COMPARATIVE ANALYSIS OF ESTIMATION OF HORIZONTAL GEOTHERMAL WELL PRODUCTIVITY

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Abstract. This work compares some of the most commonly used analytical solutions and discusses estimates of horizorital well productivity and perfomance through numerical simulation,which is a powerful tool for comparing the productivity of vertical and horizontal wells since it can account for natural fractures,heterogeneities in three dimensions,multi-phase flow and a variety of boundarv condions.The importance of this estimation for geothermal well is shown.

A worldwide interest exists today in drilling horizontal wells to increase productivity. Because of its large flow area, a horizontal well may be several times more productive than a vertical one draining the same volume. Therefore,to determine the economic feasibility of horizontal well drilling, the engineer needs a reliable method of its expected productivity estimate.

The solution of the partial-differential equation that describes the flow behavior of a horizontal well and that preserves the physics is very complex. Because of this , simplifying assumptions are frequently introduced, for example:

- filtration of incompressible viscous fluid is subjected to linear Darcy low ;
- linear Laplace equation describes potential pressures and velocities of fluid movement. in reservoir;
- drainage volume is a circular region with natural feed;
- inflow of fluid to well is the result of stationary filtration conditions;
- reservoir is an anisotropic fractured rock :
- fluid is characterized by mean of the viscosity and by mean of the formation volume factor.

These assumptions are approximate,but they *are* widely used in underground hydromechanics and are generally accepted. Drainage volume can he of any dimension. In this work inflow of oil t.o a single horiizontal well in anisotropic formation is discussed . The analysis is based upon the formulae for the flow-rate of oil of a single well and the method of filtrational resistances.

In technical literature the following relations are known :

The approximate S.Joshi's formula [2,3].

$$Q_h=\frac{2\pi k_h h\Delta P}{\mu B_0\left[\ln\left(\frac{a+\sqrt{a^2-0.25L^2}}{0.5L}\right)+\left(\frac{\beta h}{L}\right)\ln\left(\frac{\beta h}{2r_w}\right)\right]} \quad (1)$$

The exact formula of Renard and Dupuy [2,3].

$$Q_h=\frac{2\pi k_h h\Delta P}{\mu B_0\left[\cos h^{-1}\left(\frac{2a}{L}\right)+\left(\frac{\beta h}{L}\right)\ln\left(\frac{h}{2\pi r'_w}\right)\right]} \quad (2)$$

The appoximate V.G.Griguletsky's formula [2,3].

$$Q_h=\frac{2\pi\sqrt{k_h k_v}\, h\beta\Delta P}{\mu B_0\left[\ln\left(\frac{4R_c}{L}\right)+\left(\frac{\beta h}{L}\right)\ln\left(\frac{\beta h}{2\pi r_w}\right)\right]} \quad (3)$$

where: Eo-the formation volume factor:

$a=0.5L\sqrt{0.5+\sqrt{0.25+(2R_c/L)^4}}$; $\beta=\sqrt{k_h/k_v}$;

L - the length of norizontal well;r_w - the wellbore radius;R_c - the radius of circular *feed* boundary ; $r'_w=0.5(1+\beta)/\beta$; μ - the viscosity; k_h arid k_v - the horizontal and vertical permeabilities, respectively; h - the thickness of productive reservoir: ΔP - the pressure drop hetween pressure on boundary of a circular teed contour P_c and pressure on bottomhole P_b.

In plane case replacement of well with flows arid sources depending on sign of flow-rate i.e.with points with finite intensivity permit:.; to account the influence of boundaries and so on simply enough. Attempt to account the well radius finiteness be led to the problem of investigation of hydrodynamics of flow inside a well and in a horizontal filtrational zone and to taking into account of nonlinear effects. But for problems of oil output or geothermics the filtrational resistance of a well system is of prevalent importance and the hydraulic resistance of a bottomhole zone may be account, as a correction. Then the flows-sources principle may be enlarged to cover spatial case too.

The physical model, Fig. 1, consists of a well of radius r_w and length L.

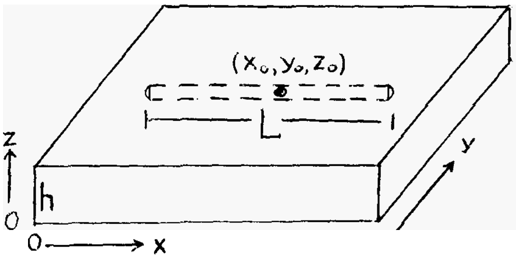

Fig. 1 The physical model.

The pressure of source in Formation for arbitrary well accordingly to Magomedov and Aliev formula [1] is

$$P-P_h=\frac{\mu Q}{4\pi kh}\left\{-\frac{1}{2}\left[\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_1^2}]}{\sqrt{r^2+z_1^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_2^2}]}{\sqrt{r^2+z_2^2}}\right]+\frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+(n+z_1)^2}]}{\sqrt{r^2+(n+z_1)^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+(n+z_2)^2}]}{\sqrt{r^2+(n+z_2)^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+(n-z_1)^2}]}{\sqrt{r^2+(n-z_1)^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+(n-z_2)^2}]}{\sqrt{r^2+(n-z_2)^2}}\right]\right\}$$

where: P_h - the initial pressure in reservoir; $\varepsilon=h/\sqrt{æt}$; æ - the coefficient of piezoconductivity; t - the time;

$$z_1=\frac{z-z_0}{2h};\quad z_2=\min\left(\frac{z+z_0}{2h};\,1-\frac{z+z_0}{2h}\right);$$

$$\bar{r}^2=\frac{(x-x_0)^2+(y-y_0)^2}{(2h)^2};$$

(x,y,z) - the coordinates system related with formation; (x_0,y_0,z_0) - the coordinates system related with well.

For rapid convergence let us introduce peculiarity for $\varepsilon\to 0$ and designating

$$\lambda_s=\frac{P-P_h}{\mu Q/(4\pi kh)}$$

we shell obtain the coefficient of filtrational resistance of pointwise source:

$$\lambda_s=\ln\frac{\pi}{4\varepsilon^2}-\frac{1}{2}\left(\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_1^2}]}{\sqrt{r^2+z_1^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_2^2}]}{\sqrt{r^2+z_2^2}}\right)+\frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{r^2+(n+z_1)^2}}+\frac{1}{\sqrt{r^2+(n+z_2)^2}}+\frac{1}{\sqrt{r^2+(n-z_1)^2}}+\frac{1}{\sqrt{r^2+(n-z_2)^2}}-\frac{4}{n}\exp\left(-\frac{2\varepsilon n}{\sqrt{\pi}}\right)\right]$$

The total resistance :

$$\lambda=1/L\int_0^L\lambda_s\,ds\approx 1/L\sum_{0}^{N}\lambda_s\,\Delta s_i$$

where:

$$\Delta s_i=\sqrt{(x_{0i+1}-x_{0i})^2+(y_{0i+1}-y_{0i})^2+(z_{0i+1}-z_{0i})^2}$$

s - the length of well arc; N - the number of sources on well. For a single horizontal well taking into account $\Delta P=P_c-P_b$ we have:

$$\frac{4\pi\sqrt{k_hk_v}h\beta\Delta P}{\mu Q_h}=-\frac{1}{2}\left(\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_1^2}]}{\sqrt{r^2+z_1^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r^2+z_2^2}]}{\sqrt{r^2+z_2^2}}\right)-$$

$$\frac{1}{2}\left(\frac{\operatorname{erfc}[\varepsilon\sqrt{r_1^2+z_1^2}]}{\sqrt{r_1^2+z_1^2}}+\frac{\operatorname{erfc}[\varepsilon\sqrt{r_1^2+z_2^2}]}{\sqrt{r_1^2+z_2^2}}\right)+\frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{r^2+(n+z_1)^2}}+\frac{1}{\sqrt{r^2+(n+z_2)^2}}+\frac{1}{\sqrt{r^2+(n-z_1)^2}}\right.$$

$$+\frac{1}{\sqrt{r^2+(n-z_2)^2}}-\frac{1}{\sqrt{r_1^2+(n+z_1)^2}}-\frac{1}{\sqrt{r_1^2+(n+z_2)^2}}$$

$$\left.-\frac{1}{\sqrt{r_1^2+(n-z_1)^2}}-\frac{1}{\sqrt{r_1^2+(n-z_2)^2}}\right]\ast\exp\left(-\frac{2\varepsilon n}{\sqrt{\pi}}\right)$$

The final Magomedov and Aliev formula for flow-rate of a single horizontal well can be represented as

$$Q_h=\frac{4\pi\sqrt{k_h k_v}\,h\beta\Delta P}{\mu B_0[\text{———}//\text{———}]} \qquad (4)$$

where:

$$\bar{r}^2=\frac{(x-x_{0i})^2+(y-y_{0i})^2}{(2\beta h)^2};\quad \bar{r}_1^2=\frac{(x-x_{0i})^2+r_w^2}{(2\beta h)^2}$$

$$z_1=\frac{z-z_{0i}}{2\beta h};\quad z_2=\min\left(\frac{z+z_{0i}}{2\beta h};\ 1-\frac{z+z_{0i}}{2\beta h}\right)$$

Choosing the coordinates system in the following way: $x=0$; because of $-L/2 \leqslant x_{0i} \leqslant L/Z$ we have: $x_{0i} = -L/2 + (2i-1)L/(2N)$; $y=R_c$; $y_{0i}=r_w$; $z=z_{0i}=h/2+r_w$.

When estimating the productivity of a horizontal well it is necessary to take into consideration the influence of anisotropy of productive reservoir permeability. The permeabilities respectively along formation and along normal to plane of layering *are* considered in this work.

In table 1 it can be observed, that variation of wellbore radius influences on total flow-rate of a horizontal well negligibly: 50% increase of wellbore radius increases oil flow-rate 3%. Thus, in practice it is not resonable to increase wellbore radius. Besides, it can be seen that in anisotropic reservoir an increase of horizontal well length considerably influences on total well flow-rate.

The results of table 1 have been obtained under the following conditions: $k_v=75\cdot10^{-15}$ m^2; $k_h=150\cdot10^{-15}$ m^2; $h=15m$; $\mu = 2.4\cdot10^{-2}$ Pa·s; $R_c=300m$; $B_0=1$; $\Delta P=1.2\cdot9.81\cdot10^4$ Pa.

Table 1 Oil flow-rates of horizontal wells of different radii and different lengths.

Formula	r_w, m	Q_h, m^3/day with L, m: 100	150	200	250
(1)	0.1	1.8772	2.3489	2.7925	3.2346
(2)	0.1	1.7244	1.9522	2.1539	2.3413
(3)	0.1	1.8540	2.3247	2.7672	3.2086
(4)	0.1	1.9729	2.4193	2.8235	3.2076
(1)	0.15	1.9292	2.4029	2.8496	3.2359
(2)	0.15	1.7682	2.2340	2.6701	3.1028
(3)	0.15	1.9047	2.3776	2.8233	3.2688
(4)	0.15	1.9796	2.4260	2.8304	3.2150

In table 2 total flow-rates of horizontal wells for isotropic and anisotropic reservoirs are compared, from where it can be observed, that for correct estimate of horizontal wells productivity it is necessary to take into consideration anisotropy of permeability of productive reservoirs: in isotropic formation estimation of total well flow-rate gives understated results.

Table 2. Oil flow-rates of horizontal wells of different radii, lengths and types of reservoir.

Reservoir, 10^{-15} m^2	r_w, m	Q_h, m^3/day with L, m: 100	150	200	*250*
Isotropic	0.1	1.0117	1.2438	1.4756	1.7021
(k=75)	0.15	1.0330	1.2713	1.4980	1.4560
Anisotrop	0.1	1.8772	2.3489	2.7925	3.2346
(k_v=75, k_h=150)	0.15	1.9292	2.4029	2.8496	3.2959

Table 3 shows that it is worthwhile to bore horizontal wells when elaborating anisotropic formation of small thickness.

For example, at h=15 m for horizoritals well with L=150 m and r_w=0.1 m the flow-rate Q_v equals to 2.3489, from which it is evident. that oil flow-rate, when comparing horizontal **and** vertical wells, increases 4.43 **times** (the flow-rate of vertical well Q_v with h=15 m and r_w=0.1 m equals to 0.5291 m/day). For obtaining the same effect in productive formation with h=150 m it is necessary to bore a horizontal well with length L=600 m, that is difficult to put into practice.

The results of table 3 have been obtained at: $k_v=75\cdot10^{-15}$ m^2; $k_h=150\cdot10^{-15}$ m^2; μ = $=2.4\cdot10^{-2}$Pa·s; R_c=300m; B_0=1; $\Delta P=1.2\cdot9.81\cdot10^4$Pa.

Table 3 .Oil flow-rates of horizontal wells of different radii, lengths and thickness of formation.

Formula	r_w,m	h,m	Q_h,m^3/day with L,m			
			100	150	200	250
(1)	0,1	15	1.87'12	2.3489	2.7925	3.2346
(2)	0.1	15	1.7244	1.9522	2.1539	2.3413
(3)	0.1	15	1.8540	2.3247	2.7672	3.2086
(4)	0.1	15	1.9729	2.4193	2.8235	3.2078
(1)	0.1	150	4.1503	5.9633	7.7131	9.4352
(2)	0.1	150	3.3687	5.018;'	6.5221	7.1312
(3)	0.1	150	4.0360	5.8062	7.5159	7.1938
(4)	0.1	150	4.7641	6.6448	8.4310	10.1542
(1)	0.15	15	1.9292	2.4029	2.8496	3.2359
(2)	0.15	15	1.7682	2.2340	2.6701	3.1023
(3)	0.15	15	1.9047	2.3776	2.8233	3.2688
(4)	0.15	15	1.9796	2.4280	2.8304	3.2150
(1)	0.15	150	4.4132	6.3243	8.1652	9.9756
(2)	0.15	150	3.6524	5.2747	6.8461	8.3947
(3)	0.15	150	4.2871	6.1515	7.9493	3.7187
(4)	0.15	150	4.8031	6.6954	8.4921	10.2252

The results of table 4 show, that the output of horizontal well at $k_v>k_h$ is much less than at $k_v<k_h$. Therefore horizontal wells are effective in formation, where vertical permeability is less than horizontal. Effect, mainly, is determined by length of horizontal well.

For comparison, the flow-rates of perfect vertical wells are: at $k_v=75\cdot10^{-15}$ m^2 and $k_h=150\cdot10^{-15}$ m^2 - Q_v= 0.5291 m^3/day; at $k_v=150\cdot10^{-15}$ m^2 and $k_h=75\cdot10^{-15}$ m^2 - Q_v= =0.5291 m^3/day; at $k_v=750\cdot10^{-15}$ m^2 and $k_h=75\cdot10^{-15}$ m^2 - Q_v= 1.1332 m^3/day.

Table 4.Oil flow-rates of horizontal wells of different lengths and permeabilities.

Formula	k_v and k_h, 10^{-15} m^2	Q_h,m^3/day with L,m			
		100	150	200	250
(1)	k_v=75, k_h=150	1.8772	2.3489	2.7925	3.2346
(2)	k_v=75, k_h=150	1.7244	1.9522	2.1539	2.3413
(3)	k_v=75, k_h=150	1.8540	2.3247	2.7672	3.2086
(4)	k_v=75, k_h=150	1.9729	2.4193	2.8235	3.2078
(1)	k_v=150, k_h=75	1.0693	1.3078	1.5359	1.7661
(2)	k_v=150, k_h=75	1.0303	1.2685	1.4950	1.7227
(3)	k_v=150, k_h=75	1.0752	1.3136	1.5420	1.7732
(4)	k_v=150, k_h=75	1.1984	1.4622	1.6877	1.9151
(1)	k_v=750, k_h=75	1.1519	1.3890	1.6192	1.8579
(2)	k_v=750, k_h=75	1.1368	1.3743	1.6043	1.8382
(3)	k_v=750, k_h=75	1.1607	1.3976	1.6283	1.8641
(4)	k_v=750, k_h=75	1.4913	1.8210	2.1343	2.4413

Table 5.Oil flow-rates of horizontal wells with different lengths and permeabilities.

k_v, 10^{-15} m^2	k_h, 10^{-15} m^2	Q_h,m^3/day with L,m	
		100	200
10	300	1.847	3.159
50	250	2.635	4.061
100	200	2.472	3.690
150	150	2.024	2.352
200	100	1.434	2.057
250	50	0.755	1.067
300	10	0.156	0.220

Thus, tables show that flow-rates of oil for horizontal wells are determined: by vertical and horizontal permeabilities, lengths and radii of horizontal wells, viscosities of fluid, pressure drop between pressure on boundary of circular feed contour ard pressure on bottomhole. It can be seen from tables, that the most simple formula (3) gives the results, more close to "exact" data, obtained by formula (2), than the other formulae. At the same time formula (4) may be used for determination of flow-rate of free situated well in anisotropic reservoir.

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