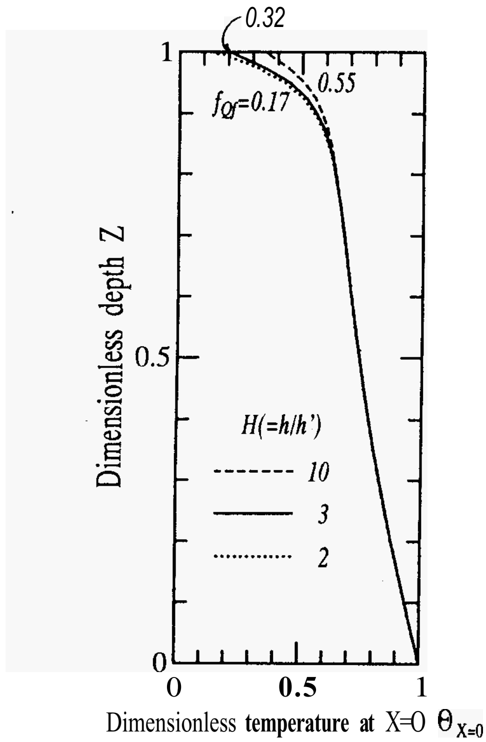

Figure 7 Temperature distribution in the vertical, i.e., depth-direction *Z* at *X=0*, and the heat ratio discharged by the fluid flow, f_{Qf} (see equation (17)).

5.CONCLUSIONS

A boundary condition with a thin water layer composed of a liquid film and a convection zone was applied to analyze a geothermal reservoir. This condition gives a more realistic estimation than when assuming a Dirichlet type boundary condition for the mass and heat discharge rates on ground surface of the reservoir. Its validity was confirmed through good agreements between the calculated results and the observed data of Takenoyu geothermal field (Kumamoto, Japan). The proposed method is applicable to analysis of a geothermal reservoir with mass and heat discharge on its ground surface.

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